The Nemeth Braille Code for Mathematics and Science Notation

2022

Braille
Authority of
North
America

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The mission of the Braille Authority of North America is to assure literacy for tactile readers through the standardization of braille and/or tactile graphics.

The purpose of BANA is to promote and to facilitate the uses, teaching, and production of braille. Pursuant to this purpose, BANA will promulgate rules, make interpretations, and render opinions pertaining to braille codes and guidelines for the provisions of literary and technical materials and related forms and formats of embossed materials now in existence or to be developed in the future for the use of blind persons in North America. When appropriate, BANA shall accomplish these activities in international collaboration with countries using English braille. In exercising its function and authority, BANA shall consider the effects of its decisions on other existing braille codes and guidelines, forms and formats; ease of production by various methods; and acceptability to readers.

For more information and resources, visit www.brailleauthority.org/.

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Foreword to the 2022 Revision

When the *Unified English Braille Code* (UEB) was adopted by the Braille Authority of North America (BANA) in 2012, it became necessary to update the *Nemeth Code for Mathematics and Science Notation, 1972 Revision* for use within the UEB code. The project was begun in the spring of 2016, culminating in adoption in 2022.

Changes have been made in an attempt to modernize the code and clarify the application of pre-existing as well as newly adopted rules for transcription of mathematics and science. The *Guidance for Transcription Using the Nemeth Code within UEB Contexts* (2018), all posted Updates to the Code (2007-2015), and errata applicable to the 1972 Revision are incorporated within the current revision to provide a comprehensive document. None of these now exists as a separate document.

Thank you to the following committee members for their diligent work updating and adjusting the Nemeth Code to align with UEB.

Dorothy Worthington, Chair
Mary Denault
Dawn Gross
Cindi Laurent
Caryn Navy
Allison O'Day
Susan Osterhaus
Jacquie Walker
Lindy Walton
Jennifer Dunnam, BANA Board Liaison

Foreword to the 1972 Revision

THE NEMETH CODE OF BRAILLE MATHEMATICS AND SCIENCE NOTATION, 1965 initiated sound principles and procedures for the presentation of braille equivalents for the complex signs and configurations of ink-print mathematical and scientific notation. The effectiveness of the Code has been amply demonstrated through its application by transcribers in producing a wealth of technical material to meet the requirements of students at all levels of educational pursuits.

At the time of publication, it was apparent that the Code would require further updating and refinement in order to assure the faithful transference from ink print to braille as new modes of scientific notation were introduced. As was anticipated, problems in interpretation and clarity were encountered when the Code was put into actual use. The comments, criticisms and suggestions from students, teachers and transcribers were taken under consideration in the revision of the Code.

Under the able tutelage of Dr. Abraham Nemeth, the members of the AAWB-AEVH Braille Authority and its Advisory Committee on Mathematical and Scientific Notation entered upon a joint effort in bringing forth a Revised Code which could withstand the test of use and time. As work progressed, however, it became increasingly evident that, because of the complexity of the subject matter and because of the many techniques employed by authors and publishers, substantial research would be required in expanding the Code to the fullest effectiveness. In recognition of this fact, the national Advisory Council to the Braille Authority applied for a planning grant from Social and Rehabilitation Services of the Department of Health, Education, and Welfare. The American Printing House for the Blind was designated as the recipient of the grant, known as the "Braille Codes Pilot Project", which is geared to bring into focus the need for fuller research in all braille codes. Upon the basis of this study, application for a research grant will be made and, if approved, all braille codes will be considered in detail in the endeavor to bring them to maximum completeness and efficiency.

The 1972 revision of THE NEMETH BRAILLE CODE FOR MATHEMATICS AND SCIENCE NOTATION provides students and transcribers with a well-drawn, logical system of braille notation which insures a faithful presentation of signs and usages employed in technical texts. The changes which have been incorporated will convey to the reader a realistic picture of the ink-print text and will equip the transcriber with the necessary signs and rules of procedure for a more exact braille transcription.

Grateful acknowledgement is accorded the following persons for their major contribution in the joint effort in developing and refining the revised Code.

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About This Book

The Nemeth Braille Code for Mathematics and Science Notation 2022 is primarily intended for use by braille transcribers and proofreaders during the production of technical materials, and by developers of translation software. Technical material includes but is not limited to the material encountered in the fields of science, technology, engineering, and mathematics (STEM).

This publication is *not* intended as a manual for learning braille. It is a *reference* that transcribers, proofreaders, and software developers will refer to frequently, and that braille readers may refer to occasionally for clarification. Good braille knowledge is vital for the successful use of this publication.

The rules are laid out by concept with each rule being accompanied by many examples. The print version uses the SimBraille font for all braille examples. The dot locator does not precede the symbols under discussion in print, however the dot locator has been added to the braille version where required. Examples in the print version are presented first in print and then in SimBraille. In the braille version the examples are shown only once. Text in parentheses following certain examples is included for the purpose of helping the reader better understand the example or point being illustrated.

In most cases, the examples are shown in mathematical context and are transcribed following the formatting directives prescribed in **Rule 26, Format**. However, the examples in Rules 3, 7, 11, 13, 14, and 15 focus on the concepts presented in those rules and are transcribed beginning in cell 1. The actual layout will depend on the surrounding text; follow Rule 26 for formatting directives.

The appendices located at the rear of the book contain a wealth of information. Take your time to review them thoroughly.

Appendix A, Code Changes

Appendix B, Placement of Code Switch Indicators

Appendix C, Combinations of Typeform, Alphabetic and Capitalization Indicators

Appendix D, Index of Nemeth Braille Symbols

Rule 1
Basic Principles

1.1 Description

1.1.1 *The Nemeth Braille Code for Mathematics and Science Notation* ("Nemeth Code") has been prepared to provide a system of symbols which will allow technical literature to be presented and read in braille. The Nemeth Code is intended to convey as accurate an impression as is possible to the braille reader of the corresponding printed text.

1.1.2 Although the Nemeth Code is intended to be as complete as possible, finality can never be achieved by any code. In the course of the rapid development in the fields of science and technology, new signs are constantly being devised and old ones modified.

1.2 Organization

1.2.1 This presentation is organized into rules. Where appropriate, each rule begins with a list of signs and their corresponding symbols for quick reference. The body of the presentation is organized into sections which are consecutively numbered and captioned. The sections contain rules, explanations, and examples of the use of the Nemeth Code. It is intended that the examples be sufficiently definitive so that they may be imitated with confidence in parallel situations. Any parenthetical descriptions below the examples are intended to supplement the actual signs shown in the print copy. The examples are drawn principally from pure mathematics. The symbols, rules, and constructions of the basic Nemeth Code apply with equal force to other technical fields. Following the rules, there is an Index of Braille Symbols, the entries of which have been arranged in braille order.

1.2.2 **"Sign" vs. "Symbol".** Throughout this presentation, the word "sign" is consistently used in referring to a character or sequence of characters in print, whereas the word "symbol" is used in referring to a character or sequence of characters in braille.

1.3 Interpretation

1.3.1 It is important that this presentation be accepted quite literally and that no meaning be imputed to the rules and principles which is not expressly stated or directly implied. It may sometimes appear quite arbitrary that a particular sign has been classified in a section which the reader's past experience or training indicates is inappropriate. For the purposes of the Nemeth Code the transcriber or teacher must accept the classification as well as the rules herewith presented, past experience or technical training notwithstanding.

1.3.2 **Uniformity.** In certain situations it may be felt that some constructions are excessively long and there may be a temptation to shorten the construction by the use of a symbol of one's own invention. *The transcriber is enjoined against yielding to this temptation*. The Nemeth Code has been formulated in such a way that the same construction gives the same information to the braille reader from elementary through the most advanced mathematics. Tampering with the constructions presented herein would have the effect of destroying this uniformity. The Nemeth Code furnishes specific braille symbols corresponding to distinct signs in print. For example, the Nemeth Code maintains a distinction between the horizontal and diagonal fraction lines, and between the dot and the cross which signify multiplication. Signs which have separate identities in print should be represented by distinct symbols in braille.

1.4 Technical Texts

1.4.1 *Literary* works which use only occasional mathematical terminology and notation are transcribed entirely in Unified English Braille (UEB).

1.4.2 In the fields of mathematics, statistics, physics, or chemistry, the symbols and rules of the Nemeth Code are used. They must also be used in works in other fields which make strong use of mathematical signs and modes of expression. If using the Nemeth Code for technical material, the entire transcription must follow Nemeth Code format rules.

1.4.3 **Transcriber-Generated Pages.** Nemeth Code symbols are not used on the title page or supplemental title pages of a book. Switch indicators and symbols devised by the transcriber are listed on the Special Symbols page. It is not necessary to list any other Nemeth symbols used. The transcriber must indicate at the beginning of each volume by means of a transcriber's note that the work has been transcribed in Nemeth Code, giving the year the code was adopted and any applicable updates.

1.4.4 **Tactile Graphics.** Any braille volume that contains one or more tactile graphics must contain a note on the transcriber’s notes page stating that the *Guidelines and Standards for Tactile Graphics* were used in the preparation of the tactile graphics, giving the year the guidelines were adopted.

1.4.5 **Chemistry.** Any braille volume transcribed using the chemistry code must contain a note on the transcriber's notes page that the work has been transcribed according to *Chemical Notation Using the Nemeth Braille Code*. The chemistry symbols must be listed on the Special Symbols page.

1.4.6 **Computer Notation.** Follow UEB rules and use UEB symbols for computer notation, such as email addresses and programming language.

1.4.7 **Format.** The Nemeth Code contains several formatting rules. These rules apply throughout a transcription, even in the UEB portions. If a format is not addressed in the Nemeth

Code, the guidelines outlined in *Braille Formats, Principles of Print to Braille Transcription* should be followed.

Rule 2
Nemeth Braille Indicators

Alphabetic Indicators
- English-Letter ⠰
- German-Letter ⠸
- Greek-Letter
  - For standard letters ⠨
  - For alternative letters ⠨⠈
- Hebrew-Letter ⠠⠠
- Russian-Letter ⠈⠈

Arrow Direction Indicators
- Depresses Nearer Arrowhead by 45 Degrees ⠰
- Elevates Nearer Arrowhead by 45 Degrees ⠘
- Makes Nearer Arrowhead Point Up ⠣
- Makes Nearer Arrowhead Point Down ⠩

Arrow Types
- Boldface ⠸

Cancellation Indicators
- Opening ⠪
- Closing ⠻

Capitalization Indicators
- Single ⠠
- Double ⠠⠠

Fraction Indicators
- Simple
  - Opening ⠹
  - Closing ⠼

Complex	
Opening	⠠⠹
Closing	⠠⠼
Hypercomplex	
Opening	⠠⠠⠹
Closing	⠠⠠⠼
Fractional Part of a Mixed Number	
Opening	⠸⠹
Closing	⠸⠼
General Reference Indicator	⠈⠹
Level Indicators	
Baseline	⠐
Superscript	⠘
Superscript with Superscript	⠘⠘
Superscript with Subscript	⠘⠰
Superscript with Superscript with Superscript	⠘⠘⠘
Superscript with Superscript with Subscript	⠘⠘⠰
Superscript with Subscript with Superscript	⠘⠰⠘
Superscript with Subscript with Subscript	⠘⠰⠰
Subscript	⠰
Subscript with Superscript	⠰⠘
Subscript with Subscript	⠰⠰
Subscript with Superscript with Superscript	⠰⠘⠘
Subscript with Superscript with Subscript	⠰⠘⠰
Subscript with Subscript with Superscript	⠰⠰⠘
Subscript with Subscript with Subscript	⠰⠰⠰

Modification Indicators	
Multipurpose	⠐
Directly Over	
First order	⠣
Second order	⠣⠣
Directly Under	
First order	⠩
Second order	⠩⠩
Superposition	⠈
Termination	⠻
Multipurpose Indicator	⠐
Nemeth Code Switch Indicators	
Opening Nemeth Code indicator (UEB indicator)	⠸⠩
Nemeth Code terminator	⠸⠱
Single-word switch indicator	⠠⠄
Numeric Indicator	⠼
Punctuation Indicator	⠸
Radical Indicators	
Index-of-Radical	⠣
Order-of-Radical	
First inner radical	⠨
Second inner radical	⠨⠨
Third inner radical	⠨⠨⠨
Termination	⠻
Regrouping Indicators	
For numbers above the arrangement (varying in length)	⠣⠶⠶

For numbers below the arrangement (varying in length)	⠩⠶⠶
Shape Indicators	
Shape	⠫
Structural Shape-Modification	⠨
Interior Shape-Modification	⠸⠫
Filled-In Shape	⠸
Shaded Shape	⠨
Termination	⠻
Keystroke Indicator	⠫⠅
Termination Indicator	⠻
Typeform Indicators for Letters and Numerals	
Boldface Type	⠸
Italic Type	⠨
Sans Serif Type	⠠⠨
Script Type	⠈
Barred Type (Blackboard or Double Struck)	⠠⠸
Typeform Indicators for Words, Phrases, and Mathematical Statements	
Single Word Boldface Type	⠸
Single Word Italic Type	⠨
Single Word Bold Italic Type	⠸⠨
Opening Boldface Type for two or more words	⠠⠄⠸ (followed by a space)
Opening Italic Type for two or more words	⠠⠄⠨ (followed by a space)
Closing Boldface Type for two or more words	⠸⠠⠄ (preceded by a space)
Closing Italic Type for two or more words	⠨⠠⠄ (preceded by a space)

Typeform Terminator	⠠⠄

2.1 Concept of Braille Indicators

Mathematical expressions are represented in print by the use of arbitrary signs among which are the digits, the lowercase and capitalized letters of several alphabets, the script, italic, and boldface forms of these same letters, as well as numerous signs of operation, signs of comparison, signs of grouping, and many other signs serving the miscellaneous requirements of mathematical and scientific expression. Furthermore, mathematical significance is imparted not only by these signs separately, but by their collective arrangement on levels above or below a reference line of writing, as well as by their disposition above or below a fraction line. With only sixty-three distinct braille characters available, sixty-four if the space is counted, the accomplishment of the Nemeth Code is to make provision for the representation of all these signs, as well as to give an indication of their arrangement.

Indicators are braille symbols which have no print equivalents, but affect the meanings and connotations of other symbols within a transcription.

Rule 3
Numeric Signs and Symbols

Numeric Indicator ⠼

Arabic Digits (Nemeth Code)

⠴	⠂	⠆	⠒	⠲	⠢	⠖	⠶	⠦	⠔
0	1	2	3	4	5	6	7	8	9

Comma (mathematical)

American , ⠠

European . ⠨

Decimal Point

American . ⠨

European , ⠠

3.1 Representation of Arabic Numerals

Digits are represented in two ways: as in Unified English Braille, and as in the Nemeth Code.

3.1.1 **Use of UEB Numerals.** The digits in Unified English Braille are represented by the letters "j" and "a" through "i". Numerals at the corners of pages and at the ends of page change indicators are transcribed as in UEB. Titles for figures, tables, sections, etc. are transcribed in UEB. UEB numerals may be used with freestanding, unmodified numbers, ordinals, and plurals. (See **Rule 4.6**)

Example 3-1: Section Title

Section 1.3.4

⠠⠎⠑⠉⠰⠝⠀⠼⠁⠲⠉⠲⠙

3.1.2 **Nemeth Numerals.** The digits in the Nemeth Code are represented by the symbols for letters "j" and "a" through "i", but which occupy the lower portion of the braille cell.

3.2 Comma, Decimal Point

3.2.1 **Variant Forms.** Variant forms of the comma and decimal point are sometimes employed, particularly in books published outside of the United States. Follow print for symbols used.

Example 3-2: American Usage of Comma and Decimal

$1,378.07

⠸⠩⠀⠈⠎⠂⠠⠒⠶⠦⠨⠴⠶⠀⠸⠱

Example 3-3: European Usage of Comma and Decimal

£1.378,07

⠸⠩⠀⠈⠇⠂⠨⠒⠶⠦⠠⠴⠶⠀⠸⠱

3.2.2 **Numeric Comma.** A comma which is interior to a modified numeral, and which is used to partition the numeral into short regular segments, is a numeric symbol.

Example 3-4: Numeric Comma

1,478.00

⠸⠩⠀⠼⠂⠠⠲⠶⠦⠨⠴⠴⠀⠸⠱

(this comma is a numeric symbol, not a mark of punctuation)

3.2.3 **Decimal Point.** The decimal point is regarded as a numeric symbol only when it is followed by a number. The multipurpose indicator is used after the decimal point symbol to indicate that the symbol which follows it is non-numeric. An omission symbol is non-numeric even when it represents a number.

Example 3-5: Numeric Decimal Point

.35

⠸⠩⠀⠼⠨⠒⠢⠀⠸⠱

Example 3-6: Numeric Decimal Point

3.14

⠸⠩⠀⠼⠒⠨⠂⠲⠀⠸⠱

Example 3-7: Numeric Decimal Point

$.2a_1a_2a_3$

⠸⠩⠀⠼⠨⠆⠁⠂⠁⠆⠁⠒⠀⠸⠱

Example 3-8: Non-numeric Decimal Point

$1.a_1a_2a_3$

⠸⠩⠀⠼⠂⠨⠐⠁⠂⠁⠆⠁⠒⠀⠸⠱

Example 3-9: Numeric and Non-numeric Decimal Points

.1 + .2 = .____

⠸⠩⠀⠼⠨⠂⠬⠨⠆⠀⠨⠅⠀⠨⠐⠤⠤⠤⠤⠀⠸⠱

Example 3-10: Numeric and Non-numeric Decimal Points

1.2 + 1.4 = 2.?

⠸⠩⠀⠼⠂⠨⠆⠬⠂⠨⠲⠀⠨⠅⠀⠼⠆⠨⠐⠿⠀⠸⠱

3.3 Use of the Numeric Indicator

The numeric indicator is used to introduce a numeral under the following circumstances:

3.3.1 The numeric indicator is used at the beginning of a braille line or after a space. It is also used after a minus symbol which occurs at the beginning of a braille line or which follows a space.

Example 3-11: Negative Number

-1

⠸⠩⠀⠤⠼⠂⠀⠸⠱

Example 3-12: Negative Decimal Number

-.3

⠸⠩⠀⠤⠼⠨⠒⠀⠸⠱

Example 3-13: Number Not Requiring a Numeric Indicator

+3

⠸⠩⠀⠬⠒⠀⠸⠱

(numeric indicator is not used with a number preceded by a plus symbol

Example 3-14: Numbers Preceded by a Space

1 + x + y = 0

⠸⠩⠀⠼⠂⠬⠭⠬⠽⠀⠨⠅⠀⠼⠴⠀⠸⠱

Example 3-15: Number Preceded by a Space

y = 2 sin x

⠸⠩⠀⠽⠀⠨⠅⠀⠼⠆⠎⠊⠝⠀⠭⠀⠸⠱

Example 3-16: Number Preceded by a Space and a Minus Sign

$$H = \frac{12}{N(N+1)}\left(\frac{R_1^2}{n_1} + \frac{R_2^2}{n_2} + \ldots + \frac{R_k^2}{n_k}\right) - 3(N+1)$$

⠸⠩

⠠⠓

⠀⠀⠨⠅⠀⠹⠂⠆⠌⠠⠝⠷⠠⠝⠬⠂⠾⠼

⠀⠀⠷⠹⠗⠂⠘⠆⠐⠌⠝⠂⠼⠬⠹⠠⠗⠆⠘⠆⠐⠌⠝⠆⠼⠬⠀⠄⠄⠄

⠀⠀⠬⠹⠠⠗⠰⠅⠘⠆⠐⠌⠝⠰⠅⠐⠼⠾

⠀⠀⠤⠼⠒⠷⠠⠝⠬⠂⠾⠀⠸⠱

Example 3-17: Number Preceded by a Space

$$y = \frac{\sin^2 x}{\cos 2x}$$

⠸⠩⠀⠽⠀⠨⠅⠀⠹⠎⠊⠝⠘⠆⠀⠭⠌⠉⠕⠎⠀⠼⠆⠭⠼⠀⠸⠱

Example 3-18: Function Name in an Enclosed List

(3sin 30º, 3cos 60º)

Example 3-19: Number Preceded by a Space

sin 1

Example 3-20: Number Preceded by a Space

$\sin^2 2x$

Example 3-21: Number Preceded by a Space

0.333 ... 3 ...

Example 3-22: Number Preceded by a Space

$\log_{10} 2$

Example 3-23: Number Preceded by a Space

$\angle 1$

Example 3-24: Equation within Grouping Symbols

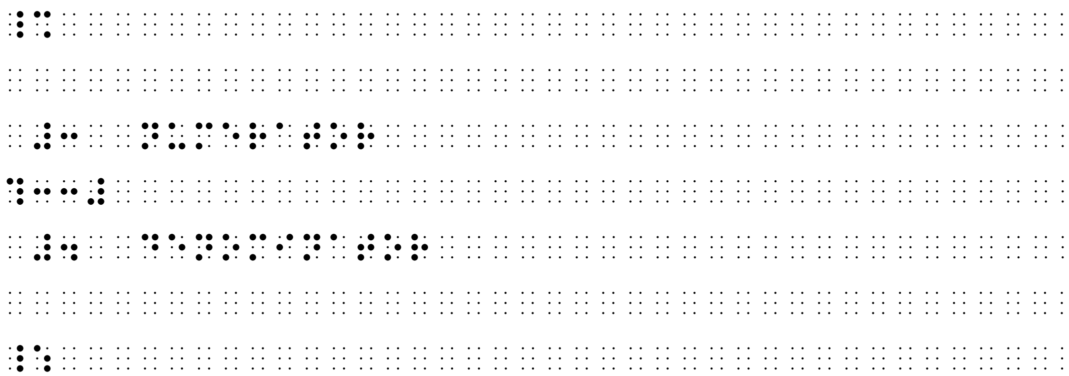
(x = 0)

Example 3-25: Simple Equation

0 = x

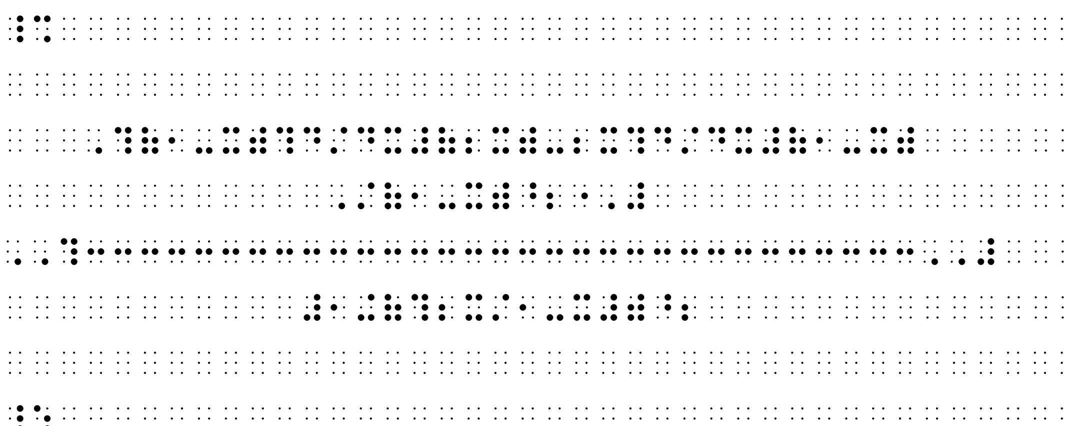
Example 3-26: Numbers Beginning a Braille Line

$$\frac{3 \quad \text{numerator}}{4 \quad \text{denominator}}$$



Example 3-27: Hypercomplex Fraction

$$\frac{\dfrac{(1-x)\dfrac{d}{dx}(2x)-2x\dfrac{d}{dx}(1-x)}{(1-x)^2}}{1+\left(\dfrac{2x}{1-x}\right)^2}$$



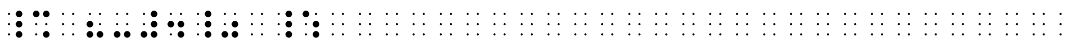
3.3.2 The numeric indicator is used after a mark of punctuation that precedes a numeral. It is also used before a number following a minus symbol that is preceded by a punctuation mark. The hyphen requires special attention (see section **3.3.8**). A grouping sign is not a mark of punctuation.

Example 3-28: Decimal Number Following a Quotation Mark

".5 oz."

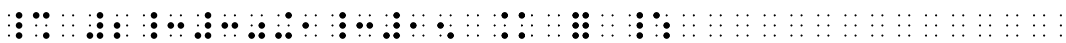
Example 3-29: Negative Number Following a Quotation Mark

"-4"



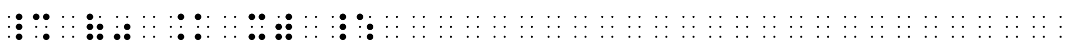
Example 3-30: Number Following a Colon in Digital Time

2:30 + 1:15 =



Example 3-31: Number Following a Left Grouping Symbol

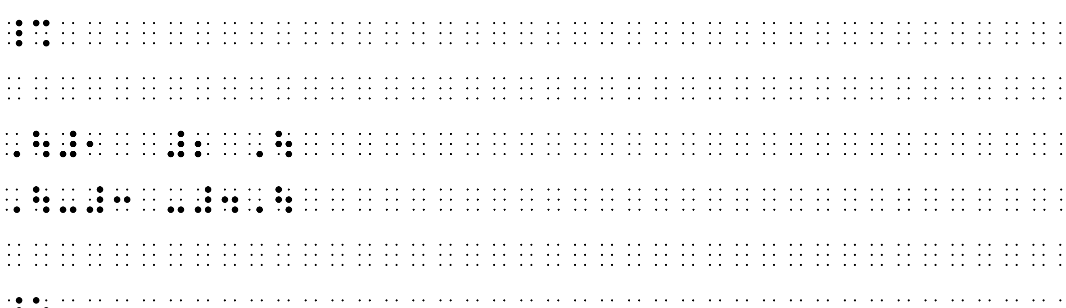
(0 = x)



3.3.3 The numeric indicator is used after a left grouping symbol which introduces a determinant or matrix. It is also used after a minus symbol which follows such a grouping symbol.

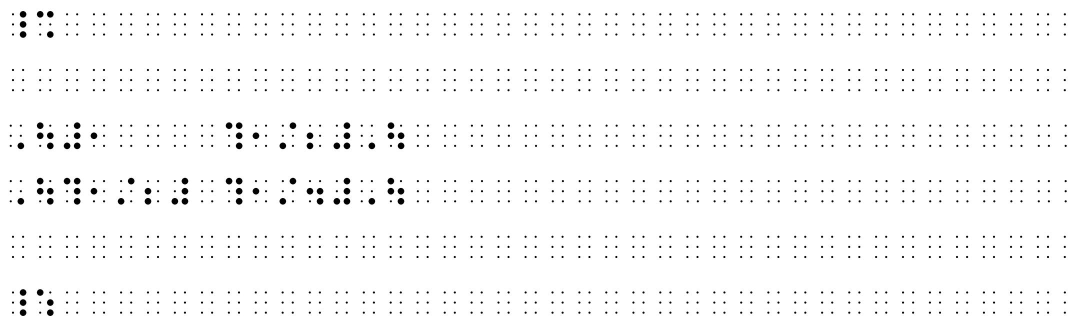
Example 3-32: Numbers in a Determinant

$$\begin{vmatrix} 1 & 2 \\ -3 & -4 \end{vmatrix}$$



Example 3-33: Numbers in a Determinant

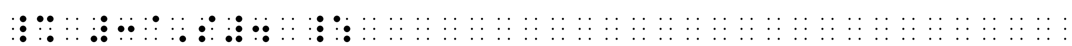
$$\begin{vmatrix} 1 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{4} \end{vmatrix}$$



3.3.4 The numeric indicator is required after a section mark, paragraph mark, crosshatch, or asterisk used as a sign of operation.

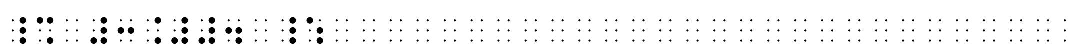
Example 3-34: Number Following Section Mark

3 § 4



Example 3-35: Number Following Crosshatch

3 # 4



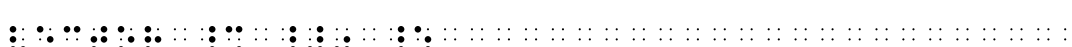
Example 3-36: Number Following Asterisk

3 * 4

3.3.5 The numeric indicator is used after a typeform indicator, or after making a transition from non-regular to regular type within the same numeral.

Example 3-37: Mathematical Number in Boldface Type

vector **0**



(a vector is a math symbol)

Example 3-38: Mathematical Number in Script Type

2

⠸⠩⠀⠈⠼⠆⠀⠸⠱

(when the script typeform has mathematical meaning)

Example 3-39: Literary Number in Script Type

2

⠈⠆⠼⠃

(when the typeform has no mathematical meaning, a script numeral is transcribed in UEB)

Example 3-40: Number with Two Different Typeforms

*43*56

⠸⠩⠀⠨⠼⠲⠒⠼⠢⠖⠀⠸⠱

3.3.6 When a numeral immediately follows a reference indicator, with or without a space, a numeric indicator is required.

3.3.7 The numeric indicator is used after the interior shape-modification indicator.

Example 3-41: Number Within a Circle

⑤

⠸⠩⠀⠫⠉⠸⠫⠼⠢⠻⠀⠸⠱

Example 3-42: Number Within a Square

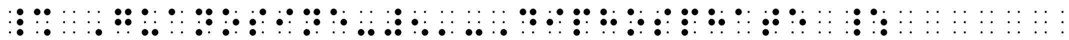
$\boxed{5}$

⠸⠩⠀⠫⠲⠸⠫⠼⠢⠻⠀⠸⠱

3.3.8 The numeric indicator is used after a hyphen when the hyphen follows a word, an abbreviation, or a mark of punctuation. A grouping sign is not considered a mark of punctuation. However, also see **3.4.3**.

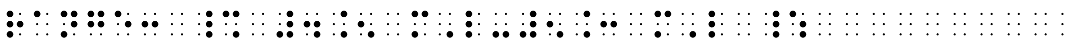
Example 3-43: Hyphen Between Word and Number

Guanosine-5'-Diphosphate



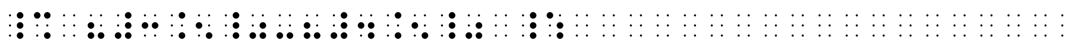
Example 3-44: Hyphen Between Abbreviation and Number

range: 4.5 mL-5.3 mL



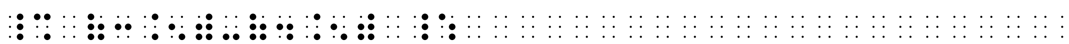
Example 3-45: Hyphen Between Punctuation and Number

"3.5"-"4.5"



Example 3-46: Hyphen Between Grouping Signs

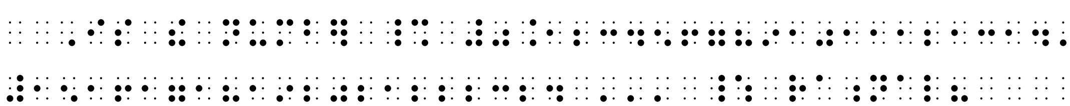
(3.5)-(4.5)



3.3.9 The numeric indicator is used at the beginning of the runover line of a long numeral. Such a division is made after a comma, if present, and a hyphen is inserted.

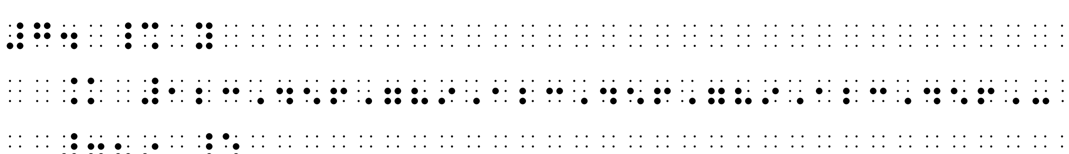
Example 3-47: Division of a Long Number with No Commas

Is the number 0.123456789101112131415161718192021222324 ... rational?



Example 3-48: Division of a Long Number with Commas

7. y = 123,456,789,123,456,789,123,456,789



3.4 Non-Use of the Numeric Indicator

It must not be assumed that because a symbol is numeric that the numeric indicator must be used with that symbol.

The numeric indicator is not used preceding a numeric symbol under the following circumstances.

3.4.1 The numeric indicator is not used with a number that is preceded unspaced by a symbol, letter, indicator, etc. See section **3.3.1** for the exception regarding a negative number.

Example 3-49: Number Follows a Plus Sign

$+3$

⠸⠩⠀⠬⠒⠀⠸⠱

Example 3-50: Number Follows Superscript Indicator

x^2

⠸⠩⠀⠭⠘⠆⠀⠸⠱

Example 3-51: Number Follows Fraction Indicator

$\frac{3}{x}$

⠸⠩⠀⠹⠒⠌⠭⠼⠀⠸⠱

Example 3-52: Number Follows Multipurpose Indicator

r5

⠸⠩⠀⠗⠐⠢⠀⠸⠱

(remainder of 5 as in a division problem)

Example 3-53: Number is Unspaced in a Math Expression

$x - 5$

⠸⠩⠀⠭⠤⠢⠀⠸⠱

Example 3-54: Second Number is Unspaced in Math Expression

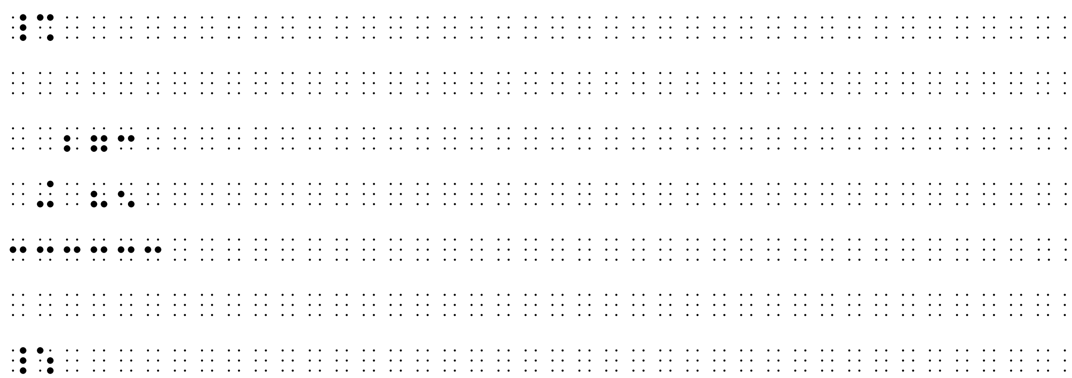
2×4

⠸⠩⠀⠼⠆⠈⠡⠲⠀⠸⠱

3.4.2 **Spatially Arranged Problems.** The numeric indicator is not used in work arranged in columns and aligned for addition, subtraction, or multiplication, or in spatial arrangements for division. When digital time is arranged spatially for computation, a numeric indicator is required following the colon.

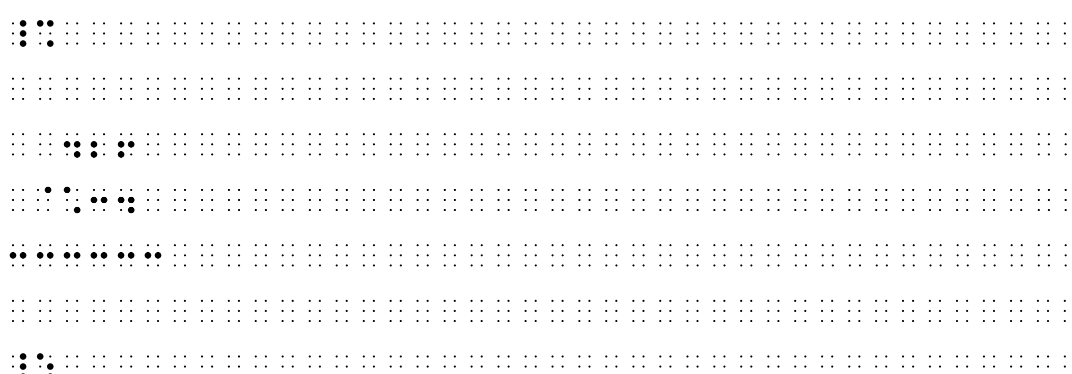
Example 3-55: Spatial Addition Problem

$$\begin{array}{r} 273 \\ +\ \ 85 \\ \hline \end{array}$$



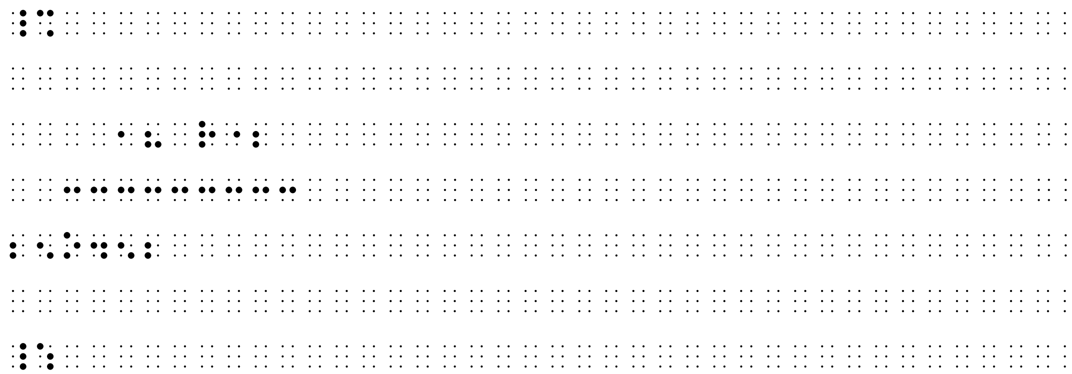
Example 3-56: Spatial Multiplication Problem

$$\begin{array}{r} 426 \\ \times\ \ 34 \\ \hline \end{array}$$



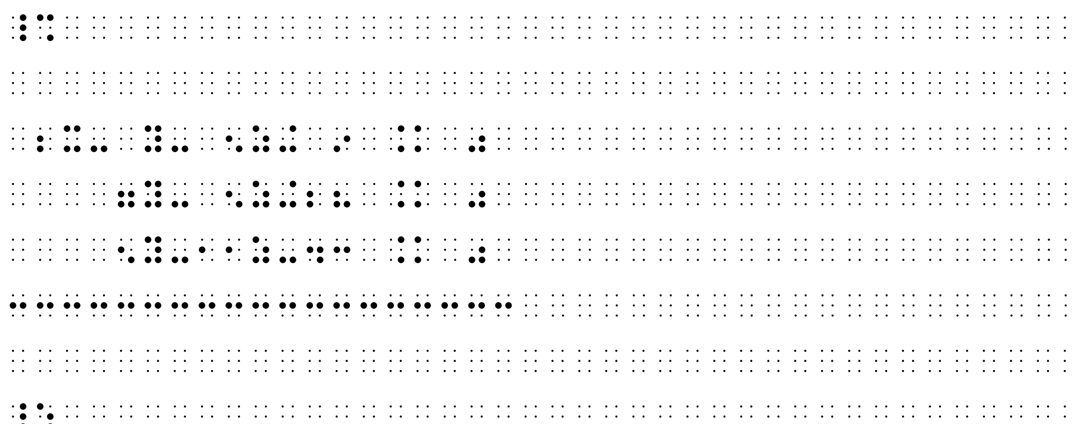
Example 3-57: Spatial Division Problem

$$\begin{array}{l} \quad 18\ r2 \\ 25\overline{)452} \end{array}$$



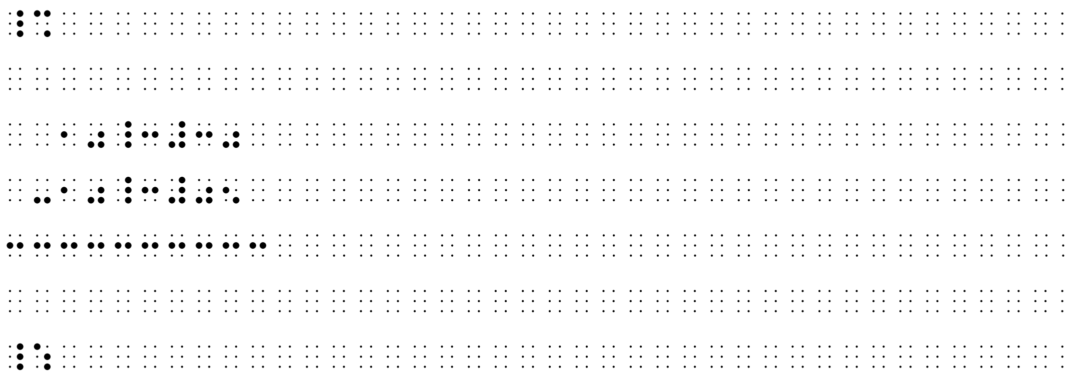
Example 3-58: Spatial System Arranged for Computation

$$\begin{array}{rrrrl} 2x - & y - & 5z + & 9 = & 0 \\ & 7y - & 5z + & 28 = & 0 \\ & 5y - & 11z - & 43 = & 0 \\ \hline \end{array}$$



Example 3-59: Digital Time Arranged for Computation

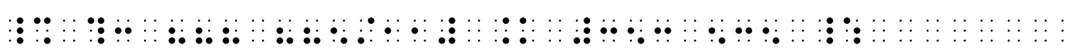
$$\begin{array}{r} 10:30 \\ -10:05 \\ \hline \end{array}$$



3.4.3 The numeric indicator is not used after a space if the purpose of the space is to partition a numeral into segments. The numeric indicator is not inserted at the beginning of the runover line of a long partitioned numeral.

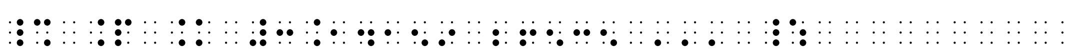
Example 3-60: Numbers Grouped by Spacing

$$\frac{3\ 888\ 885}{11} = 353\ 535$$



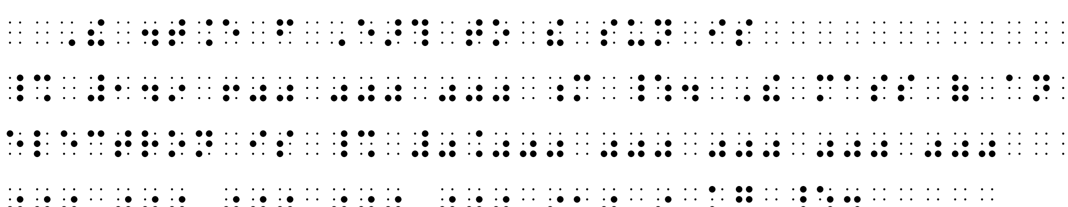
Example 3-61: Decimal Number Grouped by Spacing

$\pi = 3.14159\ 26535\ \ldots$



Example 3-62: Partitioned Number with a Runover

The distance from Earth to the sun is 149 600 000 000 m. The mass of an electron is 0.000 000 000 000 000 000 000 000 000 000 910 9 kg.



(partitioned numbers are transcribed in Nemeth Code)

3.4.4 The numeric indicator is not used after a hyphen if the hyphen follows a numeral, a letter, or other mathematical expression.

Example 3-63: Hyphen Between Numbers

6.5-7.5 cc

⠸⠩⠀⠼⠖⠨⠢⠤⠶⠨⠢⠀⠉⠉⠀⠸⠱

Example 3-64: Hyphen Between a Symbol and a Number

80%-90%

⠸⠩⠀⠼⠦⠴⠈⠴⠤⠔⠴⠈⠴⠀⠸⠱

3.5 Enclosed List

Special provision is made for the transcription of a sequence of items enclosed within grouping signs in mathematical context (enclosed list). A switch to Nemeth Code is required to transcribe an enclosed list.

3.5.1 **Definition.** An enclosed list, for the purposes of the Nemeth Code, meets the following requirements:

a. It begins and ends with a sign of grouping. These signs of grouping do not necessarily have to be of the same kind.

b. The list must have at least two items, separated only by commas. The space cannot be the sole means for separating items. A function name, an abbreviated function name, or a sign of shape and the signs which follow them (including the space) are regarded as a single item.

c. The list contains no word, abbreviation, ordinal ending, or plural ending. Exception: set notation, **3.5.4**.

d. Any sign used for omission may be an item of the list.

e. The list must not contain a mark of punctuation other than the comma or a sign used for omission.

f. No sign of comparison may appear anywhere within the list.

3.5.2 The numeric indicator is not used before a numeral at the *beginning* of an item which is part of an enclosed list as defined in **3.5.1** above, even if such an item has been run over to another line. If any item in an enclosed list is a numeral in a typeform other than regular type, that item requires the numeric indicator.

Example 3-65: Enclosed List

[0, 1]

Example 3-66: Enclosed List

(–1, –2, –3)

Example 3-67: Enclosed List

(1 + h, 2 + k, 0)

Example 3-68: Enclosed List

(0, –1, ±2)

Example 3-69: Enclosed List with Abbreviated Function Names

(2 sin 30°, 3 cos 60°)

(the numeric indicator is required before the 30 and the 60 because these are *not the beginning* of their respective items)

Example 3-70: Enclosed List with Runoverb

(1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12)

(the numeric indicator is not required before 12 even though 12 begins a new line)

Example 3-71: Enclosed List with a Bold Numeral

(x, 7, **8**, y)

(the 8 in this "enclosed list" is in boldface type)

Example 3-72: Enclosed List with Letters

(a, b]

Example 3-73: Enclosed List with Fractions

$$\left(\frac{1}{4}, \frac{1}{2} + x, \frac{3}{4} + x^2\right)$$

Example 3-74: Enclosed List with a Shape and a Function

$[\angle\ 1, \sin 1^{\circ}]$

Example 3-75: Enclosed List with Letters and an Ellipsis

(a, b, ...)

Example 3-76: Enclosed List with Omissions

(x + 1, x + 2, ?, ?, x + 5)

3.5.3 Expressions Not Qualifying as Enclosed Lists.

Example 3-77: Contains a Word

(1, 2, and 3)

Example 3-78: Contains an Abbreviation

(h ft, k in)

Example 3-79: Contains an Ordinal

(1st, 2nd, 3rd)

Example 3-80: Contains an Apostrophe

(x’s, y’s, z’s)

Example 3-81: Contains a Semicolon

(u, v; x, y)

Example 3-82: Does Not Contain a Comma Between Items

(1 2 3)

Example 3-83: Contains a Comparison Sign

(x = 1, 2, ..., 10)

(Nemeth Code is required because of the equals sign)

Example 3-84: Contains a Comparison Sign

(a = 1, b = 2, c = –4)

(Nemeth Code is required because of the comparison signs)

Example 3-85: Is Not Enclosed in Grouping Signs

$-1, -i, -2, -j$

⠸⠩⠀⠤⠼⠂⠠⠀⠤⠊⠠⠀⠤⠼⠆⠠⠀⠤⠚⠀⠸⠱

(the minus sign is a modifier so the series is transcribed in Nemeth Code)

3.5.4 **Set Notation.** Even though it may contain words, set notation that lists elements of a set enclosed in braces is transcribed in Nemeth Code following the rules for an enclosed list.

Example 3-86: Words in Set Notation

{Minnesota, Wisconsin, Iowa}

⠸⠩⠀⠨⠷⠠⠍⠊⠝⠝⠑⠎⠕⠞⠁⠂⠀⠠⠺⠊⠎⠉⠕⠝⠎⠊⠝⠂⠀⠠⠊⠕⠺⠁⠨⠾⠀⠸⠱

(a literary comma is used after a word)

3.6 Representation of Numerals to Non-Decimal Bases

3.6.1 **Use of Letters.** When a system of numeration is to a base other than 10, a common technique for providing additional digits is to use letters, either lowercase or capitalized, in addition to the ten Arabic digits. When this technique is used, the transcriber uses only lowercase letters. If capitalized letters are used in print, the transcriber must indicate this fact in a transcriber's note.

Sample transcriber's note.

t and e are capitalized in print.

Example 3-87: A Base-12 Numeral Using T and E

13TE7

⠸⠩⠀⠼⠂⠒⠞⠑⠶⠀⠸⠱

(a base-12 numeral in which T represents ten and E represents eleven)

Example 3-88: A Base-16 Numeral Using E and F

3FFE2

(a base-16 numeral in which E represents fourteen and F represents fifteen)

3.6.2 **Use of Arbitrary Signs.** Another common technique for providing additional digits is to use standard or arbitrary signs to supplement the ten Arabic digits. Authors sometimes give names to these signs. For example, *x* (dek derived from deka) may represent ten and ℰ (el derived from eleven) may represent eleven. In this case, the transcriber devises one-cell symbols for these signs, preferably chosen from among the letters of the English alphabet, and inserts a transcriber’s note to specify the meanings which have been assigned to these symbols. The transcriber's note will include drawings of the symbols used in print.

Example 3-89: A Base-12 Numeral Using X and E

13 *x*ℰ7

(a base-12 numeral in which **x** represents ten and ℰ represents eleven. Here the transcriber has assigned d to **x** and e to ℰ)

Example 3-90: Arbitrary Signs Representing Additional Digits

@%$

(an arbitrary set of signs to provide additional digits)

3.6.3 The one-cell symbols which the transcriber uses to represent the digits of a non-decimal numeration system are regarded as numeric symbols. As such, these numeric symbols are subject to the rules for transcribing numerals.

Example 3-91: Numeric Indicator Used for Numeral "t"

t2e4

⠸⠩⠀⠼⠞⠆⠑⠲⠀⠸⠱

(a base-12 numeral)

Example 3-92: Letters are Numerals in Base-12

3t.t8

⠸⠩⠀⠼⠒⠞⠨⠞⠦⠀⠸⠱

Example 3-93: Letters are Numerals in Base-16

FA9,B7C.0A

⠸⠩⠀⠼⠋⠁⠔⠠⠃⠶⠉⠨⠴⠁⠀⠸⠱

(F, A, B and C are capitalized in print)

3.7 Ordinal Endings

Unmodified digits with ordinal endings are transcribed in UEB unless they occur in mathematical expressions such as equations.

Example 3-94: Ordinal Ending in an Equation

first = 1st

⠸⠩⠀⠋⠊⠗⠎⠞⠀⠨⠅⠀⠼⠂⠎⠞⠀⠸⠱

3.8 Plural and Possessive Endings

See **8.4**.

3.9 Numerals in Diagrams

For use of numerals in diagrams, number lines, plots, graphs, etc., refer to the most recent edition of *Guidelines and Standards for Tactile Graphics*.

3.10 Numerals in Table Entries

This section applies only to the body of a table and not to the headings. In tables whose entries consist entirely of numerals, the numeric indicator may be omitted. This is used only as a space-saving option. When the numeric indicator is omitted from an entire table, the table is transcribed in Nemeth Code. The UEB numeric passage indicator is not used. In tables whose entries are a mixture of words, numerals, letters, or other mathematical signs, the numeric indicator is used. The minus symbol is non-numeric so, if it occurs in a table, the numeric indicator is used throughout the table. If guide dots occur in a table, the numeric indicator is used throughout the table. Determinants and matrices are not to be regarded as tables.

3.11 Roman Numerals

3.11.1 **Roman Numerals Transcribed in Nemeth Code.** When Roman numerals have mathematical meaning or are used as identifiers within Nemeth switch indicators, the following rules apply.

a. **Uppercase Roman Numerals.** Capitalized Roman numerals are transcribed using the single capital letter indicator before one letter and the double capital letter indicator before more than one letter. For the use of the English letter indicator with Roman numerals, see **6.5**.

Example 3-95: Roman Numerals in an Enclosed List

(I, II, III, IV, V)

⠸⠩⠀⠷⠠⠊⠠⠀⠠⠠⠊⠊⠠⠀⠠⠠⠊⠊⠊⠠⠀⠠⠠⠊⠧⠠⠀⠠⠧⠾⠀⠸⠱

(an enclosed list is transcribed in Nemeth Code)

Example 3-96: Roman Numerals in an Equation

VII + V = XII

⠸⠩⠀⠠⠠⠧⠊⠊⠬⠠⠧⠀⠨⠅⠀⠠⠠⠭⠊⠊⠀⠸⠱

Example 3-97: Roman Numerals in an Equation

(I + II) + V = I + (II + V)

⠸⠩⠀⠷⠠⠊⠬⠠⠠⠊⠊⠾⠬⠠⠧⠀⠨⠅⠀⠠⠊⠬⠷⠠⠠⠊⠊⠬⠠⠧⠾⠀⠸⠱

Example 3-98: Roman Numeral with a Prime Sign

II'

⠸⠩⠀⠠⠠⠊⠊⠄⠀⠸⠱

(prime sign follows the Roman numeral)

Example 3-99: Roman Numeral with a Bar Over

$\overline{\mathrm{M}}$

⠸⠩⠀⠠⠍⠱⠀⠸⠱

Example 3-100: Roman Numerals in Equations

V = 5, L = 50

⠸⠩⠀⠠⠧⠀⠨⠅⠀⠼⠢⠠⠀⠠⠇⠀⠨⠅⠀⠼⠢⠴⠀⠸⠱

Example 3-101: Roman Numerals as Identifiers

1. Work each problem.

I. $\frac{4}{9} \cdot \frac{1}{6}$

II. $\frac{4}{9} \div \frac{1}{6}$

⠼⠁⠲⠀⠠⠐⠺⠀⠑⠁⠡⠀⠏⠗⠕⠃⠇⠑⠍⠲⠀⠸⠩
⠀⠀⠰⠠⠊⠸⠲⠀⠹⠲⠌⠔⠼⠡⠹⠂⠌⠖⠼
⠀⠀⠠⠠⠊⠊⠸⠲⠀⠹⠲⠌⠔⠼⠨⠌⠹⠂⠌⠖⠼⠀⠸⠱

b. **Lowercase Roman Numerals.** When a Roman numeral consists of one or more lowercase letters it is treated as though it were a "single letter" and, as such, the English letter indicator is used or is not used in accordance with the rules governing the English-letter indicator (see **6.5**).

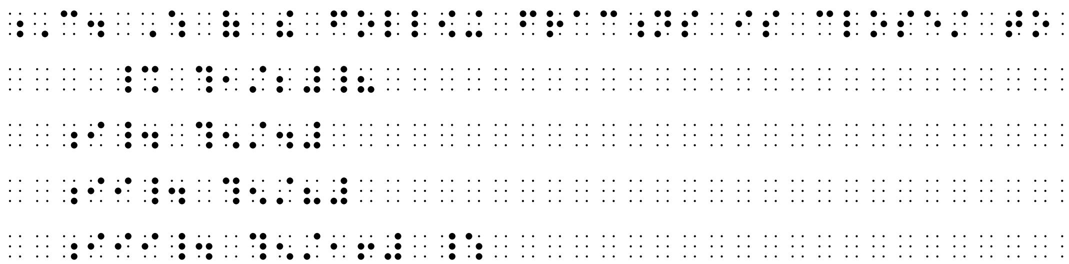
Example 3-102: Lowercase Roman Numerals as Identifiers

C. Which of the following fractions is closest to $\frac{1}{2}$?

i. $\frac{5}{4}$

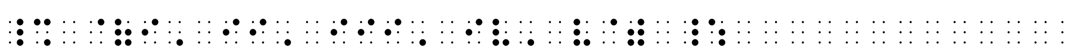
ii. $\frac{5}{8}$

iii. $\frac{5}{16}$



Example 3-103 Roman Numerals in an Enclosed List

[i, ii, iii, iv, v]



Example 3-104: Roman Numerals in an Equation

vi + iv = x

Example 3-105: Roman Numerals in Equations

i = 1, v = 5, and x = 10.

3.11.2 **Roman Numerals Transcribed in UEB.** Roman numerals standing alone may be transcribed in UEB.

Example 3-106: Roman Numerals in UEB Context

(1) The current year (2019) is MMXIX.

(2) The third graders learned that X means 10 in Roman numerals.

Example 3-107: Lowercase Roman Numerals as Identifiers

C. Which of the following numbers is closest to 100?

i. 94

ii. 107

When it is questionable that a letter combination is a Roman numeral, treat the combination as if it is not a Roman numeral.

Example 3-108: Letters May or May Not Be Roman Numerals

CL

Example 3-109: Letters May or May Not Be Roman Numerals

mix

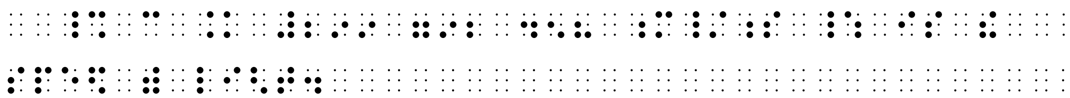
3.11.3 **Punctuation of Roman Numerals.** For punctuation of Roman numerals in mathematical context, see **8.2.3**.

3.12 Spacing with Numerals

Spaces within numerals must be retained when it is necessary to partition a numeral into short regular segments, or to achieve alignment. Do not insert a numeric indicator.

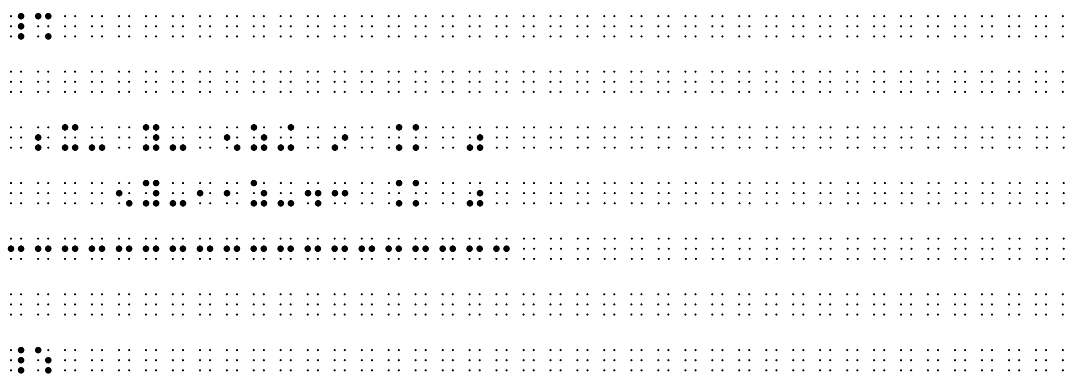
Example 3-110: Large Number Grouped by Spacing

c = 299 792 458 m/s is the speed of light.



Example 3-111: Alignment Achieved by Spacing

$$\begin{array}{r}
2x - y - 5z + 9 = 0 \\
5y - 11z - 43 = 0 \\
\hline
\end{array}$$



Rule 4
Code Switching

Nemeth Code Switch Indicators

Opening Nemeth Code indicator ⠸⠩

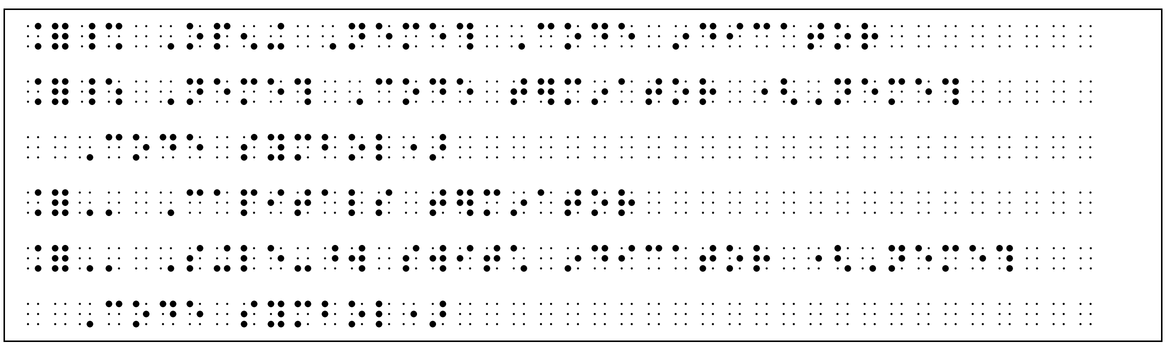
Nemeth Code terminator ⠸⠱

Single-word switch indicator ⠰⠄

4.1 Special Symbols Page

The switch indicators are listed on the special symbols page in braille order. The Nemeth Code terminator and single-word switch indicator are Nemeth symbols and must be identified as such in the special symbols list. The identifications of each must be followed by the phrase (Nemeth Code symbol) in parentheses.

Example 4-1: Special Symbols Page Entries



4.2 Code Switch Indicators

The purpose of code switching is to indicate text which is transcribed in a braille code other than UEB. The opening Nemeth Code indicator followed by a space is placed before the math symbol, expression, or passage to which it applies. Its effect is terminated by the Nemeth Code terminator preceded by a space. Use of the single-word switch indicator allows a single UEB word to be within a Nemeth passage without the need to explicitly terminate Nemeth Code. See **4.6.8.c** for rules regarding use of the single-word switch indicator.

UEB symbols are not used within the switch indicators for Nemeth Code. Nemeth symbols are not used outside the switches.

4.3 Considerations Regarding Use of the Nemeth Code

Making the decision regarding which braille code to use is based on the content and purpose of the entire text. UEB symbols for mathematical signs are used in purely literary texts. When a math or science text contains mathematical characters, it is referred to as "technical text" and Nemeth Code should be used.

4.4 When Nemeth is Required

In the transcription of technical text, a mathematical expression or chemical formula is transcribed in Nemeth Code. This includes isolated signs and fragmentary expressions (e.g., parts of formulas, incomplete equations). A mathematical expression is understood to contain at least one mathematical symbol. Additionally, the following items are transcribed in Nemeth wherever they occur.

4.4.1 **Abbreviated Function Name.** An abbreviated function name is a mathematical expression and is transcribed in Nemeth Code. Transcription of a function name that is not abbreviated depends on context. See **4.6.8.a**.

Example 4-2: Abbreviated Function Names within Text

sin and cos are circular functions.

⠸⠩ ⠎⠊⠝ ⠠⠄⠯ ⠉⠕⠎ ⠸⠱ ⠜⠑ ⠉⠊⠗⠉⠥⠇⠜ ⠋⠥⠝⠉⠰⠝⠎⠲

4.4.2 **Chemical Notation.** An element symbol such as C, O, H, Na, is a scientific symbol and is transcribed inside the Nemeth Code switches. See *Chemical Notation Using the Nemeth Braille Code* for rules regarding the transcription of a chemistry text and for depiction of molecular diagrams.

Example 4-3: Chemical Element Symbol

The symbol for carbon is C; for silver, Ag; and for bromine, Br.

(no English letter indicator is used on chemical element symbols)

4.4.3 **Fraction.** A fraction is transcribed in Nemeth Code.

Example 4-4: Fraction

Mara spent 1/2 of her practice time on her scales.

Example 4-5: Fraction

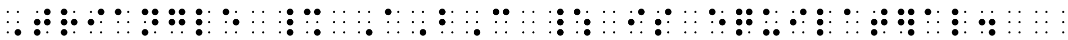
The class was right $\frac{9}{10}$ of the time.

4.4.4 **Letters in Mathematical Context.**

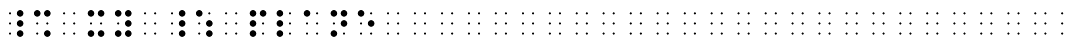
a. **Series of Letters.** A series of two or more letters in mathematical context is transcribed in Nemeth Code (e.g., Triangle ABC, line EF). The name of the figure is not included within the switches unless the name is an abbreviation. A hyphenated term involving letters should be entirely enclosed within the switches.

Example 4-6: More than One Letter (in Technical Context)

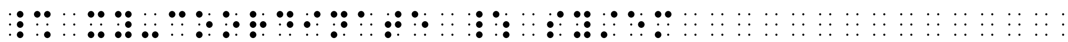
(1) Triangle ABC is equilateral.



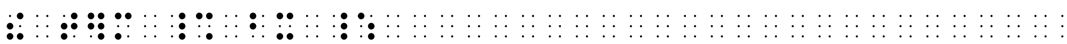
(2) xy plane



(3) xy-coordinate system



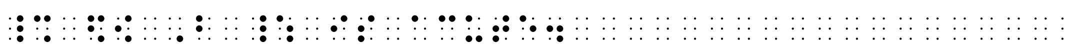
(4) the term bx



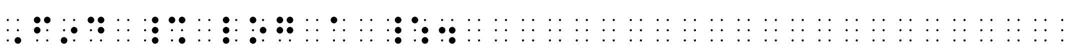
b. **Letter Associated with a Shape or Abbreviated Function Name.** A letter associated with a shape or with an abbreviated function name is transcribed in Nemeth Code.

Example 4-7: Letter with Shape or Abbreviated Function Name

(1) ∠ B is acute.



(2) Find log a.



c. **Modified Letter.** A modified letter is transcribed in Nemeth Code. See **4.6.6** for the distinction between modified and unmodified letters.

4.4.5 **Modified Number.** A modified number is transcribed in Nemeth Code. See **4.6.4** for the distinction between modified and unmodified numbers.

4.4.6 **Nemeth Code Symbols.** To maintain consistency throughout a book, signs and symbols such as percent sign, degree sign, prime, monetary symbols, non-Latin letters — whether in a math expression, within narrative, or freestanding — are transcribed in Nemeth Code. This includes symbols consisting of the same dot formation in either code, such as lower-case Greek letters. When the location of a dot or dots making up a freestanding symbol may be unclear to the reader, a transcriber's note should be included explaining the dot configuration, as shown in Example **4-10**. A transcriber's note must be located outside the Nemeth switches.

Example 4-8: Freestanding Percent Sign in Narrative

Convert the fraction to a %.

Example 4-9: Degree Sign

Express your answer in ˚F.

Example 4-10: Prime

The prime symbol ' represents feet (ft), as in 6' tall.

Example 4-11: Dollar Sign

Student A has a $20 bill.

Example 4-12: Freestanding Greek Letter in Narrative

The symbol β represents beta.

4.4.7 **Number Line.** A number line is transcribed in Nemeth Code. Number line symbols are listed on the Special Symbols page in braille order under a separate Nemeth Horizontal Number Line heading. Refer to the current version of *Guidelines and Standards for Tactile Graphics* for rules regarding number lines.

4.4.8 **Partitioned Number.** Partitioned numbers are numbers whose segments are defined by spaces. They are transcribed in Nemeth Code.

Example 4-13: Partitioned Number

The astronomical unit is defined as exactly 149 597 870 700 m.

4.4.9 **Probability Statement.** Letters and words in the context of a probability statement are transcribed in Nemeth Code.

Example 4-14: Probability Statement

P(green)

4.4.10 **Slash in Mathematical Context.** A slash meaning per, over, or divided by is mathematical and is transcribed as a fraction line according to the rules of the Nemeth Code. An associated value is included inside the switches.

Example 4-15: Decimal Number and Slash Meaning "Per"

1.5 mi/h

(the slash in mi/h means "per" and indicates a fraction; fractions are transcribed in Nemeth Code)

Example 4-16: Whole Number and Slash Meaning "Per"

15 mi/h

4.5 When UEB is Used

In addition to regular narrative text, the following items are transcribed in UEB when encountered within a Nemeth transcription.

4.5.1 **Slash in Literary Context.** A slash that does not mean per, over, or divided by is literary and is transcribed according to the rules of UEB.

Example 4-17: Slash in a Date (UEB)

10/31/2019

⠼⠁⠚⠸⠌⠼⠉⠁⠸⠌⠼⠃⠚⠁⠊

(not a fraction; the slash does not mean per, over, or divided by)

4.5.2 **Titles and Labels.** Titles or labels for figures, tables, sections, state standards, etc., including their identifying numbers, are transcribed in UEB. UEB punctuation such as a decimal point, hyphen, or dash are used in these identifying numbers. If mathematical material occurs within the title or label, a switch to Nemeth Code is required for the mathematical expression within the title or label.

Example 4-18: Decimal Number Label (UEB)

Section 1.4

⠠⠎⠑⠉⠰⠝⠀⠼⠁⠲⠙

Example 4-19: Title Containing a Mathematical Expression

Table 6.1 Critical Values for Chi-Square (χ^2)

⠀⠀⠀⠀⠠⠞⠁⠃⠇⠑⠀⠼⠋⠲⠁⠀⠠⠉⠗⠊⠞⠊⠉⠁⠇⠀⠠⠧⠁⠇⠥⠑⠎⠀⠿

⠀⠀⠀⠀⠀⠀⠀⠀⠠⠡⠊⠤⠠⠎⠟⠥⠜⠑⠀⠸⠩⠀⠷⠨⠯⠘⠆⠐⠾⠀⠸⠱

Example 4-20: State Standard in Brackets

Long Division [4.a, 5.c]

⠠⠇⠰⠛⠀⠠⠙⠊⠧⠊⠨⠝⠀⠨⠣⠼⠙⠲⠰⠁⠂⠀⠼⠑⠲⠰⠉⠨⠜

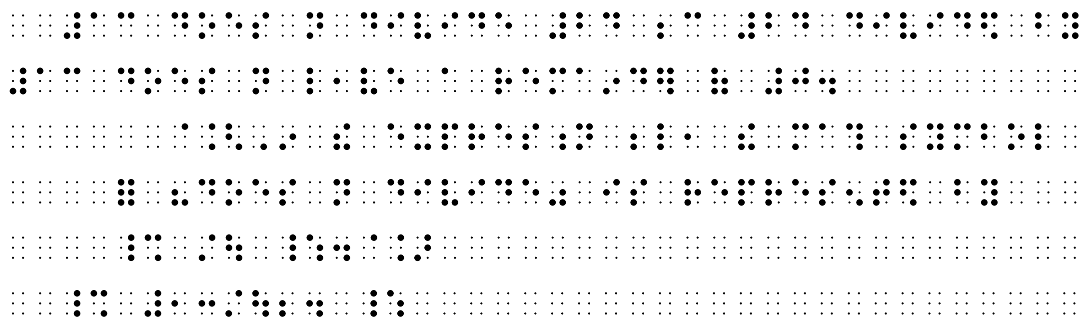
4.5.3 **Transcriber's Note.** A transcriber's note is written outside of the Nemeth Code switch indicators, using the transcriber's note indicators of UEB and following *Braille Formats* guidelines. When the note itself contains mathematical material, code switching occurs within the note. Nemeth

Code must be terminated before the closing transcriber's note indicator.

Example 4-21: Transcriber's Note

13 does not divide 24 because 24 divided by 13 does not leave a remainder of 0.

$13 \nmid 24$



4.6 When Code Switching Depends on Context

To avoid excessive code switching, the following situations allow for flexibility depending upon context.

4.6.1 **Abbreviated Unit of Measure.** An abbreviated unit of measure (e.g., ft., cm) adjacent to its related value is part of the technical expression. The associated abbreviation is transcribed in the same code as its related value. If the unit of measure requires a switch to Nemeth Code (e.g., µg), the related value is transcribed in Nemeth Code. If the measurement consists of more than one part, one of which is an abbreviation, all parts must be contained within the switches. If a value is transcribed in Nemeth Code, then its adjacent related abbreviated unit of measure (if it has one) is also transcribed in Nemeth Code, and if an abbreviated unit of measure is transcribed in Nemeth Code, then its adjacent related value (if it has one) is also transcribed in Nemeth Code.

Example 4-22: Abbreviated Unit of Measure

(1) The centipede is 1/2 in. long.

(2) This millipede is 7 cm long.

(3) 4 mcg can also be notated as 4 μg.

(4) The compost pile had a volume of 3.5 cu. yards.

4.6.2 **Digital Time.** Digital time is transcribed in UEB unless the time is involved in computation or is part of a number line.

Example 4-23: Digital Time in UEB Context

The time of the concert is 9:00-12:30.

Example 4-24: Digital Time in a Math Expression

3:30 + 15 minutes = 3:45.

4.6.3 **Identifiers.** Alphabetical or numerical identifiers may be transcribed in UEB or in Nemeth. In a numbered or lettered series of math problems that are transcribed in Nemeth Code, keep Nemeth Code in effect for the identifiers to avoid excessive switching, even though these identifiers are not technically part of the math. See **4.8.4** for an example.

4.6.4 **Numbers.** An unmodified number, defined below, may be transcribed in either code – in UEB within UEB context or in Nemeth within Nemeth context. A modified number must be transcribed in Nemeth Code.

a. **Unmodified Number.** A number that is standing alone is considered to be unmodified. A number with an ordinal, with a plural ending, or with an internal comma is considered to be unmodified. Roman numerals are included in this definition.

Example 4-25: Unmodified Number in UEB Context

(1) The sum of their squares is 90.

(2) Color the 4^{th} animal red.

(3) How to teach the 9's facts. (9s facts)

(4) I have II togas and my friend has IV.

Example 4-26: Unmodified Number in Nemeth Context

Find the median of 7, –4, 9, –7, –2, 5.

(to avoid frequent switching, in this example, all numbers, modified and unmodified, could be transcribed in Nemeth Code)

b. **Modified Number.** A number that is combined with anything other than an ordinal or an internal comma, is not part of a hyphenated term or is anything other than a plural is considered modified and is transcribed in Nemeth Code. For example, a number with a minus sign, a dollar sign, a decimal point, or an internal low line indicating omission is a modified number. Typeform applied to a number is considered a modification if the typeform is mathematically significant. See **4.6.7**.

Example 4-27: Modified Numbers (Nemeth)

(1) Make 50¢ using at least 1 quarter.

(2) The number line goes from -5 to 5.

or

(3) 20°

4.6.5 **Punctuation.** Sentence punctuation follows Nemeth Code rules inside the code switches, and UEB rules outside the code switches.

a. **Inside the Code Switches.** Mathematical items are punctuated using the mathematical comma or with literary punctuation preceded by the punctuation indicator of the Nemeth Code. Words and abbreviations are punctuated in literary mode.

Parentheses, brackets, braces, or quotation marks that enclose only Nemeth Code material are transcribed in Nemeth Code.

See **Rule 8** for further details regarding punctuation.

Example 4-28: Math Comma Between Numbers

3.5, 4, 4.5, 5

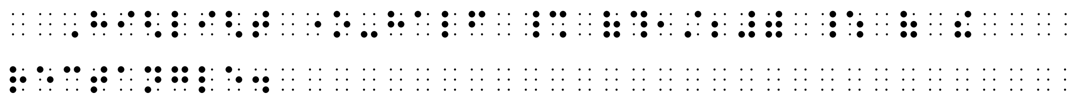
Example 4-29: Literary Punctuation Inside the Switches

3.5 ft., 4 ft., 4.5 ft., 5 ft.

(the abbreviations use literary punctuation)

Example 4-30: Parentheses Enclosing a Fraction

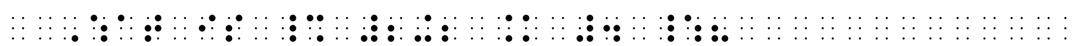
Highlight one-half ($\frac{1}{2}$) of the rectangle.



b. **Outside the Code Switches.** Punctuation that is logically associated with surrounding text is transcribed according to UEB. When Nemeth Code ends and sentence punctuation follows, the punctuation is usually transcribed immediately following the Nemeth Code terminator without an intervening space.

Example 4-31: Punctuation Outside the Nemeth Terminator

What is $2 + 2 = 4$?



(the question mark applies to the sentence structure)

4.6.6 **Single Letter.** An unmodified letter, defined below, may be transcribed in either code – in UEB within UEB context or in Nemeth within Nemeth context. A modified letter must be transcribed in Nemeth Code.

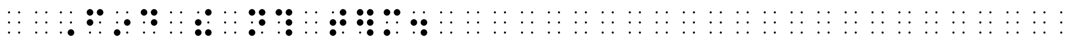
Two or more unspaced mathematical letters are transcribed in Nemeth Code. See **4.4.4.a**. A mathematical letter in an alphabet other than English must be transcribed in Nemeth Code.

a. **Unmodified Letter.** A single mathematical English letter that is standing by itself or in a hyphenated term (such as x-axis) is considered to be unmodified. A letter with an ordinal or with a plural ending is considered to be unmodified. Roman numerals are included in this definition.

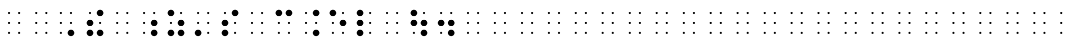
Example 4-32: Unmodified Letter (UEB)

(1) This is a linear expression in *x*.

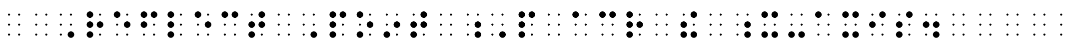
(2) Find the *n*th term.



(3) The *z*'s cancel out.



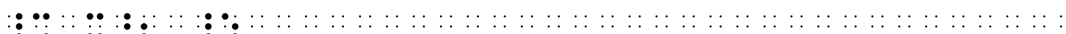
(4) Reflect Point P across the x-axis.



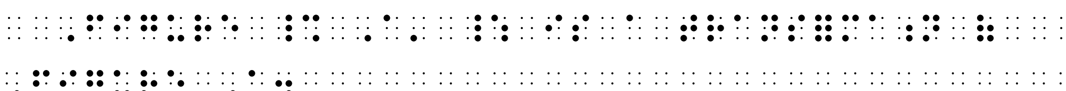
b. **Modified Letter.** A mathematical letter that is combined with anything other than an ordinal or a plural is considered modified and is transcribed in Nemeth Code.

Example 4-33: Modified Letter (Nemeth)

(1) x^2



(2) Figure A' is a transformation of Figure A.



c. **Single Letter Associated with a Math Item.** A single letter associated with a sign of shape or an abbreviated function name is transcribed in Nemeth Code. A single letter associated with a word may be transcribed in UEB (e.g., Circle O). If there is other math surrounding the phrase, the transcriber may decide, for consistency, that he/she wants to do all such letters in Nemeth.

Example 4-34: Letter Associated with a Math Item (Nemeth)

(1) $\angle X$ is 90º.

(2) Given csc A, find the hypotenuse.

Example 4-35: Letter(s) Associated with a Word (UEB)

(1) Label Circle O.

(2) Triangle ABC, Circle O, Line AB

4.6.7 **Typeform.** Typeform applied to a number or letter is considered a modification if the typeform is mathematically significant. A switch to Nemeth Code is required. Typeform applied to content other than a mathematical symbol is transcribed according to the rules of UEB. For rules regarding typeform in mathematical context, see **Rule 7**.

Example 4-36: Number and Letter with Typeform Modification

In p**v** = **0**, **v** is a vector and **0** is the null vector.

(boldface is retained on vectors)

A switch from UEB to Nemeth Code terminates the effect of a UEB typeform indicator. No UEB typeform terminator is required. This does not apply when switching from Nemeth to UEB. Capitalization is not a typeform attribute and must be terminated.

Example 4-37: Math Expression within Bold Phrase

In this equation, ***ab* = *ba***, **the operation is commutative**.

(typeform in the equation is disregarded when it is not mathematically significant)

Example 4-38: Capitalization with UEB and Nemeth

SOLVE THE EQUATION x-4y = 12 can be solved for y.

4.6.8 Words.

a. **Words in Literary Context.** When more than one word follows mathematical material, Nemeth Code is terminated before transcribing the words. For the purpose of this code, words joined by a slash or separated by a dash are considered to be more than one word. A hyphenated word is considered to be one word. A function name is transcribed in UEB unless the word is part of a mathematical expression that requires a switch to Nemeth Code. When words are used to replace technical symbols (e.g., "plus", "equals", "times", etc.), the words are transcribed in UEB. For words in a table, see **4.8.10**.

Example 4-39: Two Words Between Math Expressions (UEB)

2.5 inches plus 3.5 inches equals 6 inches.

Example 4-40: Function Name within Text (UEB)

Sine and cosine are circular functions.

Example 4-41: Two Words Separated by a Slash

We can use + and/or ± in this problem.

Example 4-42: Two Words Separated by a Dash

9.4 works—but 9.5 does not.

b. **One or More Words in Mathematical Context.** When one or more words are part of an equation or math expression, they are as much a part of the technical notation as are the variables (letters), numbers, signs of operation, etc. The entire expression is placed inside the Nemeth switches. No contractions are used in the words. Spacing rules of the Nemeth Code are followed.

Example 4-43: Words in a Math Equation

20 bags of red marbles + 10 bags of blue marbles = ? total marbles

Example 4-44: One Word in a Math Equation

25% of \$2.00 = \$.50

Example 4-45: Function Name in Math Context

$$\text{Sine } \theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

c. **The Single-Word Switch Indicator.** When only one narrative word, hyphenated word, or abbreviation without an associated value occurs between two Nemeth expressions or symbols, the single-word switch indicator (⠸⠆) is used to indicate that the word is in UEB. The indicator is unspaced from the affected word. Contractions are used in the subsequent word as needed. The indicator is used whether or not the word contains contractions. A UEB typeform indicator may be used with a word that is preceded by the single-word switch indicator. The effect of the single-word switch indicator is terminated by a space.

The single-word switch indicator cannot be used on words joined by a slash or separated by a dash. (See examples **4-41** and **4-42**.) The single-word switch cannot be used immediately before a UEB grouping symbol or quotation mark. (See example **4-52**.)

Example 4-46: One Word Between Math Expressions

10^7 or 10^{-7}

Example 4-47: Single-Word Switch Between Math Expressions

25% of $2.00 is $.50

Example 4-48: Word Instead of Math Symbol

2.5 in. plus 3.5 in. equals 6 in.

(the first two abbreviations are part of the math)

Example 4-49: Hyphenated Word Between Math Expressions

1/5, one-fifth; 1/6, one-sixth

Example 4-50: Abbreviation Between Math Expressions

10.0 yd. vs. 30.0 ft.

(vs. is an abbreviation with no associated value)

Example 4-51: Bold Word Between Math Expressions

ac = 12, bd = −10, **and** ad + bc = 7

Example 4-52: Single Word with Grouping Symbol

17.3 (or) 1.73

4.7 Components That Do Not Require Switching

Code switching is not necessary in the following situations.

4.7.1 Running heads, running footers, page change indicators, box lines, note separation lines, page numbers, and column separation lines do not interrupt the effect of the Nemeth Code indicators. The effect of the opening Nemeth Code indicator is not terminated by transition to a new braille or print page.

Example 4-53: Page Change Indicator

Similarly, 15 : 9, 10 : 6,

— — — — — — — — [print page turn for page 24] — — — — — — — —

and 5 : 3 are equivalent ratios.

(the code in place before the page change indicator remains in effect following the page change indicator)

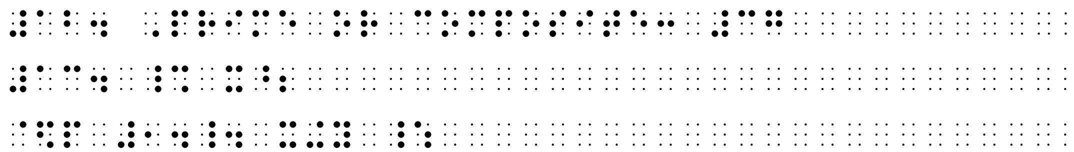
4.7.2 Icons created using the UEB transcriber-defined shape indicator may be used in either UEB or Nemeth context without the insertion of switch indicators.

Example 4-54: Icon in Nemeth Context

12. Prime or composite: 37

13. x^2

14. $x+y$



(alignment of identifiers is not required)

4.8 Placement of Switch Indicators

In general, switch indicators are kept on the same line as the mathematics to which they apply. Exceptions occur with certain displayed material, itemized material, lists, spatial material, and tables, as outlined in this section.

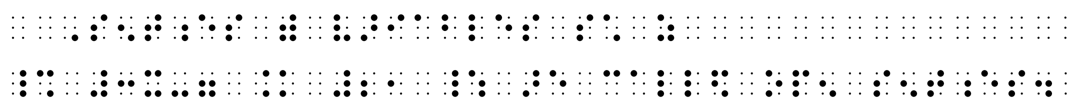
4.8.1 **Placement of Switches after a Cell-5 or Cell-7 Heading.** An opening switch indicator may be placed at the end of a cell-5 or cell-7 heading, or in the runover position for the heading. Rules for placement of switches with embedded mathematical material (**4.8.2**), nonspatial displayed mathematical material (**4.8.3**), and spatial arrangements (**4.8.5**) take precedence over this option. The opening switch indicator cannot be placed at the end of a centered heading.

4.8.2 **Placement of Switches with Embedded Mathematical Material.** If the mathematical material and its switch indicators will fit on one braille line within current margins, this is the preferred layout. If not, the priority is to not divide the math expression. When only one code switch indicator will fit on the line with the math expression, either indicator may be placed on the line with the math. That is, the opening Nemeth Code indicator may be placed on the previous line, or the Nemeth Code terminator may be placed on the following line. The terminator and related punctuation may be placed on the following line if there is not room on the line with the math expression.

Text and switch indicators are aligned with the main line in an embedded spatial expression. See **26.6.2** for details.

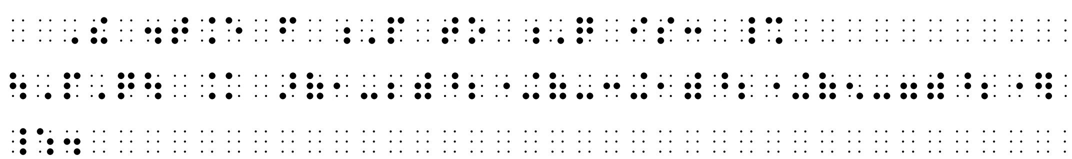
Example 4-55: Code Switching for Embedded Math

Sentences with variables such as 3x - 7 = 21 are called open sentences.



Example 4-56: Placement of Terminator with Punctuation

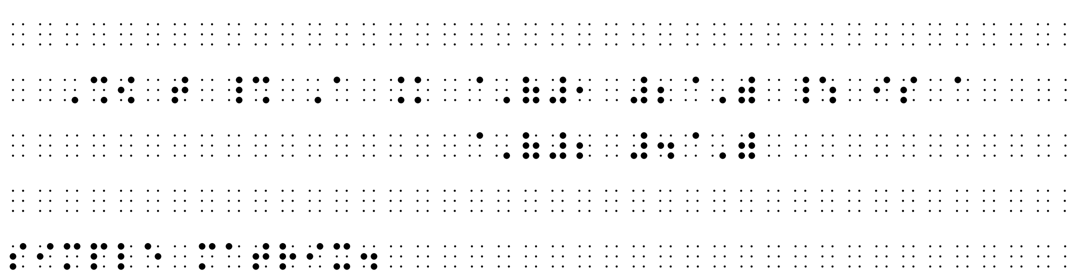
The distance from P to Q is: $|PQ| = \sqrt{(1-2)^2 + (-3+1)^2 + (5-7)^2}$.



(because the math expression takes up the full available line width, both the opening Nemeth Code indicator and the Nemeth Code terminator fall on lines separate from the math)

Example 4-57: Embedded Matrix

Show that $A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$ is a simple matrix.

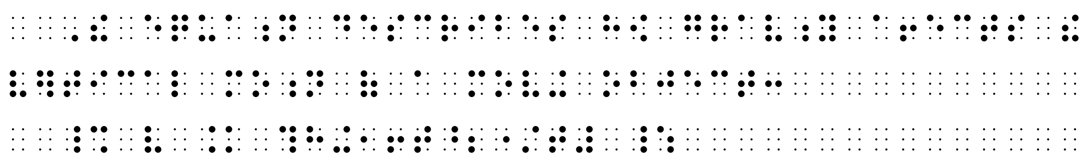


4.8.3 **Placement of Switches with Nonspatial Displayed Mathematical Material.** If the nonspatial displayed mathematical material and its switch indicators will fit on one braille line within current margins, that is the preferred layout. If not, the opening indicator is placed at the end of the previous line of text or in the runover position if there is not room on that line. An opening switch may follow a closing transcriber's note indicator if it fits on the same line. The Nemeth Code terminator is placed at the end of the displayed material, in the runover position if there is not room on the current line. For rules regarding displayed spatial material, see **4.8.5**.

Example 4-58: Code Switching for Short Displayed Math

The equation describes how gravity affects the vertical motion of a moving object:

$$v = \frac{h + 16t^2}{t}$$

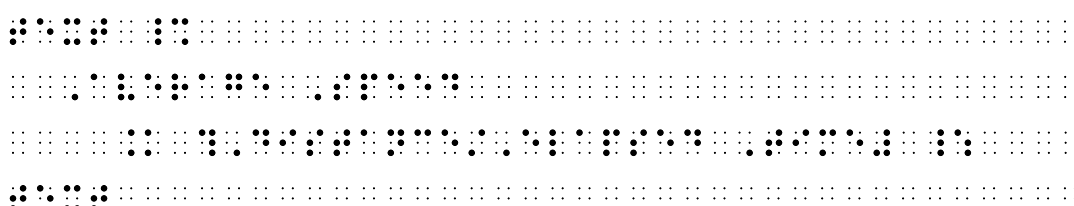


Example 4-59: Code Switching for Displayed Math with Runover

text

$$\text{Average Speed} = \frac{\text{Distance}}{\text{Elapsed Time}}$$

text



4.8.4 **Code Switching Considerations with Identifiers.** To assure that identifiers align in the proper cell, the opening Nemeth Code indicator is placed at the end of the text that precedes the itemized material. If there is not room at the end of the braille line, the opening indicator is placed in the runover position of the text.

Example 4-60: Opening Code Switch Before Identifiers

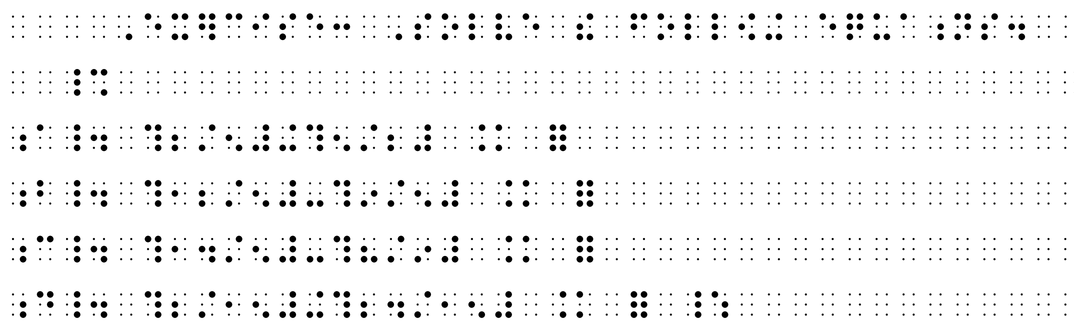
Exercise: Solve the following equations.

a. $\frac{2}{5} + \frac{5}{2} =$

b. $\frac{12}{5} - \frac{9}{5} =$

c. $\frac{14}{5} - \frac{8}{9} =$

d. $\frac{2}{15} + \frac{24}{15} =$



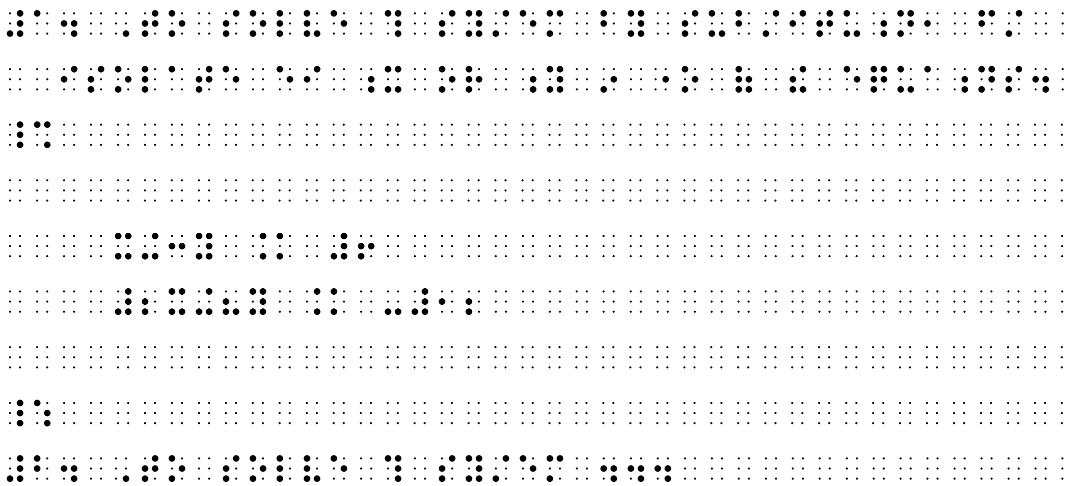
4.8.5 **Code Switching with Spatial Problems.** An opening Nemeth Code indicator that precedes a spatial problem is placed on the same line as the end of the text above the problem if it fits. If there is not room on that line, the opening Nemeth Code indicator is placed on the next line in cell 1. The required blank line follows the opening Nemeth Code indicator. If there is no identifier or text preceding the spatial problem, the opening indicator is placed in cell 1 on a line by itself followed by the requisite blank line. When Nemeth Code is closed after a spatial problem, the terminator is placed in cell 1 on a line by itself and is preceded by the required blank line.

Example 4-61: Code Switches and Spatial Math

1. To solve this system by substitution, first isolate either x or y in one of the equations.

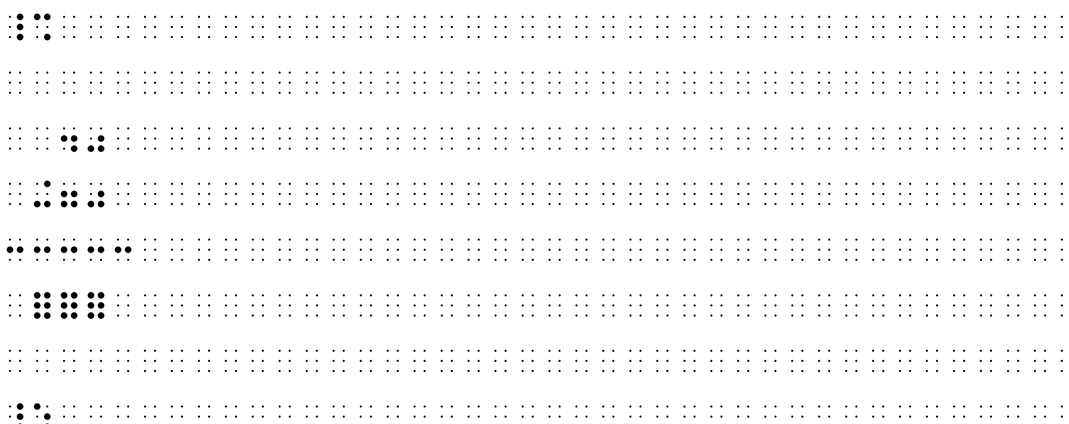
$$x + 3y = 6$$
$$2x + 8y = -12$$

2. To solve this system …



Example 4-62: Code Switches and Spatial Math

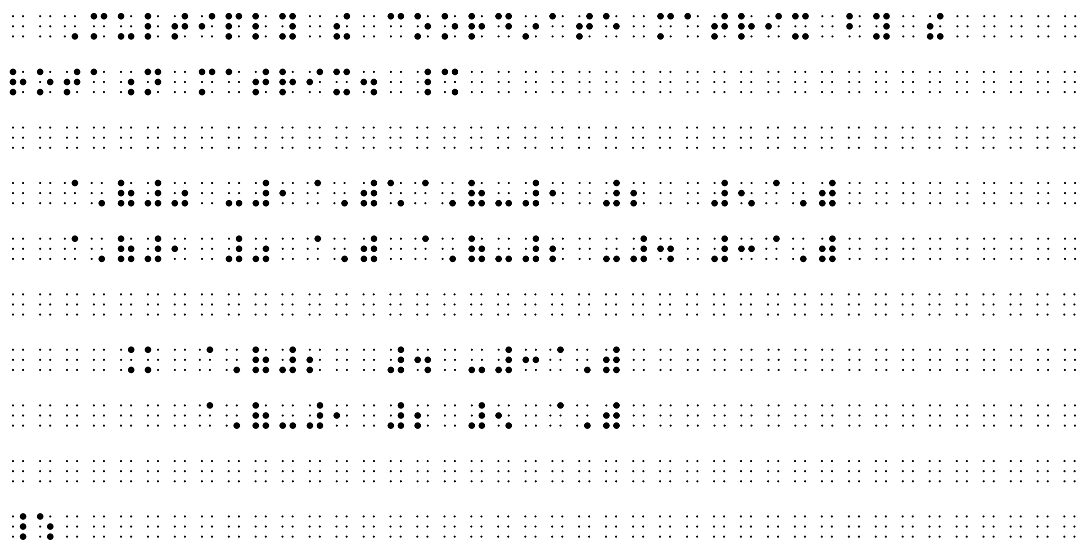
$$\begin{array}{r} 40 \\ \underline{+70} \\ ??? \end{array}$$



Example 4-63: Code Switches and Spatial Math

Multiply the coordinate matrix by the rotation matrix.

$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \bullet \begin{bmatrix} -1 & 2 & 5 \\ -2 & -4 & 3 \end{bmatrix} = \begin{bmatrix} 2 & 4 & -3 \\ -1 & 2 & 5 \end{bmatrix}$$

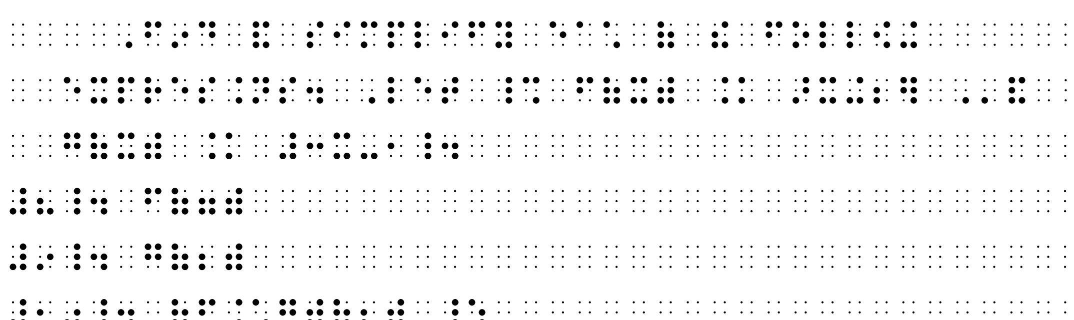


4.8.6 **Code Switching and Instructions.** If instructions end with an expression transcribed in Nemeth Code and the subsequent math problem starts with Nemeth Code, Nemeth Code is left in effect between the end of the instructions and the start of the problem.

Example 4-64: Nemeth Code Continues

Find and simplify each of the following expressions. Let $f(x) = \sqrt{x+2}$ and $g(x) = 3x - 1$.

8. $f(7)$ 9. $g(2)$ 10. $(f \circ g)(2)$



4.8.7 **Code Switching with Grouping Symbols.** Left and right grouping symbols must be transcribed in the same code. For math entirely enclosed between grouping symbols, see **4.6.5.a**.

Example 4-65: Grouping Symbols in the Same Code

... which gives 2x = xy and so (since x ≠ 0) y = 2z.

⠲⠲⠲⠀⠱⠀⠛⠊⠧⠑⠎⠀⠸⠩⠀⠼⠆⠭⠀⠨⠅⠀⠭⠽⠀⠸⠱⠀⠯⠀⠎

⠸⠩⠀⠷⠠⠄⠎⠔⠉⠑⠀⠭⠀⠌⠨⠅⠀⠼⠴⠾⠀⠽⠀⠨⠅⠀⠼⠆⠵⠀⠸⠱⠲

... which gives 2x = xy and so (given that x ≠ 0) y = 2z.

⠲⠲⠲⠀⠱⠀⠛⠊⠧⠑⠎⠀⠸⠩⠀⠼⠆⠭⠀⠨⠅⠀⠭⠽⠀⠸⠱⠀⠯⠀⠎⠀⠐⠣⠛⠊⠧⠢⠀⠞

⠸⠩⠀⠭⠀⠌⠨⠅⠀⠼⠴⠀⠸⠱⠐⠜⠀⠸⠩⠀⠽⠀⠨⠅⠀⠼⠆⠵⠀⠸⠱⠲

(note the different wording in the two examples)

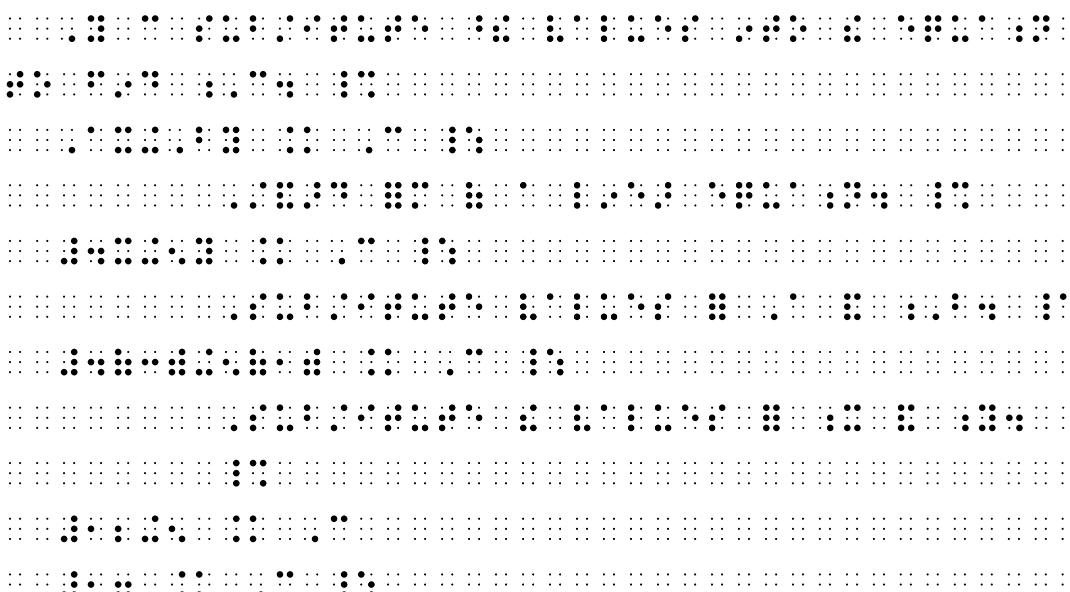
4.8.8 **Code Switching After Transcriber's Notes.** An opening Nemeth Code indicator may follow a closing transcriber's note indicator if they fit on the same line.

4.8.9 **Code Switching with Comments.** To maintain the alignment in multi-line displayed expressions interrupted by author's comments, place the opening Nemeth Code indicator at the end of the line above, or in the runover position as needed.

Example 4-66: Remarks in UEB

You can substitute these values into the equation to find C.

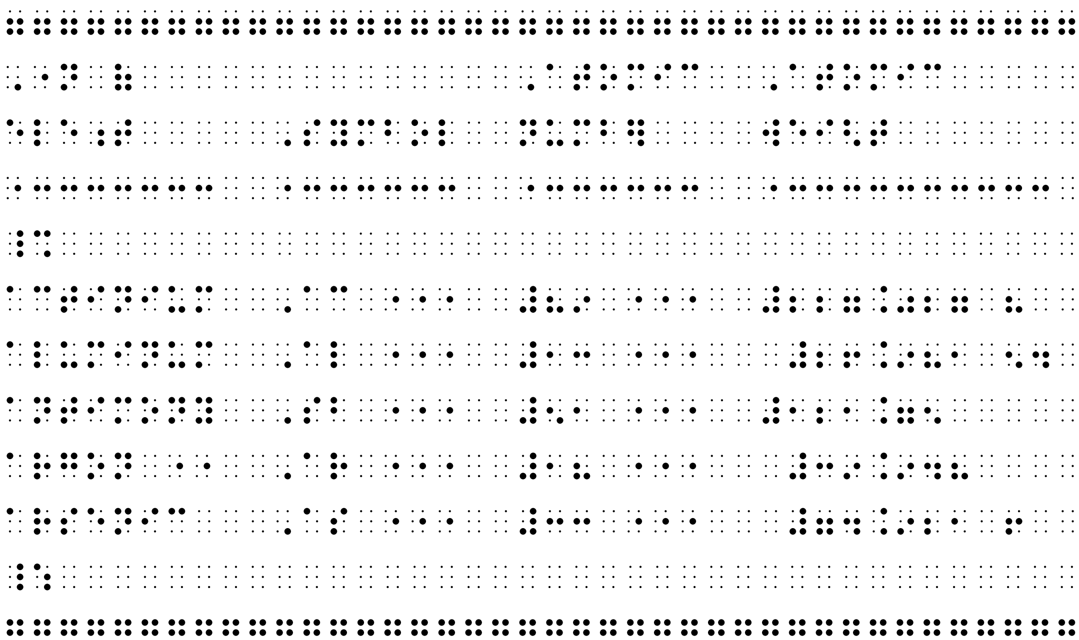
$Ax + By = C$	*Standard form of a linear equation.*
$4x + 5y = C$	*Substitute values for A and B.*
$4(3) + 5(1) = C$	*Substitute the values for x and y.*
$12 + 5 = C$	
$17 = C$	



4.8.10 **Code Switching in Tables.** Tables consisting entirely of words or unmodified letters/numbers are transcribed in UEB. When table entries contain technical material but the row headings are words, the whole table is considered technical material, excluding the table title and column headings. The opening Nemeth Code indicator is placed at the left margin (cell 1) of the line following the column separation line with the first row heading or first entry on the next line. The Nemeth Code terminator follows the last line of entries, placed at the left margin (cell 1), followed by the bottom box line, if needed, on the next braille line. Because the row headings are included inside the switches, words in the row headings are not contracted. If the row heading consists of one word, the single-word switch indicator is not used.

Example 4-67: Body of the Table in Nemeth Code

Name of element	Symbol	Atomic number	Atomic weight
actinium	Ac	89	227.027 8
aluminum	Al	13	26.981 54
antimony	Sb	51	121.75
argon	Ar	18	39.948
arsenic	As	33	74.921 6

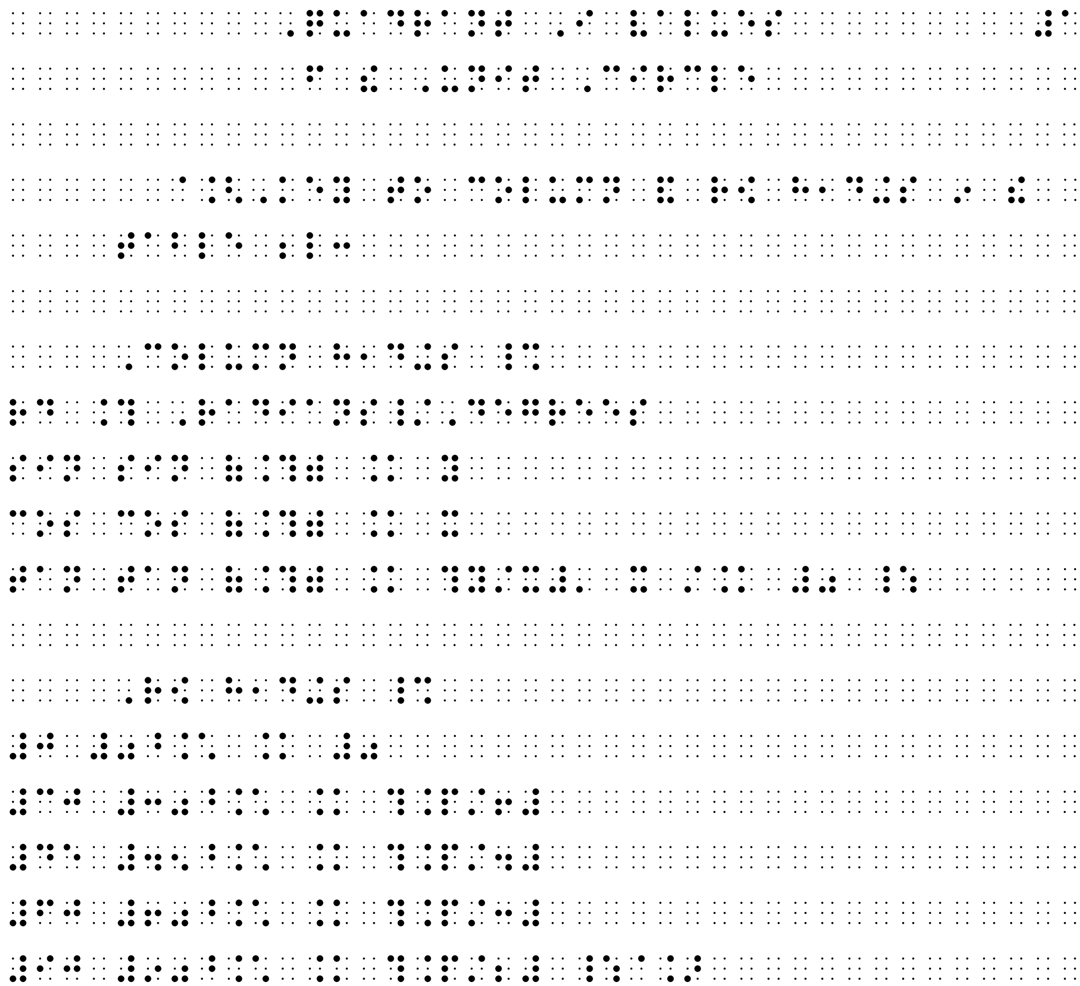


a. **Tables Within Boxes.** For a box transcribed entirely in Nemeth Code, the opening Nemeth Code indicator is placed at the beginning of the top box line, followed by a space, and the Nemeth Code terminator is placed at the end of the bottom box line, preceded by a space. This technique is not used if technical material immediately precedes or follows the box. In that case, begin Nemeth Code before the box and terminate Nemeth Code after the box.

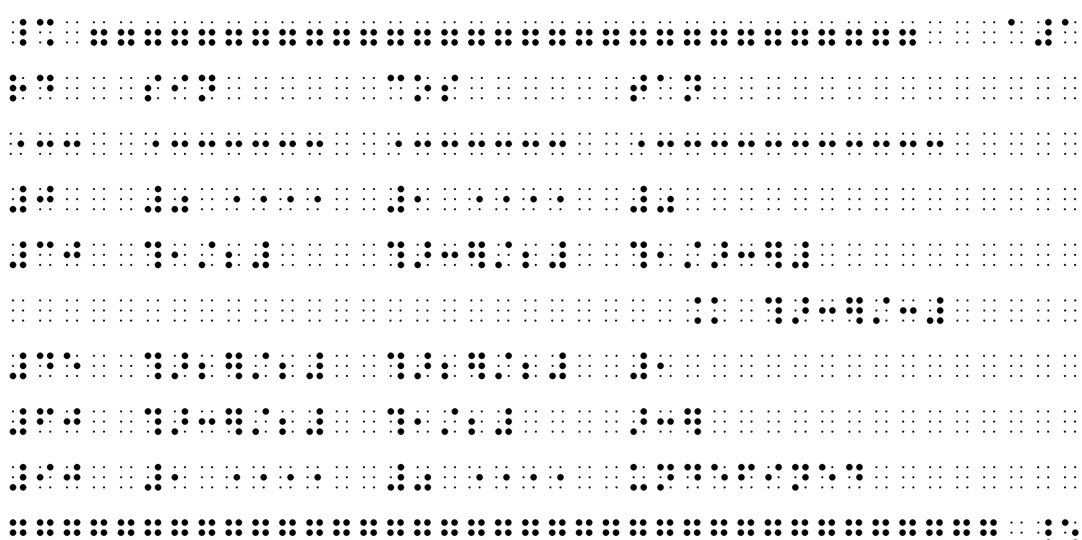
Example 4-68: Table in Nemeth Code

Quadrant I values from the Unit Circle

θ Radians/Degrees	$\sin(\theta) = y$	$\cos(\theta) = x$	$\tan(\theta) = \frac{y}{x}, x \neq 0$
$0^{\circ} = 0$	0	1	0
$30^{\circ} = \frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$
$45^{\circ} = \frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
$60^{\circ} = \frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
$90^{\circ} = \frac{\pi}{2}$	1	0	undefined



– New Braille Page –

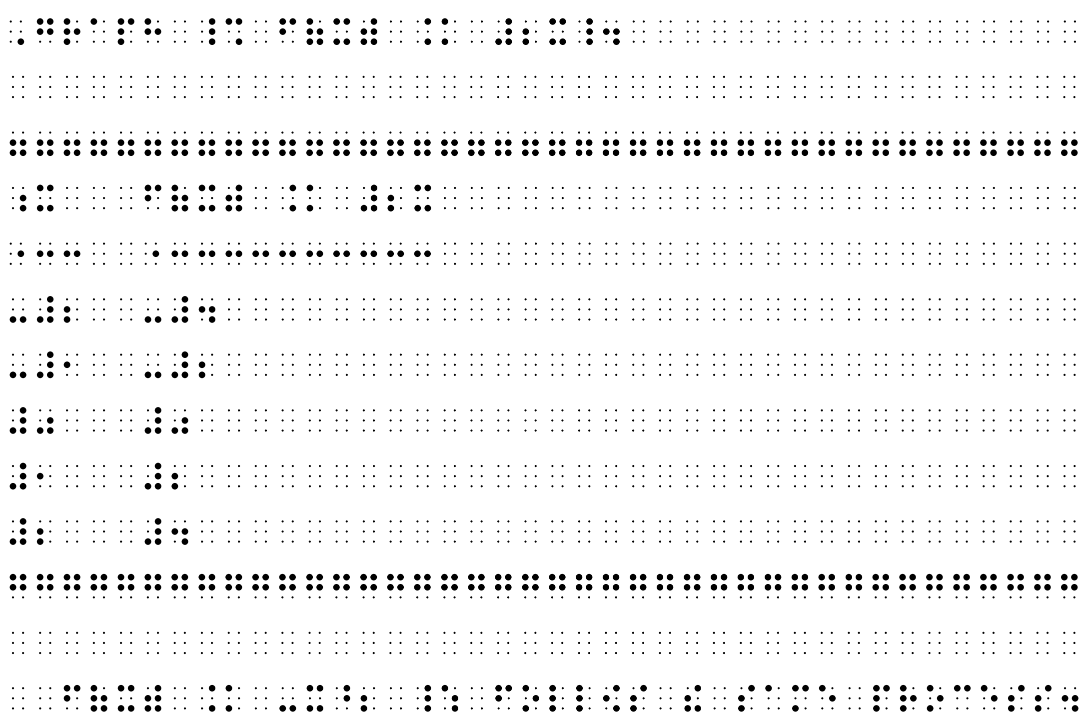


Example 4-69 Table Preceded and Followed by Math

Graph $f(x) = 2x$.

x	**f(x) = 2x**
-2	-4
-1	-2
0	0
1	2
2	4

$f(x) = -x^2$ follows the same process.



4.8.11 **Code Switches at Page Turns.** The effect of the opening Nemeth Code indicator is not terminated by transition to a new braille or print page. The opening Nemeth Code indicator is placed on the same braille page as the beginning of the mathematical material. The Nemeth Code terminator is placed on the same braille page as the end of the mathematical material. Placement of the switches before or after the print page indicator is at the transcriber's discretion.

Rule 5
Capitalization

Capitalization Indicators

Single ⠠

Double ⠠⠠

(For combinations of capitalization, alphabetic, and typeform indicators, see **Appendix C**)

5.1 Use of the Capitalization Indicator

5.1.1 The capitalization indicator is used to indicate the capitalization of a letter from any of the alphabets listed in **Rule 6**, except the Hebrew alphabet whose letters do not possess a capitalized form. This indicator precedes the letter concerned.

Example 5-1: German Capitalized Ah

𝔄

⠸⠩⠀⠸⠠⠁⠀⠸⠱

Example 5-2: Greek Capitalized Gamma

Γ

⠸⠩⠀⠸⠠⠛⠀⠸⠱

5.1.2 **Roman Numerals.** For capitalized Roman numerals, see **3.11.1**.

5.1.3 **Abbreviations.** For the capitalization of abbreviations, see **10.1-10.2**.

5.2 Non-Use of the Capitalization Indicator

Capitalization is not used with a letter just because it begins a sentence if the corresponding letter in print is uncapitalized.

Example 5-3: Sentence Beginning with Lowercase Letter

x – 1 is a number between 2 and 3.

5.3 Effectiveness of the Capitalization Indicator

5.3.1 **Letters in Mathematical Context.** The effectiveness of the single capitalization indicator extends only to the letter which follows it, so that if each single letter in a sequence requires capitalization, the capitalization indicator is used with each of these letters individually.

Example 5-4: Three-Letter Sequence

△ ABC

5.3.2 **Roman Numerals and Abbreviations.** The effectiveness of the double capitalization indicator in Roman numerals and in abbreviations extends to all of the letters which immediately follow it. A symbol other than a letter terminates its effect.

Example 5-5: Abbreviation with Capital Letters

SD = 1.9

(SD means Standard Deviation)

Example 5-6: Capitalized Roman Numerals

III + V

Rule 6
Alphabets

Alphabetic Indicators

English Letter	⠰
German Letter	⠸
Greek Alternative Letters	⠨⠈
Greek Standard Letter	⠨
Hebrew Letter	⠠⠠
Russian (Cyrillic) Letter	⠈⠈

(For combinations of capitalization, alphabetic, and typeform indicators, see **Appendix C**.)

6.1 Non-English Letters Commonly Used in Technical Material

(For additional letters from the Hebrew and Russian alphabets see "World Braille Usage", available for download at Perkins.org.)

6.1.1 German Fraktur Alphabet

Name of letter	Lowercase	Capital	Braille
ah	𝖆	𝕬	⠁
beh	𝖇	𝕭	⠃
tseh	𝖈	𝕮	⠉
deh	𝖉	𝕯	⠙
eh	𝖊	𝕰	⠑
eff	𝖋	𝕱	⠋
gheh	𝖌	𝕲	⠛
hah	𝖍	𝕳	⠓
ee	𝖎	𝕴	⠊
yaht	𝖏	𝕵	⠚

Name of letter	Lowercase	Capital	Braille
kah	𝔨	𝔎	⠅
ell	𝔩	𝔏	⠇
em	𝔪	𝔐	⠍
en	𝔫	𝔑	⠝
oh	𝔬	𝔒	⠕
peh	𝔭	𝔓	⠏
koo	𝔮	𝔔	⠟
err	𝔯	ℜ	⠗
ess	𝔰	𝔖	⠎
teh	𝔱	𝔗	⠞
oo	𝔲	𝔘	⠥
fao	𝔳	𝔙	⠧
veh	𝔴	𝔚	⠺
iks	𝔵	𝔛	⠭
ypsilon	𝔶	𝔜	⠽
tset	𝔷	ℨ	⠵

6.1.2 Letter from the Hebrew Alphabet

Name of letter	Sign	Braille
aleph	א	⠁

6.1.3 Letters from the Russian (Cyrillic) Alphabet

Name of letter	Lower-case	Capital	Braille
ell	л	Л	⠇
sha	ш	Ш	⠱

6.1.4 Greek Alphabet (Standard)

Name of letter	Lowercase	Capital	Braille
alpha	α	Α	⠁
beta	β	Β	⠃
gamma	γ	Γ	⠛
delta	δ	Δ	⠙
epsilon	ϵ or ε	Ε	⠑
zeta	ζ	Ζ	⠵
eta	η	Η	⠱
theta	θ	Θ	⠹
iota	ι	Ι	⠊
kappa	κ	Κ	⠅
lambda	λ	Λ	⠇
mu	μ	Μ	⠍
nu	ν	Ν	⠝
xi	ξ	Ξ	⠭
omicron	ο	Ο	⠕
pi	π	Π	⠏
rho	ρ	Ρ	⠗
sigma	σ	Σ	⠎
tau	τ	Τ	⠞
upsilon	υ	Υ	⠥
phi	ϕ	Φ	⠋
chi	χ	Χ	⠯
psi	ψ	Ψ	⠽
omega	ω	Ω	⠺

6.1.5 Greek Alphabet (Alternative Lowercase)

Name of letter	Sign	Braille
alpha	∝	⠨⠈⠁
beta	ϐ	⠨⠈⠃
theta	ϑ	⠨⠈⠹
sigma	ς	⠨⠈⠎
phi	φ	⠨⠈⠋

6.2 Non-English Alphabetic Indicators - German, Greek, Hebrew, and Russian

6.2.1 **Code Switching.** A switch to Nemeth Code is required when letters of non-English alphabets are used, whether in a mathematical expression or freestanding. The appropriate alphabetic indicator is used to specify the alphabet to which the letter belongs. If the letter is lowercase, the alphabetic indicator directly precedes the letter; if the letter is capitalized, the alphabetic indicator precedes the capitalization indicator.

Example 6-1: Greek Letters

α	⠸⠩⠀⠨⠁⠀⠸⠱⠀	(alpha)
Σ	⠸⠩⠀⠨⠠⠎⠀⠸⠱	(Sigma)
π	⠸⠩⠀⠨⠏⠀⠸⠱⠀	(pi)

Example 6-2: German Letters

𝔳	⠸⠩⠀⠸⠧⠀⠸⠱⠀	(fao)
𝔙	⠸⠩⠀⠸⠠⠧⠀⠸⠱	(Fao)

Example 6-3: Hebrew Letter

$\aleph_0$	⠸⠩⠀⠠⠠⠁⠴⠀⠸⠱	(alef with subscript zero)

Example 6-4: Russian Letters

л	⠸⠩⠀⠈⠈⠇⠀⠸⠱	(ell)
Л	⠸⠩⠀⠈⠈⠠⠇⠀⠸⠱	(Ell)

6.2.2 **Alternative Forms of Greek Letters.** Some letters of the lowercase Greek alphabet possess an alternative form. (See the Greek Alphabet table above.) The alternative forms are used only when the author has assigned distinct meanings to the standard and alternative forms of the same letter. When alternative forms occur *instead of* the standard forms throughout a text, the symbols for the standard forms are used in braille. The transcriber should call attention to this usage by a transcriber's note.

Example 6-5: Alternate Greek Letter Form

φ	⠸⠩⠀⠨⠈⠋⠀⠸⠱	(alternate form of phi)

6.2.3 **Extent of Effect.** The effectiveness of a non-English alphabetic indicator extends only to the letter which follows it. When an alphabetic indicator is required, it is used with each individual letter of a sequence of letters.

Example 6-6: Greek Lowercase Letters

αβ

⠸⠩⠀⠨⠁⠨⠃⠀⠸⠱

(Greek lowercase alpha followed by Greek lowercase beta)

Example 6-7: German and Greek Letters in a Math Expression

$\mathfrak{G}\gamma + \mathfrak{B}\beta$

⠸⠩⠀⠸⠠⠛⠨⠛⠬⠸⠠⠧⠨⠃⠀⠸⠱

(German capital gheh followed by Greek lowercase gamma plus German capital fao followed by Greek lowercase beta)

6.3 The English-Letter Indicator

6.3.1 **Single Letters.** An English-letter indicator is required with a single English letter unless prohibited by other rules of the Nemeth Code.

Single letter criteria:

a. is in regular type

b. is unmodified

c. is not an abbreviation (see **10.3** for use of the English-letter indicator with abbreviations)

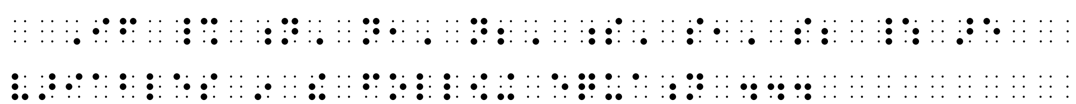
d. is preceded by a space or a punctuation mark

e. is followed by a space or a punctuation mark

Note: In mathematical context, a grouping sign is not considered a mark of punctuation.

Example 6-8: Single letter in a Mathematical Series

If n, n_1, n_2, s, s_1, s_2 are variables in the following equation ...



6.3.2 **Identifiers.** An English-letter indicator is required when a letter is used as an identifier in itemized material. Letters A, a, I, i, O, and o are included in this rule. *Exception*: English-letter identifiers which are entirely enclosed in grouping signs do not require a letter indicator (see **6.4.4**).

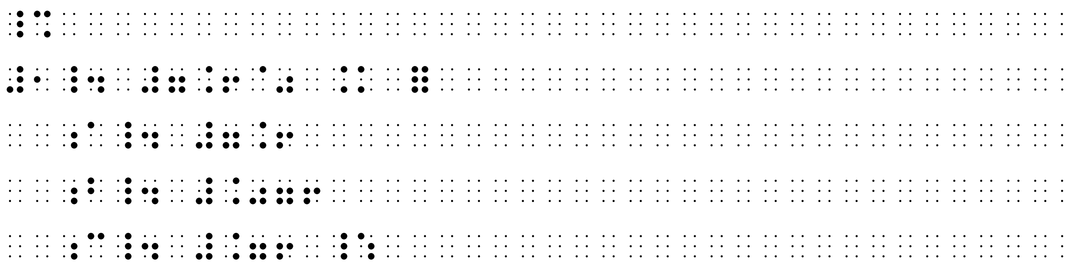
Example 6-9: Single-Letter Identifiers

1. 7.6% =

    a. 7.6

    b. .076

    c. .76



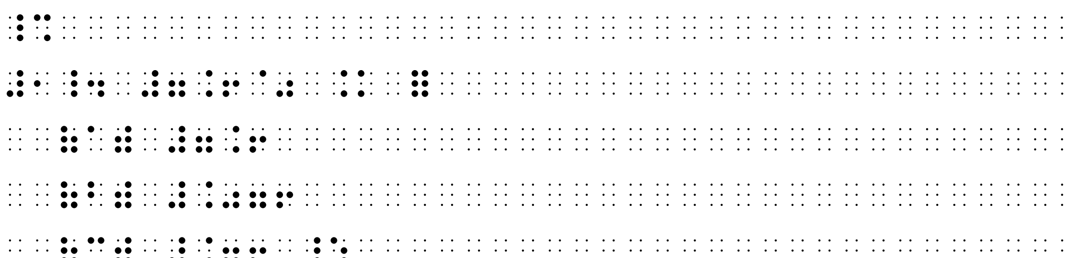
Example 6-10: Single-Letter Identifiers in Parentheses

1. 7.6% =

    (a) 7.6

    (b) .076

    (c) .76



6.3.3 **Spatially Arranged Fractions.** When there is special need, such as introduction to fractions, a fraction may be represented spatially. See **13.10.2**. An English-letter indicator is required when a single letter is the numerator or denominator of a spatially arranged fraction. *Exception*: Spatially arranged fractions within a determinant or matrix do not use an English letter-indicator. See **6.4.4**.

Example 6-11: Single Letter Numerator in a Spatial Fraction

$$\frac{x}{xy}$$

⠸⠩

⠀⠰⠭
⠹⠒⠒⠼
⠀⠭⠽

⠸⠱

6.3.4 **Nonregular Type.** An English-letter indicator is required for an English letter in nonregular type. (See also **Rule 7, Typeforms**.) The English-letter indicator directly precedes the letter; if the letter is capitalized, the indicator precedes the capitalization indicator.

Example 6-12: English Letters in Several Typeforms

AB

⠸⠩⠀⠸⠰⠠⠁⠸⠰⠠⠃⠀⠸⠱ (bold English A and B)

ab

⠸⠩⠀⠨⠰⠁⠨⠰⠃⠀⠸⠱ (italic English a and b)

$\mathcal{ef}$

⠸⠩⠀⠈⠰⠑⠈⠰⠋⠀⠸⠱ (script English e and f)

Example 6-13: Bold X (Nemeth) vs. Regular x (UEB)

X is a vector; x is a scalar.

⠀⠀⠸⠩⠀⠸⠰⠠⠭⠀⠸⠱⠀⠊⠎⠀⠁⠀⠧⠑⠉⠞⠕⠗⠆⠀⠰⠭⠀⠊⠎⠀⠁⠀⠎⠉⠁⠇⠜⠲

6.4 Non-Use of the English-Letter Indicator

6.4.1 **Letter Combinations.** The English-letter indicator is not used with an unspaced combination of letters unless typeform is being applied. See **7.2**.

Example 6-14: Letter Combinations

ab is not equal to cd.

⠀⠀⠸⠩⠀⠁⠃⠀⠸⠱⠀⠊⠎⠀⠝⠀⠑⠟⠥⠁⠇⠀⠞⠕⠀⠸⠩⠀⠉⠙⠀⠸⠱⠲

6.4.2 **Function Names.** An English-letter indicator is not used with a letter that follows a function name or its abbreviation.

Example 6-15: Letter Following an Abbreviated Function Name

cos A

⠸⠩⠀⠉⠕⠎⠀⠠⠁⠀⠸⠱

Example 6-16: Letter Following Function Name

cosine A

⠸⠩⠀⠉⠕⠎⠊⠝⠑⠀⠠⠁⠀⠸⠱

Example 6-17: Letter Following an Abbreviated Function Name

$e^{\sin x}$

⠸⠩⠀⠑⠘⠎⠊⠝⠀⠭⠀⠸⠱

6.4.3 **Shapes.** An English-letter indicator is not used with a letter following a sign of shape provided that the sign of shape does not have a plural or possessive ending. The English-letter indicator is not used with a letter which precedes a sign of shape when that sign of shape is also a sign of omission.

Example 6-18: Letter Following a Sign of Shape

$\angle$ C

⠸⠩⠀⠫⠪⠀⠠⠉⠀⠸⠱

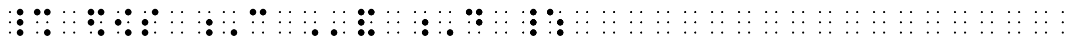
Example 6-19: Shape as Sign of Omission

x □ y

⠸⠩⠀⠭⠀⠫⠲⠀⠽⠀⠸⠱

Example 6-20: Sign of Shape with Plural Ending

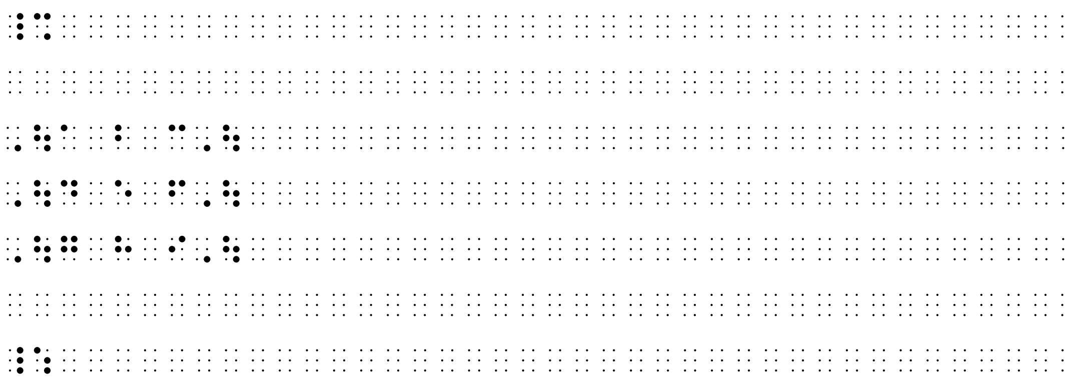
$\angle$s C and D



6.4.4 **Arrays.** The English-letter indicator is not used with a letter in a determinant or matrix.

Example 6-21: Single Letters in a Determinant

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$$



Example 6-22: Single-Letter Numerator in a Determinant

$$\begin{vmatrix} \dfrac{d}{du} & \dfrac{d}{dv} \\ \dfrac{d}{dx} & \dfrac{d}{dy} \end{vmatrix}$$

```
⠸⠩

⠠⠳⠀⠙⠀⠀⠀⠀⠙⠀⠀⠀⠠⠳
⠠⠳⠹⠒⠒⠼⠀⠹⠒⠒⠼⠠⠳
⠠⠳⠀⠙⠥⠀⠀⠀⠙⠧⠀⠀⠠⠳
⠠⠳⠀⠀⠀⠀⠀⠀⠀⠀⠀⠠⠳
⠠⠳⠀⠙⠀⠀⠀⠀⠙⠀⠀⠀⠠⠳
⠠⠳⠹⠒⠒⠼⠀⠹⠒⠒⠼⠠⠳
⠠⠳⠀⠙⠭⠀⠀⠀⠙⠽⠀⠀⠠⠳

⠸⠱
```

6.4.5 **Enclosed Lists.** The English-letter indicator is not used with a single letter which is an item in an enclosed list. For the definition of an enclosed list see **3.5**.

Example 6-23: Single Letters in an Enclosed List

(a, 2x, b)

```
⠸⠩⠀⠷⠁⠠⠀⠆⠭⠠⠀⠃⠾⠀⠸⠱
```

Example 6-24: Single Letters in an Enclosed List

(0, a, 1, b, 2)

```
⠸⠩⠀⠷⠴⠠⠀⠁⠠⠀⠂⠠⠀⠃⠠⠀⠆⠾⠀⠸⠱
```

Example 6-25: Single Letters in Set Notation

{a, b, c, d}

```
⠸⠩⠀⠨⠷⠁⠠⠀⠃⠠⠀⠉⠠⠀⠙⠨⠾⠀⠸⠱
```

6.4.6 **Plurals.** An English-letter indicator is not used for the letter "s" in plural forms or when it is part of the apostrophe-s combination.

Example 6-26: Plural Form of "Triangles"

$\triangle$ ABC and A'B'C' are similar.

⠀⠀⠸⠩ ⠫⠞⠎ ⠠⠁⠠⠃⠠⠉ ⠠⠄⠯ ⠠⠁⠄⠠⠃⠄⠠⠉⠄ ⠸⠱ ⠜⠑
⠎⠊⠍⠊⠇⠜⠲

Example 6-27: Apostrophe-s

Find the sum of the n ∠'s.

⠀⠀⠠⠋⠔⠙ ⠮ ⠎⠥⠍ ⠷ ⠮ ⠰⠝ ⠸⠩ ⠫⠪⠸⠄⠎ ⠸⠱⠲

6.4.7 **Comparison Sign.** An English-letter indicator is not used with a letter which is preceded or followed by a comparison sign.

Example 6-28: Letters Next to a Comparison Sign

"x = y"

⠸⠩ ⠦⠭ ⠨⠅ ⠽⠸⠴ ⠸⠱

Example 6-29: Letter Next to a Comparison Sign

i = 1, 2, …, n

⠸⠩ ⠊ ⠨⠅ ⠼⠂⠠ ⠼⠆⠠ ⠄⠄⠄⠠ ⠰⠝ ⠸⠱

Example 6-30: Letters Next to a Comparison Sign

(a, 2x, y = z)

⠸⠩ ⠷⠰⠁⠠ ⠼⠆⠭⠠ ⠽ ⠨⠅ ⠵⠾ ⠸⠱

Example 6-31: Letters Next to a Comparison Sign

a = b

⠸⠩ ⠁ ⠨⠅ ⠃ ⠸⠱

Example 6-32: Letters Next to a Comparison Sign

If $a = b$, then $ac = bc$.

Example 6-33: Letter Next to a Comparison Sign

$\{x \mid x$ has the property $R\}$

Example 6-34: Letters Next to a Comparison Sign

$a = b$, but $c \neq b$.

Example 6-35: Letter Next to a Comparison Sign

30% of $N = 63$

Example 6-36: Letter Next to a Comparison Sign

In $x = 5$, x is the unknown.

Example 6-37: Letter Next to a Comparison Sign

For some value of s, $d = st$.

Example 6-38: Letter Next to a Comparison Sign

a and b are integers, $b \neq 0$.

Example 6-39: Letter Next to a Comparison Sign

$e \times e =$ e-squared

Example 6-40: Letter Next to a Comparison Sign

Solve for x (x > y).

Example 6-41: Letter Not Next to a Comparison Sign

"x" = "y"

(the quotation marks separate the letters from the comparison sign)

6.4.8 An English-letter indicator is not used for a single letter which is in contact with both left and right signs of grouping.

Example 6-42: Letter Enclosed in Signs of Grouping

7. Write each number as a fraction.

(a) 0.06

(b) 0.7

(c) 0.008

Example 6-43: Letter Enclosed in Signs of Grouping

$|x|, [x], \|f\|$

6.4.9 When a letter is in direct contact with only its left or right grouping sign, the English-letter indicator is used or not used as though the grouping signs are not present. If the grouping sign has a prime, subscript, or superscript, the English-letter indicator is not used.

Example 6-44: Letter Next to a Sign of Grouping

P(A or B) = P(A) + P(B) - P(A and B)

(in probability notation, the words "or" and "and" are a mathematical part of the expression so are transcribed in Nemeth Code without using the single-word switch indicator)

Example 6-45: Letter Next to Right Sign of Grouping

(j = 1, 2, …, n)

Example 6-46: Grouping Sign with Subscript/Superscript

$R]_a^b$

6.4.10 When a single letter has a plural, possessive, or ordinal ending, the English-letter indicator is used or not used as though such endings were not present.

Example 6-47: Single letter with Possessive Ending

△ C's measurement

6.4.11 The English-letter indicator is not used with any letter which is not a single letter in situations not specifically covered in section **6.4**.

Example 6-48: English Modified Letter

$x', x'', x_1, x_a, x^2, \bar{x}$

Example 6-49: Not a Single Letter

x + y and –a

Example 6-50: Not a Single Letter

x%

Example 6-51: Letter Following a Sign of Shape

m∠B

Example 6-52: Series of Modified and Unmodified Letters

If n, n_1, n_2, …

6.4.12 **Chemical Elements.** The English-letter indicator is not used with a single-letter chemical element symbol such as C, H, or O. The element symbols are transcribed in Nemeth Code.

Example 6-53: Chemical Symbol

H stands for hydrogen.

6.5 Roman Numerals in Mathematical Context

Roman numerals follow the same rules as any other English letter. The English-letter indicator is used or is not used in accordance with the rules for any letter (see **6.3-6.4**). When a letter indicator is required, a lowercase Roman numeral is treated as consisting of one letter even when it is formed with more than one letter.

6.6 Letters in Diagrams

When a single English letter in regular type is used as a label in a diagram, the English-letter indicator is required if the letter is in lowercase, but is omitted if the letter is capitalized. See the latest edition of *Guidelines and Standards for Tactile Graphics*.

6.7 Letters in Tables

When letters appear in tables, whether as entries or headings, the English-letter indicator is used or is not used in accordance with the rules contained in **6.3-6.4**.

Rule 7
Typeforms

Typeform Indicators for Letters and Numerals	
Boldface Type	⠸
Italic Type	⠨
Sans Serif Type	⠠⠨
Script Type	⠈
Barred Type (Blackboard or Double Struck)	⠠⠸
Typeform Indicators for Words, Phrases, and Mathematical Expressions	
Boldface Type for a single word	⠸
Italic Type for a single word	⠨
Boldface Italic Type for a single word	⠸⠨
Opening Boldface Type for two or more words	⠠⠠⠸ (followed by a space)
Opening Italic Type for two or more words	⠠⠠⠨ (followed by a space)
Closing Boldface Type for two or more words	⠸⠠⠠ (preceded by a space)
Closing Italic Type for two or more words	⠨⠠⠠ (preceded by a space)
Typeform Terminator	⠠⠠

(For combinations of capitalization, alphabetic, and typeform indicators, see **Appendix C**)

Note: The following rules have been developed for transcription when it has been determined that the typeform shown in print is essential to the meaning of mathematical or technical expressions. Typeforms representing only emphasis within the switches are ignored in braille.

7.1 Typeforms

UEB typeform indicators are used in the non-technical text. Nemeth typeform indicators are used only if it is necessary to indicate distinction within the Nemeth Code switches. UEB typeform is terminated by the opening Nemeth indicator.

Specific provision is made in this Code for six typeforms: boldface, italic, regular, sans serif, script, and barred (double struck). Except for regular type, these typeforms are specified by the appropriate typeform indicator.

7.2 Use of Typeform Indicators with Letters and Numerals

7.2.1 **Letters.** Subject to the provisions of **7.4**, if typeform is retained, the appropriate typeform indicator is used to express the typeform of a letter. The typeform indicator for a letter must always be followed by an alphabetic indicator. When a typeform indicator is used with letters, its effectiveness extends only to the letter which follows it. Thus, except for regular type, a typeform indicator is used with each individual letter of a sequence of letters.

Example 7-1: Single Letter in Italic Type

t

⠸⠩ ⠨⠰⠞ ⠸⠱

Example 7-2: Single Letter in Barred Type

$\mathbb{R}$

⠸⠩ ⠠⠸⠰⠠⠗ ⠸⠱

Example 7-3: Letters in Boldface Type

j	⠸⠩⠀⠸⠰⠚⠀⠸⠱	
T	⠸⠩⠀⠸⠰⠠⠞⠀⠸⠱	
curl **F**	⠸⠩⠀⠉⠥⠗⠇⠀⠸⠰⠠⠋⠀⠸⠱	(**F** is a vector)
α	⠸⠩⠀⠸⠨⠁⠀⠸⠱	(boldface Greek alpha)
αβ	⠸⠩⠀⠸⠨⠁⠸⠨⠃⠀⠸⠱	(boldface Greek alpha beta)
ш	⠸⠩⠀⠸⠈⠈⠱⠀⠸⠱	(boldface Russian sha)

Example 7-4: Single Letters in Script Type

$\mathscr{a}$	⠸⠩⠀⠈⠰⠁⠀⠸⠱	(script English a)
$\mathscr{A}$	⠸⠩⠀⠈⠰⠠⠁⠀⠸⠱	(script uppercase English A)

Example 7-5 Single Letters in Sans Serif Type

h	⠸⠩⠀⠠⠨⠰⠓⠀⠸⠱	(sans serif English h)
H	⠸⠩⠀⠠⠨⠰⠠⠓⠀⠸⠱	(sans serif English H)

Example 7-6: Mixed Typeforms with Letters

a*b***c**$\mathscr{d}$e

⠸⠩⠀⠁⠸⠰⠃⠸⠰⠉⠈⠰⠙⠑⠀⠸⠱

(regular English a, italic b, boldface c, script d, regular e)

7.2.2 **Numerals.** Subject to the provisions of **7.4**, if typeform is retained, the appropriate typeform indicator is used to express the typeform of a numeral. When a typeform indicator is used with numerals only, it is effective until terminated by a space, a numeric indicator, the typeform termination indicator or another typeform indicator.

The typeform indicator for a numeral is always followed by the numeric indicator. If a numeral to be transcribed by using typeform indicators contains more than one digit, and is all of one typeform, the typeform indicator and the numeric indicator are used only before the first digit. If there is a transition from one typeform to another non-regular typeform within the same numeral, the new typeform indicator followed by the numeric indicator is used before the first digit of the new typeform. If the transition is to regular type, only the numeric indicator is used. In UEB context, a freestanding numeral, where there is no mathematical meaning to the typeform, may be transcribed in UEB using UEB typeform indicators. **Note:** A numeral composed of digits of more than one typeform is transcribed in Nemeth Code. *The examples that follow are assumed to be in technical context.*

Example 7-7: Numeral in Boldface Type

vector **0**

Example 7-8: Numeral in Script Type

2

Example 7-9: Three-Digit Number in Boldface Type

345

Example 7-10: Decimal Number in Italic Type

3.5

Example 7-11: Mixed Typeforms with Numerals

*3***4***5*

Example 7-12: Mixed Typeforms with Numerals

435

7.3 Use of Typeform Indicators with Words, Phrases, and Mathematical Expressions

7.3.1 **Typeform Retained.** Typeform is retained for mathematical material if it is mathematically significant or is required for instructional purposes.

7.3.2 **One Word in Non-Regular Typeform.** Use the appropriate typeform indicator for a single word in mathematical context.

Example 7-13: Word in Italic Type

6 *pieces* of pizza ÷ 2 people = 3 *pieces* per person

7.3.3 A hyphen does not terminate typeform in a hyphenated expression. If there is a change in typeform after the hyphen to regular type, the hyphen must be preceded by the typeform termination symbol. If there is a change in typeform after the hyphen to non-regular type, only the appropriate typeform indicator must be used after the hyphen.

Example 7-14: Hyphenated Expression in Italic Type

4.5-ohm

Example 7-15: Hyphenated Expression in Boldface Type

4.5-ft

Example 7-16: Hyphenated Expression in Two Typeforms

4.5-ohm

Example 7-17: Hyphenated Expression in Two Typeforms

*4.5-**ohm***

7.3.4 **More Than One Word in Non-Regular Typeform.** Use the appropriate typeform indicator for a phrase of two or more words.

Example 7-18: Phrase in italics

6 pieces of pizza ÷ 2 people = 3 *pieces per person*

7.3.5 **A Mathematical Expression in Boldface or Italic Typeform.** Use open and close boldface or italic typeform indicators for an expression containing mathematical symbols in bold or italic type.

Example 7-19: Expression in Boldface Type

The **median** score was only **59%**.

(retain if typeform of a math expression is considered to be significant)

7.4 Non-Use of Typeform Indicators

7.4.1 A typeform indicator is not used when a letter or numeral is printed in regular type.

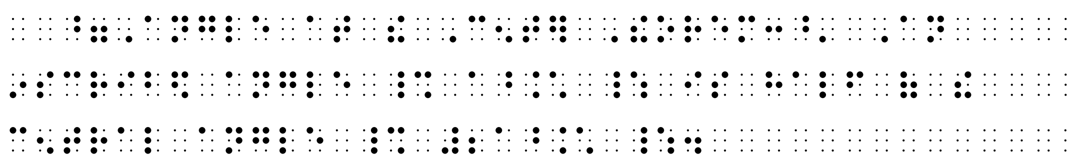
7.4.2 When technical material is printed in non-regular type that has no mathematical significance or instructional purpose, the variant typeform is not represented in the transcription. A variant typeform is often used, particularly at the lower grade levels, for the sole purpose of attracting the reader's attention. Such variant typeforms are not represented in the transcription.

7.4.3 Frequently the letters which represent variables, constants, and formulas throughout a book are printed in italicized type. This practice is not carried over to the transcription unless the author has specifically distinguished between two meanings of the same letter, assigning one meaning to the letter in regular type and another to the letter in italic type.

7.4.4 In mathematical statements such as Lemmas, Theorems, Definitions, Corollaries, Axioms, to avoid excessive use of typeform indicators in the body of the statement, omit the typeform indication when all such statements in the text are in the same typeform unless doing so would change the meaning of the statement. Follow print for the capitalization and typeform of the label. When print shows the label as fully capitalized and emphasized, retain the capitalization and ignore the typeform.

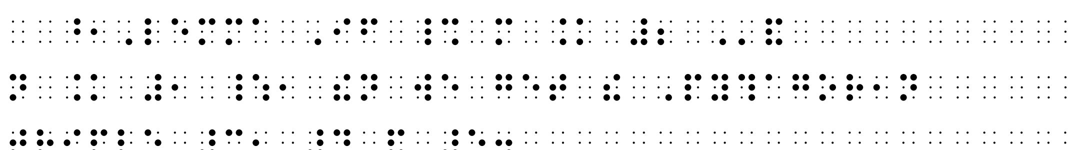
Example 7-20: Bold Label; Italicized Statement

Angle at the Center Theorem: *An inscribed angle a° is half of the central angle 2a°.*



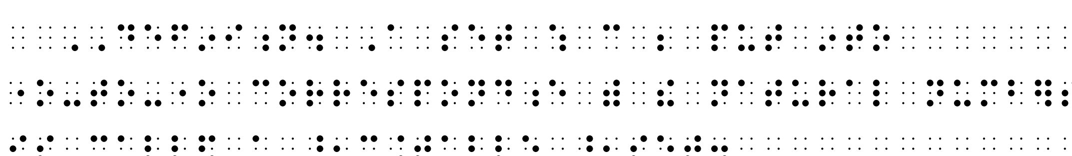
Example 7-21: Bold Label; Italics Disregarded in the Statement

Lemma *If m = 2 and n = 1, then we get the Pythagorean triple 3, 4 and 5.*



Example 7-22: Uppercase Bold Label; Italicized Statement

DEFINITION. *A set which can be put into one-to-one correspondence with the natural numbers is called a* ***countable set****.*



Example 7-23: Math Expression within Boldface Phrase

In this equation, ***ab* = *ba***, **the operation is commutative**.

⠀⠀⠘⠶⠠⠔⠀⠹⠀⠑⠟⠥⠁⠰⠝⠂⠀⠸⠩⠀⠁⠃⠀⠨⠅⠀⠃⠁⠀⠸⠱⠂⠀⠘⠶⠮⠀⠀⠀⠀

⠕⠏⠻⠁⠰⠝⠀⠊⠎⠀⠉⠕⠍⠍⠥⠞⠁⠞⠊⠧⠑⠲⠘⠄⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀

(typeform in the equation is disregarded when it is not mathematically significant)

7.5 Symbols in Boldface Type

7.5.1 **Restricted Use.** When certain signs are printed in boldface type, this Code employs the device of placing ⠸ before the corresponding symbol. The specific signs to which this technique applies are listed in the appropriate sections (**Rule 20** Signs and Symbols of Operation, **Rule 21** Signs and Symbols of Comparison, and **Rule 19** Signs and Symbols of Grouping). The transcriber must not use this technique with any other sign. In such cases, dots 456 is not regarded as the boldface typeform indicator but as an integral part of the symbol to which it belongs. The technique is only used when the distinction between the regular and boldface forms of the same sign has mathematical significance.

7.5.2 **Vectors.** Boldface type, used in many texts to identify letters as vectors, is preserved in the transcription. When both boldface type and arrows of uniform construction are used in conjunction to represent vectors, the arrows themselves are omitted from the transcription unless the author calls special attention to them as a notational device. A transcriber's note is included indicating their presence in the print copy.

Sample transcriber's note:

> The arrows above boldface vectors in print are omitted in braille.

Example 7-24: Arrow Omitted Above Vector Notation

On the Transcriber's Notes page:

⠀⠀⠀⠀⠀⠀⠀⠀⠠⠠⠞⠗⠁⠝⠎⠉⠗⠊⠃⠻⠄⠠⠎⠀⠠⠠⠝⠕⠞⠑⠎⠀⠀⠀⠀⠀⠀⠀⠀⠀

⠀⠀

⠀⠀⠠⠮⠀⠜⠗⠪⠎⠀⠁⠃⠧⠀⠃⠕⠇⠙⠋⠁⠉⠑⠀⠧⠑⠉⠞⠕⠗⠎⠀⠔⠀⠏⠗⠔⠞⠀⠜⠑

⠕⠍⠊⠞⠞⠫⠀⠔⠀⠃⠗⠇⠲⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀

The elements of $\vec{\boldsymbol{E}}$ are called translations.

⠀⠀⠠⠮⠀⠑⠇⠑⠰⠞⠎⠀⠷⠀⠸⠩⠀⠸⠰⠠⠑⠀⠸⠱⠀⠜⠑⠀⠉⠁⠇⠇⠫⠀⠀⠀⠀⠀⠀⠀

⠞⠗⠁⠝⠎⠇⠁⠰⠝⠎⠲⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀

Rule 8
Punctuation Signs and Symbols

Punctuation Indicator		⠸
Punctuation Marks		
Apostrophe	'	⠄
Colon	:	⠒
Comma		
Literary (used with words and abbreviations)	,	⠂
Mathematical	,	⠠
Dash		
Short	_	⠤⠤
Long	___	⠤⠤⠤⠤
Ellipsis	...	⠄⠄⠄
Exclamation Point	!	⠖
Hyphen	-	⠤
Period	.	⠲
Question Mark	?	⠦
Quotation Marks		
Opening double	"	⠦
Closing double	"	⠴
Opening single	'	⠠⠦
Closing single	'	⠠⠴
Opening directional	“	⠦
Closing directional	”	⠴

Left-pointing double angle « ⠦

Right-pointing double angle » ⠴

Semicolon ; ⠆

8.1 Modes of Punctuation

Since numerals are represented by symbols in the lower part of the cell, and since these symbols also serve as punctuation marks, it is necessary to formulate rules concerning punctuation so that the meanings of such symbols are unambiguous. This Code employs two modes of punctuation — mathematical and literary.

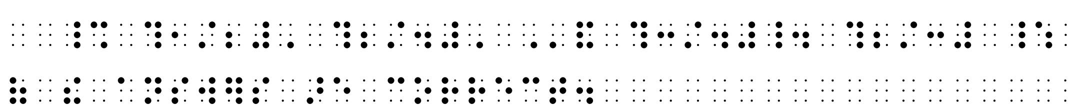
8.2 Use of the Punctuation Indicator

The punctuation indicator is used *before* a punctuation mark and *after* any symbol of the type listed below *when Nemeth mode has not been terminated*. In all these circumstances, the mode of punctuation is considered to be mathematical. (In some of the following examples, an ellipsis indicates that the technical material continues.)

8.2.1 The punctuation indicator is used after any braille indicator.

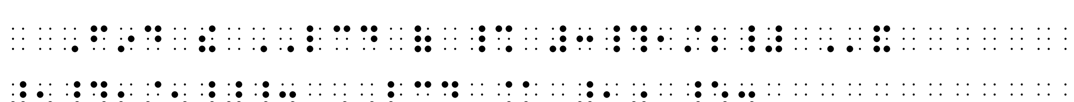
Example 8-1: Closing Fraction Indicator

$\frac{1}{2}$, $\frac{2}{4}$, and $\frac{3}{4}$. $\frac{2}{3}$ of the answers are correct.



Example 8-2: Closing Mixed Number Indicator

Find the LCD of $3\frac{1}{2}$ and $5\frac{2}{5}$. LCD = 10.



Example 8-3: Modification Terminator

$\overline{\text{velocity}}$.

⠸⠩⠀⠐⠧⠑⠇⠕⠉⠊⠞⠽⠣⠱⠻⠸⠲⠀⠄⠄⠄⠀⠸⠱

(the ellipsis indicates that technical material continues following the period)

8.2.2 The punctuation indicator is used after any numeric symbol written inside the switches.

Example 8-4: Decimal Number Followed by a Period

98.6.

⠸⠩⠀⠼⠔⠦⠨⠖⠸⠲⠀⠄⠄⠄⠀⠸⠱

(the ellipsis indicates that technical material continues following the period)

Example 8-5: Closing Quotation Mark Inside the Switches

"4.9"

⠸⠩⠀⠦⠼⠲⠨⠔⠸⠴⠀⠄⠄⠄⠀⠸⠱

(the ellipsis indicates that technical material continues following the period)

8.2.3 The punctuation indicator is used after a Roman numeral written inside the switches.

Example 8-6: Nemeth Roman Numeral Followed by a Period

I + II = III.

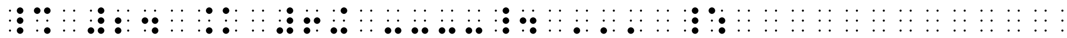
⠸⠩⠀⠠⠊⠬⠠⠠⠊⠊⠀⠨⠅⠀⠠⠠⠊⠊⠊⠸⠲⠀⠄⠄⠄⠀⠸⠱

(the ellipsis indicates that technical material continues following the period)

8.2.4 The punctuation indicator is used after a long dash or ellipsis, when these occur in a mathematical context.

Example 8-7: Period Following a Dash

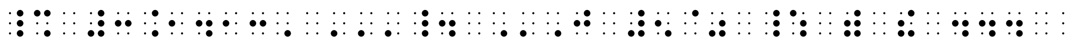
24 = 6 + __.



(the ellipsis indicates that technical material continues following the period)

Example 8-8: Period Following an Ellipsis

3.1413, ... Just 5% of the …



(first ellipsis indicates that technical material continues following the period)

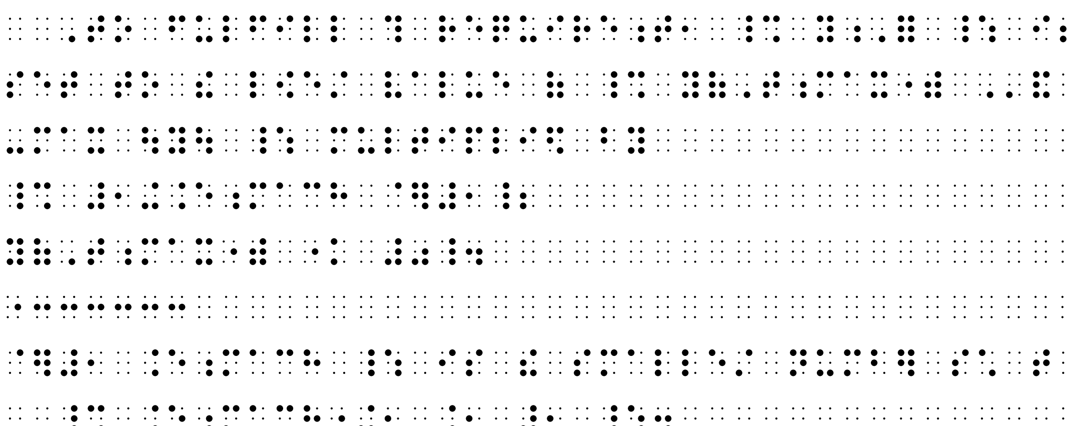
8.2.5 The punctuation indicator is used after any Nemeth reference symbol.

Example 8-9: Semicolon Following a Reference Indicator

To fulfill this requirement, y_{∞} is set to the lowest value of $y(T_{\max})$ and $-\max|y|$ multiplied by $1+\varepsilon_{mach}$[1]; $y(T_{\max})<0$.

---

[1] ε_{mach} is the smallest number such that $\varepsilon_{mach}+1>1$.



8.2.6 The punctuation indicator is used after the general omission symbol.

Example 8-10: Period Following a General Omission Symbol

$5 \times 3 = ?.$

⠸⠩⠀⠼⠢⠈⠡⠒⠀⠨⠅⠀⠿⠸⠲⠀⠄⠄⠄⠀⠸⠱

(the ellipsis indicates that technical material continues following the period)

8.2.7 The punctuation indicator is used after a "single letter".

Example 8-11: Period Following a Single Letter

a-b, c.

⠸⠩⠀⠁⠤⠃⠠⠀⠰⠉⠸⠲⠀⠄⠄⠄⠀⠸⠱

(the ellipsis indicates that technical material continues following the period)

Example 8-12: Period Following a Letter

$27 - 3 = y.\ y = 24$

⠸⠩⠀⠼⠆⠶⠤⠒⠀⠨⠅⠀⠽⠸⠲⠀⠽⠀⠨⠅⠀⠼⠆⠲⠀⠸⠱

8.2.8 The punctuation indicator is used after a sequence of more than one letter in which each letter has a separate identity, provided that such a sequence is not an abbreviation.

Example 8-13: Period Following a Three-Letter Sequence

$\triangle ABC.$

⠸⠩⠀⠫⠞⠀⠠⠁⠠⠃⠠⠉⠸⠲⠀⠄⠄⠄⠀⠸⠱

(the ellipsis indicates that technical material continues following the period)

8.2.9 The punctuation indicator is used after any word or abbreviation which is not on the baseline, if the punctuation which follows is on the baseline.

Example 8-14: Period Following a Subscript Word

$13_{\text{seven}}.$

⠸⠩⠀⠼⠂⠒⠰⠎⠑⠧⠑⠝⠸⠲⠀⠄⠄⠄⠀⠸⠱

(the ellipsis indicates that technical material continues following the period)

8.2.10 The punctuation indicator is used after any modified expression.

Example 8-15: Period Following a Contracted Bar Modifier

$\bar{x}$.

(the ellipsis indicates that technical material continues following the period)

8.2.11 The punctuation indicator is used after the radical symbol.

Example 8-16: Closing Quotation Mark Following a Radical Sign

"$\sqrt{\ }$ " means "square root."

8.2.12 The punctuation indicator is used after any symbol of shape or shape modification, operation, or comparison.

Example 8-17: Closing Quotation Mark Following a Triangle

"□ + ○ = △"

(quotation marks enclosing a math expression may be transcribed as part of the expression)

Example 8-18: Closing Quotation Mark Following a Plus Sign

The "+" is used for addition.

Example 8-19: Quotation Mark Following Comparison Signs

Real numbers may be compared by "<", "=", or ">".

8.2.13 The punctuation indicator is used after any symbol of grouping whether transcribed or drawn in.

Example 8-20: Quotation Marks and Parentheses

(".8").
⠸⠩ ⠷⠸⠦⠼⠨⠦⠸⠴⠾⠸⠲ ⠄⠄⠄ ⠸⠱
(the ellipsis indicates that technical material continues following the period)

Example 8-21: Period Following Closing Parenthesis

(five + seven).
⠸⠩ ⠷⠋⠊⠧⠑⠬⠎⠑⠧⠑⠝⠾⠸⠲ ⠄⠄⠄ ⠸⠱
(the ellipsis indicates that technical material continues following the period)

8.2.14 The punctuation indicator is used after any abbreviated function name.

Example 8-22: Quote Marks After Abbreviated Function Names

"sin" and "cos" are circular functions.
⠀⠀⠸⠩ ⠦⠎⠊⠝⠸⠴ ⠠⠄⠯ ⠦⠉⠕⠎⠸⠴ ⠸⠱ ⠜⠑ ⠉⠊⠗⠉⠥⠇⠜
⠋⠥⠝⠉⠰⠝⠎⠲

8.2.15 The punctuation indicator is used after any of the miscellaneous symbols of **Rule 23**.

Example 8-23: Period Following a Percent Symbol

100%.
⠸⠩ ⠼⠂⠴⠴⠈⠴⠸⠲ ⠄⠄⠄ ⠸⠱
(the ellipsis indicates that technical material continues following the period)

Example 8-24: Period Following the Cent Symbol

The cost was 48¢. 16¢ × 3 = 48¢.
⠀⠀⠠⠮ ⠉⠕⠌ ⠴ ⠸⠩ ⠼⠲⠦⠈⠉⠸⠲
⠼⠂⠖⠈⠉⠈⠡⠒ ⠨⠅ ⠼⠲⠦⠈⠉ ⠸⠱⠲

8.2.16 The punctuation indicator is used after a comma, hyphen, or dash, provided that if these were removed and the space which they occupy were not present, one of the conditions **8.2.1-8.2.15** would apply.

Example 8-25: Closing Quotation Mark Following a Comma

3y,"

⠸⠩ ⠼⠒⠽⠠⠸⠴ ⠄⠄⠄ ⠸⠱

(the ellipsis indicates that technical material continues following the period)

Example 8-26: Closing Quotation Mark Following a Dash

(—".2")

⠸⠩ ⠷⠤⠤⠸⠦⠼⠨⠆⠸⠴⠾ ⠄⠄⠄ ⠸⠱

(the ellipsis indicates that technical material continues following the period)

8.3 Non-Use of the Punctuation Indicator

It must not be assumed that because a punctuation mark occurs that the punctuation indicator must be used. The punctuation indicator is not used under any of the circumstances listed below.

8.3.1 The punctuation indicator is not used at the beginning of a braille line or after a space.

Example 8-27: Quotation Mark at the Beginning of a Line

"2.4" is a decimal number.

⠸⠩ ⠦⠼⠆⠨⠲⠸⠴ ⠸⠱ ⠊⠎ ⠁ ⠙⠑⠉⠊⠍⠁⠇ ⠝⠥⠍⠃⠻⠲

8.3.2 The punctuation indicator is not used after a word or abbreviation provided that the punctuation is at the same level as that word or abbreviation.

Example 8-28: Period After an Abbreviation

1/4 c.

⠸⠩ ⠼⠂⠸⠌⠲ ⠸⠉⠲ ⠸⠱

Example 8-29: Period After an Abbreviation

2 mi./min.

Example 8-30: Period and Comma After an Abbreviation

{Wed., Thurs., Fri.}

(set notation as indicated by the braces is transcribed in Nemeth Code)

Example 8-31: Period After an Abbreviation in Subscript

$\triangle_{\text{reg. polygon}}$

Example 8-32: Period After an Abbreviation

$\frac{1}{2}$-ft.

Example 8-33: Period After a Word

rate $\times$ time.

(the ellipsis indicates that technical material continues following the period)

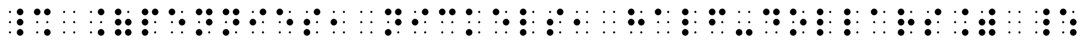
8.3.3 The punctuation indicator is not used before a comma, hyphen, dash, or ellipsis.

Example 8-34: Before a Comma Following a Number

(0, 1, 2)

Example 8-35: Before a Comma Following a Word

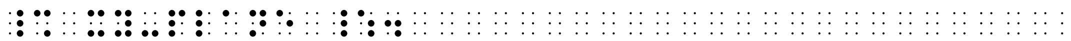
{pennies, nickels, half-dollars}



(set notation is transcribed in Nemeth Code)

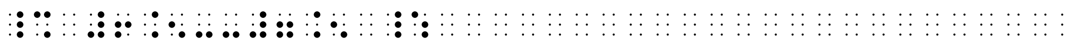
Example 8-36: Before a Hyphen Following a Letter Combination

xy-plane.



Example 8-37: Before a Short Dash Following a Number

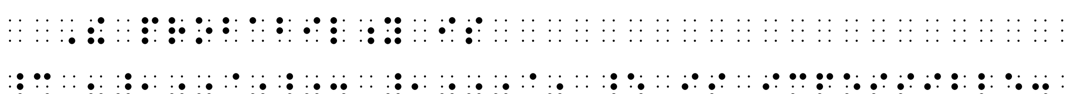
6.5–7.5



8.3.4 The punctuation indicator is not used before any additional punctuation marks in a sequence of punctuation marks. Only the first punctuation mark in the sequence requires a punctation indicator.

Example 8-38: Before a Period Following a Quotation Mark

The probability is "100%". 1000% is impossible.

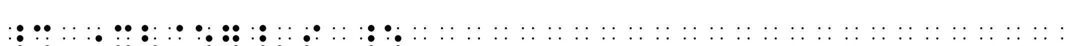


8.4 Plural and Possessive Endings

The letter s or the apostrophe-s combination may be joined to numerals, letters, and other mathematical expressions to form their plurals or possessives.

Example 8-39: Plural Math Symbol (Apostrophe-s Added)

x̃’s



(the plural of x with a superscribed tilde)

Example 8-40: Plural Math Symbol (Apostrophe-s Added)

$\bar{x}$'s

Example 8-41: Plural Shape ("s" Added)

$\angle$s 1 and 2

Example 8-42: Plural Shape ("s" Added)

$\triangle$s ABC and DEF

Example 8-43: Plural Math Symbol (Apostrophe-s Added)

x^2's

(the plural of x squared)

Example 8-44: Plural Math Symbol (Apostrophe-s Added)

c_i's

(the plural of c sub i)

Example 8-45: Plural Math Symbols (Apostrophe-s Added)

The c_1's, c_2's, ..., c_n's.

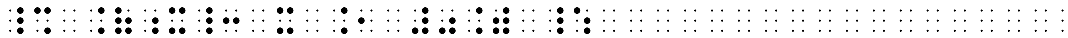
8.5 Colon

A braille colon is not preceded by a space even when used as a sign of comparison. Follow print for spacing after a colon.

Note: Do not misread a ratio sign as a colon. See **Rule 21**.

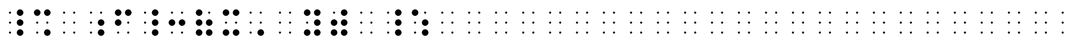
Example 8-46: Colon Meaning "Such That"

$\{x: x > 0\}$



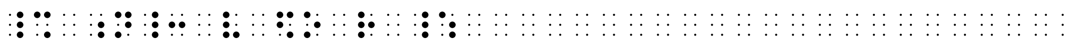
Example 8-47: Unspaced Colon

f:(x, y)



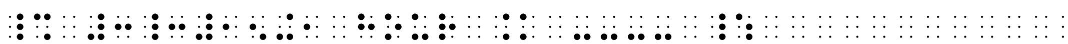
Example 8-48: Colon in Mapping Notation

n : v ⟶ r



Example 8-49: Colon in Digital Time

3:15 + 1 hour = ___



8.6 Comma

8.6.1 **Mathematical Comma.** When a comma is used as a mark of punctuation in a situation in which the mode of punctuation is mathematical, the comma is referred to as the mathematical comma.

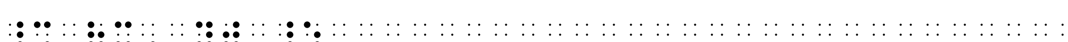
8.6.2 **Spacing.** A space is left following a comma that separates two or more items. *Exception*: See contracted comma in **14.7**. No space is left after a numeric comma within a number except for the purpose of achieving alignment.

Example 8-50: Numeric Comma

$1,000,000

Example 8-51: Comma Between Two Letters

(x, y)



Example 8-52: Comma Between Two Numbers

(-3,2)

(no space after the comma in print)

Example 8-53: Vertical Arrangement

2,375.4

2,	375	.4
thousands	ones	tenths

8.7 Short Dash

Follow print for spacing of the short dash.

8.8 Dash (Long) and Ellipsis

A long dash is represented in braille by four cells of dots 36. An ellipsis is represented in braille by three dot 3s regardless of the number of dots printed. Exceptions apply for matrices or determinants.

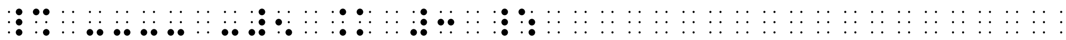
8.8.1 **Spacing and Punctuation.** The long dash and ellipsis are punctuated mathematically. They are preceded and followed by a space unless provisions in **8.8.2** apply.

Example 8-54: Ellipsis

$0.993939... \times 10^{20\text{-}15}$

Example 8-55: Long Dash

___ − 5 = 3



8.8.2 **Spacing Exceptions.** No space is left between the long dash and ellipsis and any of the items listed below, provided these items apply to the long dash or ellipsis.

a. Symbols of punctuation other than the hyphen.

b. Braille indicators.

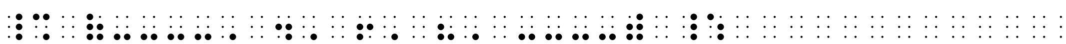
c. Symbols of grouping.

d. Dash internal to a number.

e. Decimal, percent, primes, and monetary symbols.

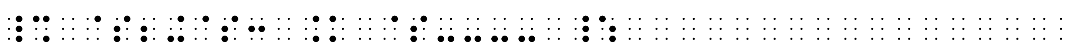
Example 8-56: Grouping Symbol Applies to the Long Dash

(___, 4, 6, 8, ___)



Example 8-57: Dollar Symbol Applies to the Long Dash

$2 + $3 = $_____



Example 8-58: Cent Symbol Applies to the Long Dash

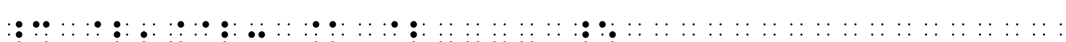
2¢ + 3¢ = _____¢

Example 8-59: Percent Symbol Applies to the Long Dash

2% + 3% = ___%

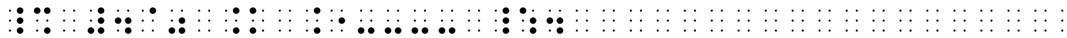
Example 8-60: Pound Symbol Applies to the Long Dash

£2 + £3 = £_____



Example 8-61: Decimal Point Applies to the Long Dash

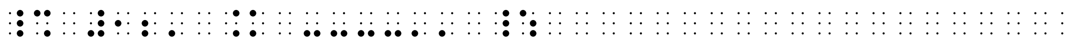
$4\% = .\_\_\_\_.$



(See **Rule 24** for use of the multipurpose indicator.)

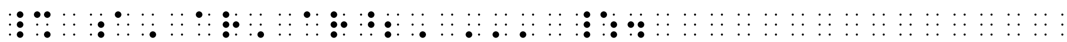
Example 8-62: Double Prime Symbol Applies to the Long Dash

$12' = \_\_\_\_''$



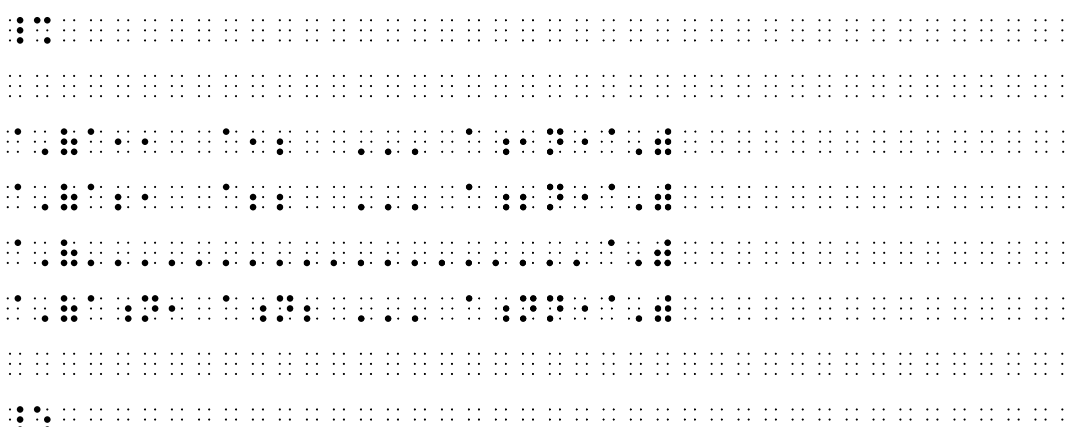
Example 8-63: Ellipsis

$a, ar, ar^2, \ldots .$



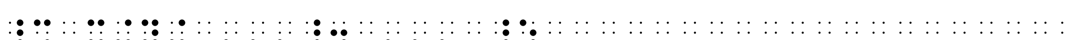
Example 8-64: Ellipses in an Array

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ . \quad . & . \quad . & . \quad . & . \quad . \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$$



Example 8-65: Period Applies to the Ellipsis

$x + y + \ldots.$



Example 8-66: Ellipsis Followed by a Termination Indicator

$\sqrt{a + b + c + ...}$

⠸⠩⠀⠜⠁⠬⠃⠬⠉⠬⠀⠄⠄⠄⠻⠀⠸⠱

Example 8-67: Ellipsis with Related Grouping Symbol

(..., -1, 0, 1, ...)

⠸⠩⠀⠷⠄⠄⠄⠠⠀⠤⠂⠠⠀⠴⠠⠀⠂⠠⠀⠄⠄⠄⠾⠀⠸⠱

Example 8-68: Ellipsis with Related Cent Symbol

12¢ + 14¢ = ... ¢

⠸⠩⠀⠼⠂⠆⠈⠉⠬⠂⠲⠈⠉⠀⠨⠅⠀⠄⠄⠄⠈⠉⠀⠸⠱

Rule 9
Reference Signs, Symbols, and Icons

Asterisk	*	⠈⠼
Dagger		
Single	†	⠸⠻
Double	‡	⠸⠸⠻
General Reference Indicator		⠈⠻
Star	☆	⠫⠎

9.1 Reference Signs and Symbols

Within the Nemeth Code switches, the reference signs of this section must be represented by the symbols listed above and Unified English Braille symbols must not be used. Some of these signs are also used as signs of operation and in that case the rules governing signs of operation apply (see **Rule 20**). When it is certain that a symbol in the above list is to be used for reference purposes, the superscript position, if indicated in print, must be ignored in the transcription.

When a reference sign occurs for which no provision exists in this Code, such as pictures, icons, etc., the transcriber must devise a suitable symbol with an explanatory transcriber's note or a listing in the Special Symbols list. Whether a reference symbol exists in the Nemeth Code or has been devised by the transcriber, such symbols are subject to the rules for signs and symbols of reference.

The reference symbol used with the note must be the same reference symbol used in the body of the text.

9.2 General Reference Indicator

When reference to a footnote is denoted by a numeral or letter, usually in the superscript position, and no other reference sign is employed, the general reference indicator immediately followed by the numeral or letter is used in the transcription. The superscript position is ignored.

Example 9-1: Footnote Denoted by a Letter

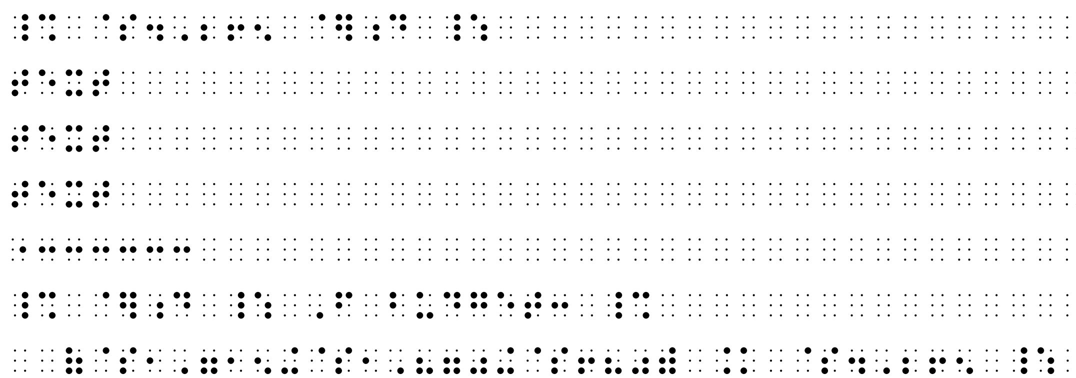
$4,265[d]

text

text

text

[d] From budget: ($1,715+$1,870+$680) = $4,265

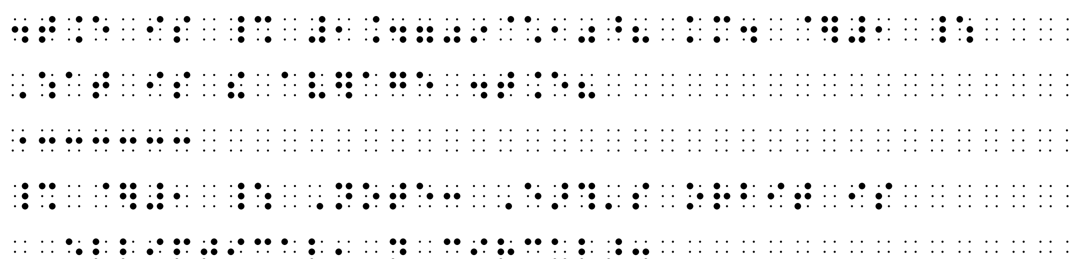


Example 9-2: Footnote Denoted by a Numeral

distance is 1.4709 × 10^8 km.[1] What is the average distance?

----

[1]Note: Earth's orbit is elliptical, not circular.



9.3 Spacing with Symbols of Reference

9.3.1 Follow print for location of reference symbols. If there is punctuation which applies to such a reference, no space should be left between the punctuation mark and the reference symbol to which it is adjacent.

Example 9-3: Dagger Footnote Marker in Several Layouts

† 2.6	⠸⠩⠀⠸⠻⠀⠼⠆⠨⠖⠀⠸⠱⠀⠀⠀⠀⠀⠀
†2.6	⠸⠩⠀⠸⠻⠀⠼⠆⠨⠖⠀⠸⠱⠀⠀⠀⠀⠀⠀
2.6†	⠸⠩⠀⠼⠆⠨⠖⠀⠸⠻⠀⠸⠱⠀⠀⠀⠀⠀⠀
2.6†?	⠸⠩⠀⠼⠆⠨⠖⠀⠸⠻⠸⠦⠀⠄⠄⠄⠀⠸⠱
2.6?†	⠸⠩⠀⠼⠆⠨⠖⠸⠦⠀⠸⠻⠀⠸⠱⠀⠀⠀⠀

(the ellipsis in the fourth example represents additional Nemeth content)

9.3.2 All reference symbols are spaced away from the words, letters, or numbers to which they apply. This is true whether or not they call attention to or introduce a footnote.

Example 9-4: Asterisk Precedes Marked Nemeth Item

1. _____

*2. _____

⠸⠩⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀

⠼⠂⠀⠸⠲⠀⠤⠤⠤⠤⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀

` # :

.

⠼⠆⠀⠸⠲⠀⠤⠤⠤⠤⠀⠸⠱⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀

(asterisk denotes a problem for extra study and is followed by the problem number)

Example 9-5: Asterisk Precedes Punctuation

1. _____

2*. _____

⠸⠩⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀

⠼⠂⠀⠸⠲⠀⠤⠤⠤⠤⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀

⠼⠆⠀⠈⠼⠸⠲⠀⠤⠤⠤⠤⠀⠸⠱⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀

(asterisk denotes a problem for extra study and precedes the period)

Example 9-6: Asterisk Follows Punctuation

1. ____

2. * ____

⠸⠩

⠼⠂⠸⠲⠀⠤⠤⠤⠤

⠼⠆⠸⠲⠀⠈⠼⠀⠤⠤⠤⠤⠀⠸⠱

(asterisk denotes a problem for extra study and follows the period)

9.3.3 **Format.** The placement and margins used for footnotes are subject to the rules for footnotes in the most current edition of *Braille Formats, Principles of Print to Braille Transcription*.

9.4 Icons

Icons that are created using the UEB transcriber-defined shape indicator (⠈⠫) may be used in either UEB or Nemeth context without the insertion of switch indicators.

Example 9-7: Icon

Solve the following problems.

✎ **75**. x^4 - y^2

✎ **76**. $x^2 + 5y – 112$

⠀⠀⠀⠀⠠⠎⠕⠇⠧⠑⠀⠮⠀⠋⠕⠇⠇⠪⠬⠀⠏⠗⠕⠃⠇⠑⠍⠎⠲⠀⠸⠩

⠈⠫⠏⠀⠼⠶⠢⠸⠲⠀⠭⠘⠲⠐⠤⠽⠘⠆

⠈⠫⠏⠀⠼⠶⠖⠸⠲⠀⠭⠘⠆⠐⠬⠢⠽⠤⠂⠂⠆⠀⠸⠱

Rule 10
Abbreviations

10.1 Abbreviations

10.1.1 Definition. Abbreviations must be regarded in a broad sense to include the following items:

a. Universal literary abbreviations of the type commonly listed in a dictionary.

Example 10-1: Literary Abbreviation for "versus"

1/x vs. 2/x

⠸⠩ ⠼⠂⠸⠌⠭ ⠠⠄⠧⠎⠲ ⠼⠆⠸⠌⠭ ⠸⠱

Example 10-2: Literary Initialism for "Side-Angle-Side"

SAS

⠠⠠⠎⠁⠎

Example 10-3: Literary Acronym for "First Outside Inside Last"

FOIL

⠠⠠⠋⠕⠊⠇

Example 10-4: Literary Initialism in Math Context

SD = Standard Deviation

⠸⠩ ⠠⠠⠎⠙ ⠨⠅ ⠠⠎⠞⠁⠝⠙⠁⠗⠙ ⠠⠙⠑⠧⠊⠁⠞⠊⠕⠝ ⠸⠱

b. Abbreviations of measurement, including any superscript or subscript to that abbreviation.

Example 10-5: Measurement Abbreviation for "yard"

1/2 yd.

⠸⠩ ⠼⠂⠸⠌⠆ ⠽⠙⠲ ⠸⠱

Example 10-6: Measurement Abbreviation for "gram"

9.80 g

⠸⠩ ⠼⠔⠨⠦⠴ ⠰⠛ ⠸⠱

Example 10-7: Measurement Abbreviation for "meter"

100.0 m

Example 10-8: Measurement Abbreviation for "square inch"

2.5 sq. inch

Example 10-9: Measurement Abbreviation for "miles per hour"

10.5 m.p.h.

Example 10-10: Measurement Abbreviation for "miles/hour"

60 mi./hr.

Example 10-11: Measurement Abbreviation for "foot-pounds"

6.3 ft.-lbs.

Example 10-12: Measurement Abbreviation for "foot" squared

5 ft^2

10.1.2 Exceptions. When a letter or sequence of letters does not represent a word or phrase, it is not considered to be an abbreviation. Abbreviated function names are also not considered to be abbreviations. When there is doubt as to whether or not a construction is an abbreviation, it must be treated as if it were not.

Example 10-13: Letters Representing Blood Types

Blood types are A, B, AB, and O.

⠀⠀⠠⠃⠇⠕⠕⠙⠀⠞⠽⠏⠑⠎⠀⠜⠑

⠸⠩⠀⠰⠠⠁⠠⠀⠰⠠⠃⠠⠀⠠⠁⠠⠃⠠⠀⠠⠄⠯⠀⠰⠠⠕⠀⠸⠱⠲

or

⠀⠀⠠⠃⠇⠕⠕⠙⠀⠞⠽⠏⠑⠎⠀⠜⠑⠀⠠⠁⠂⠀⠰⠠⠃⠂⠀⠸⠩⠀⠠⠁⠠⠃⠀⠸⠱⠂⠀⠯

⠠⠕⠲

(a series of two or more letters that is not an abbreviation is transcribed in Nemeth)

Example 10-14: Abbreviated Function Name

sin x

⠸⠩⠀⠎⠊⠝⠀⠭⠀⠸⠱

10.2 Capitalization with Abbreviations

A non-mathematical series of numbers/letters, as in a serial number, is transcribed in UEB.

Example 10-15: Non-Mathematical Number/Letter Combination

MP3 player

⠠⠠⠍⠏⠼⠉⠀⠏⠇⠁⠽⠻

Example 10-16: Non-Mathematical Number/Letter Combination

QE2

⠠⠠⠟⠑⠼⠃

Example 10-17: Non-Mathematical Number/Letter Combination

Standard 4NS2

⠠⠌⠯⠜⠙⠀⠼⠙⠠⠠⠝⠎⠼⠃

10.3 Use of the English-Letter Indicator with Abbreviations

The English-letter indicator is used before an abbreviation which consists of one letter whether or not there is an associated period. The use or non-use of the English-letter indicator with abbreviations does not depend upon the braille symbols with which the abbreviation may happen to be in contact, such as grouping symbols, braille indicators, fraction lines, the hyphen, or the slash. **Exception:** The English-letter indicator is not used before a single-letter abbreviation shown with a left superscript or subscript. See example **10-58**.

Example 10-18: Single-Letter Abbreviation with No Period

10 g + 10 g = 20 g

⠸⠩⠀⠼⠂⠴⠀⠰⠛⠀⠬⠂⠴⠀⠰⠛⠀⠨⠅⠀⠼⠆⠴⠀⠰⠛⠀⠸⠱

Example 10-19: Single-Letter Abbreviation with No Period

lat. 30°20' N

⠸⠩⠀⠇⠁⠞⠲⠀⠼⠒⠴⠘⠨⠡⠐⠆⠴⠄⠀⠰⠠⠝⠀⠸⠱

Example 10-20: Single-Letter Abbreviation with No Period

☐ L = 1000 cc

⠸⠩⠀⠫⠲⠀⠰⠠⠇⠀⠨⠅⠀⠼⠂⠴⠴⠴⠀⠉⠉⠀⠸⠱

Example 10-21: Single-Letter Abbreviation with No Period

(.5 m)

⠸⠩⠀⠷⠨⠢⠀⠰⠍⠾⠀⠸⠱

(the presence of the parentheses has no effect upon the decision that the English-letter indicator must be used with the abbreviation)

Example 10-22: Abbreviation with a Superscript

☐ $m^2 = 100\ cm^2$

⠸⠩⠀⠫⠲⠀⠰⠍⠘⠆⠀⠨⠅⠀⠼⠂⠴⠴⠀⠉⠍⠘⠆⠀⠸⠱

(the presence of the superscript indicator has no effect upon the decision that the English-letter indicator must be used)

Example 10-23: Abbreviation in a Fraction

$$\frac{\text{m}}{\text{cm}} \times \frac{\text{cm}}{\text{mm}}$$

(the presence of the fraction indicator and fraction line has no effect upon the decision that the English-letter indicator must be used)

Example 10-24: Single-Letter Abbreviation with No Period

1 km = 1000 m

Example 10-25: Single-Letter Abbreviation with a Period

1 c. + 1 c. = 2 c.

Example 10-26: Single-Letter Abbreviation with a Period

(5 m.)(5 m.) = 25 m.^{2}

(the presence of the parentheses has no effect upon the decision that the English-letter indicator is used; m. is an abbreviation for mile)

Example 10-27: Single-Letter Abbreviation with a Period

$$\frac{\text{w.}}{\text{v.}}$$

(the presence of the fraction indicators and fraction line has no effect upon the decision that the English-letter indicator must be used)

Example 10-28: Single-Letter Abbreviation with a Period

1000 yd.^{2} = ? m.^{2}

10.4 Punctuation with Abbreviations

Abbreviations are punctuated in the literary mode, provided that the punctuation is at the same level as the abbreviation. (See **8.2** and **8.3**)

Example 10-29: Abbreviation with a Period

9 ft.^{2}

Example 10-30 Period in a Subscript

$\triangle_{\text{reg. polygon}\cdot}$

Example 10-31: Abbreviation with a Period and a Comma

$3\frac{1}{2}$ gal., $2\frac{1}{4}$ qt., $1\frac{1}{8}$ pt.

Example 10-32: Abbreviation with a Period

(a. = a.)

(abbreviated form for angle = angle)

Example 10-33: Abbreviation with a Period in a Fraction

60 mph = 88 ft./sec.

10.5 Contractions in Abbreviations

No contractions are used in an abbreviation which is within the Nemeth switches.

Example 10-34: Set Notation with Abbreviations

{Ariz., Ark., Conn.}
(set notation is transcribed in Nemeth Code)

Example 10-35: Abbreviations with a Slash Meaning "Over"

6 min./360 sec.
(a slash meaning over, divided by is mathematical)

Example 10-36: Abbreviations in a Fraction

$$\frac{1\text{ hr}}{60\text{ min}}$$

Example 10-37: Abbreviations in a Subscript

$S_{\text{part. sum}}$
(period at the subscript level)

Example 10-38: Abbreviations with a Slash Meaning "Over"

statvolt-cm/statamp-oersted

10.6 Spacing with Abbreviations

10.6.1 A space must be left on either side of an abbreviation in all situations except as stated in **10.6.2** and **10.6.3**. This applies to abbreviations within UEB context as well as within the Nemeth text.

A superscript/subscript which is associated with an abbreviation is part of the abbreviation. The space comes after the superscript/subscript.

If the abbreviation is preceded by an opening cancellation indicator and/or followed by a closing cancellation indicator, the required spaces come before the opening indicator and after the closing indicator.

Example 10-39: Abbreviation Not Spaced in Print

26.4mpg

Example 10-40: Abbrev. with Parenthesis and Level Indicator

$(3\ \text{yd})^2 = 9\ \text{yd}^2$

Example 10-41: Spacing with a Termination Indicator

$\sqrt{60\ \text{ft}}$

Example 10-42: Spacing with a Fraction Line and Indicator

$\frac{1\ \text{hr.}}{60\ \text{min.}}$

Example 10-43: Spacing with a Subscript Indicator

$\triangle_{\text{reg. polygon}}$

Example 10-44: Spacing with a Symbol of Comparison

3 ft. = 1 yd.

Example 10-45: Spacing with a Symbol of Operation

$Q_1 = U_1 - \text{p.e.}$

Example 10-46: Spacing with a Symbol of Operation

6 yds – 2 ft

⠸⠩ ⠼⠖ ⠽⠙⠎ ⠤⠼⠆ ⠋⠞ ⠸⠱

Example 10-47: Spacing with a Single-Letter Abbreviation

N 35° W

⠸⠩ ⠰⠠⠝ ⠼⠒⠢⠘⠨⠡ ⠰⠠⠺ ⠸⠱

Example 10-48: Abbrev. with Related Superscript Indicator

5 ft^2 + 4 ft^2 + 2 ft^2

⠸⠩ ⠼⠢ ⠋⠞⠘⠆ ⠬⠲ ⠋⠞⠘⠆ ⠬⠆ ⠋⠞⠘⠆ ⠸⠱

Example 10-49: Spacing of Abbreviation Following Parenthesis

(2x – 3y) mi.

⠸⠩ ⠷⠆⠭⠤⠒⠽⠾ ⠍⠊⠲ ⠸⠱

(the closing parenthesis does not apply to the abbreviation; the abbreviation must be preceded by a space)

Example 10-50: Abbreviation and Fraction Indicator Spacing

$\frac{1}{2}$ hr.

⠸⠩ ⠹⠂⠌⠆⠼ ⠓⠗⠲ ⠸⠱

(the closing fraction indicator does not apply to the abbreviation; the abbreviation must be preceded by a space)

Example 10-51: Spacing with Cancellation Indicators

$$7888\ \cancel{\text{in.}^3} \cdot \frac{1\ \text{ft}^3}{1728\ \cancel{\text{in.}^3}}$$

or

10.6.2 No space is left between two components of an abbreviation when they are unspaced in print. No space is left between an abbreviation and its period, a grouping symbol, an indicator, punctuation, slash, or fraction line which applies to the abbreviation. No space is left before or after a multiplication dot when the second abbreviation has no related value.

Example 10-52: Abbreviation with Two Spaced Components

1.2 sq. ft.

Example 10-53: Abbreviation with Two Unspaced Components

1.2 sq.ft.

(no space between two adjacent abbreviations printed unspaced from each other)

Example 10-54: Abbreviation with a Slash Meaning "Per"

60 s/min

(no space between an abbreviation and its related slash; Nemeth is required when the slash means "per")

Example 10-55: Abbreviations in a Fraction

$$\frac{60\ \text{min}}{\text{h}}$$

⠸⠩⠀⠹⠖⠴⠀⠍⠊⠝⠌⠰⠓⠼⠀⠸⠱

(no space between the abbreviation h and the closing fraction indicator)

Example 10-56: Abbreviations and Dot within Parentheses

1×10^2 watt-hour (W·h)

⠸⠩⠀⠼⠂⠈⠡⠂⠴⠘⠆⠀⠺⠁⠞⠞⠤⠓⠕⠥⠗⠀⠷⠰⠠⠺⠡⠰⠓⠾⠀⠸⠱

(no space between the abbreviation h and the right grouping symbol; the W and the left grouping symbol; no space around a multiplication dot when the following abbreviation has no related value)

Example 10-57: Abbreviations with Parentheses, Dots, Slashes

1 joule = 1J = 1 N · m = 1(kg · m/s^2) · m = 1 kg · m^2/s^2

⠸⠩⠀⠼⠂⠀⠚⠕⠥⠇⠑⠀⠨⠅⠀⠼⠂⠀⠰⠠⠚⠀⠨⠅⠀⠼⠂⠀⠰⠠⠝⠡⠰⠍
⠀⠀⠨⠅⠀⠼⠂⠀⠷⠅⠛⠡⠰⠍⠸⠌⠰⠎⠘⠆⠐⠾⠡⠰⠍
⠀⠀⠨⠅⠀⠼⠂⠀⠅⠛⠡⠰⠍⠘⠆⠐⠸⠌⠰⠎⠘⠆⠀⠸⠱

10.6.3 Follow print spacing for representation of degrees Celsius and degrees Fahrenheit. C and F are punctuated mathematically when unspaced from the degree symbol.

Example 10-58: Degrees Celsius

20°C	⠸⠩⠀⠼⠆⠴⠘⠨⠡⠐⠠⠉⠀⠸⠱
20° C	⠸⠩⠀⠼⠆⠴⠘⠨⠡⠀⠰⠠⠉⠀⠸⠱
20 °C	⠸⠩⠀⠼⠆⠴⠀⠘⠨⠡⠐⠠⠉⠀⠸⠱
°C	⠸⠩⠀⠘⠨⠡⠐⠠⠉⠀⠸⠱
20°C, 30°C	⠸⠩⠀⠼⠆⠴⠘⠨⠡⠐⠠⠉⠠⠀⠼⠒⠴⠘⠨⠡⠐⠠⠉⠀⠸⠱
20° C, 30° C	⠸⠩⠀⠼⠆⠴⠘⠨⠡⠀⠰⠠⠉⠂⠀⠼⠒⠴⠘⠨⠡⠀⠰⠠⠉⠀⠸⠱

Rule 11
Omissions

General Omission Symbol ⠿

11.1 Omissions

11.1.1 **General Omission Symbol.** A large number of signs are employed in print to denote omitted mathematical material. When a blank space or a question mark, either by itself or in combination with any number of hyphens or dashes, is employed in a linear expression to denote an omission, one general omission symbol is used in the transcription.

Example 11-1: Question Mark Used in Print

$(?)^3 = 27$

Example 11-2: Question Mark Used in Print

92 in. = ? ft. ? in.

Example 11-3: Question Mark Used in Print

$7 \times 2 \ ? \ 14$

Example 11-4: Question Mark Used in Print

$? + ? = 10$

Example 11-5: Underlined Question Mark Used in Print

$7 - \underline{\ ?\ } = 5$

Example 11-6: Question Mark with Hyphens Used in Print

9 – 5 = -?-

Example 11-7: Blank Space Used in Print

(5,)

Example 11-8: Blank Space Used in Print

5 × 4 =

11.1.2 **Long Dash.** When a dash or underscore is used to denote omission in print, the long dash (⠤⠤⠤⠤) is used in the transcription. **Exception:** If a dash or underscore is internal to a number, the general omission symbol may be used instead of the long dash.

Example 11-9: Long Dash Indicating Omission

92 in. = ___ ft. ___ in.

Example 11-10: Internal Long Dash or Underscore

The number 35__,862 rounds to 350,000 when rounded to the nearest ten thousand. What numbers, 0, 1, 2, 3, 4,5, 6, 7, 8, or 9 make this true?

11.1.3 **Representation of Additional Omission Signs.** Other than as noted above, the omission symbol which is used must correspond to the sign which appears in print. If an omission sign is used in print for which this Code provides no representation, this sign may be represented by drawing it in, or the transcriber may devise a braille symbol to represent it. A symbol devised by the transcriber is listed on the Special Symbols page.

Example 11-11: Ellipsis Used in Print

(2, 4, 6, ..., 12)

Example 11-12: Shapes Used in Print

□ + ○ = 5

Example 11-13: Picture Used to Show Omission

5 × 4 = ☀

(omission symbol is a starburst, represented by a shape indicator/letter b)

11.1.4 **Spacing and Punctuation.** An omission symbol should be spaced in the same manner as the material which it replaces. Other omission symbols must be spaced in accordance with the rules governing the spacing of those symbols, such as an ellipsis or long dash. An omission symbol inside the switches is punctuated mathematically.

Example 11-14: Dash Used in Print

five × ___ = fifteen

Example 11-15: Shapes Used to Show Omission

□ – △ = 2

(the square and triangle are unspaced from the sign of operation)

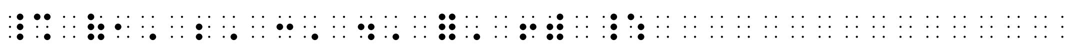
Example 11-16: Blank Space Showing Omission

16÷4   4

(the general omission symbol represents a sign of comparison which is spaced)

Example 11-17: Omission Symbol Punctuated Mathematically

(1, 2, 3, 4, ?, 6)



11.1.5 **Spatial Arrangements.** In a spatial arrangement, only the general omission symbol is used in braille regardless of the symbol denoted in print. Follow print for placement of the general omission symbol. When an ellipsis or a long dash indicates the print omission, one general omission symbol is right justified in the present alignment. (However, see **Rule 25** for information regarding an ellipsis in a matrix.)

Example 11-18: Question Marks in a Spatial Arrangement

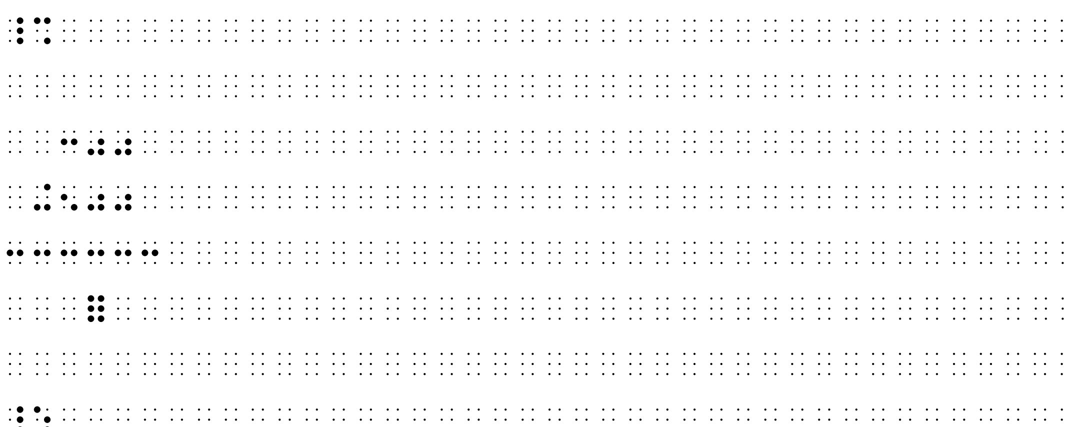
$$\begin{array}{r} 40 \\ +70 \\ \hline ??? \end{array}$$

Example 11-19: Question Marks in a Spatial Arrangement

$$\begin{array}{r} 642 \\ \underline{-???} \\ 452 \end{array}$$

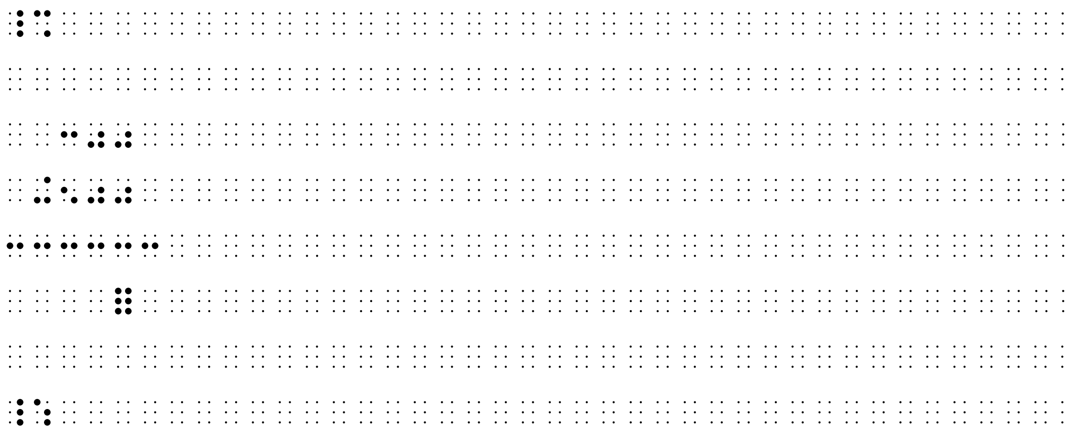
Example 11-20: One Question Mark Showing Omission

$$\begin{array}{r} 300 \\ \underline{+500} \\ ? \end{array}$$



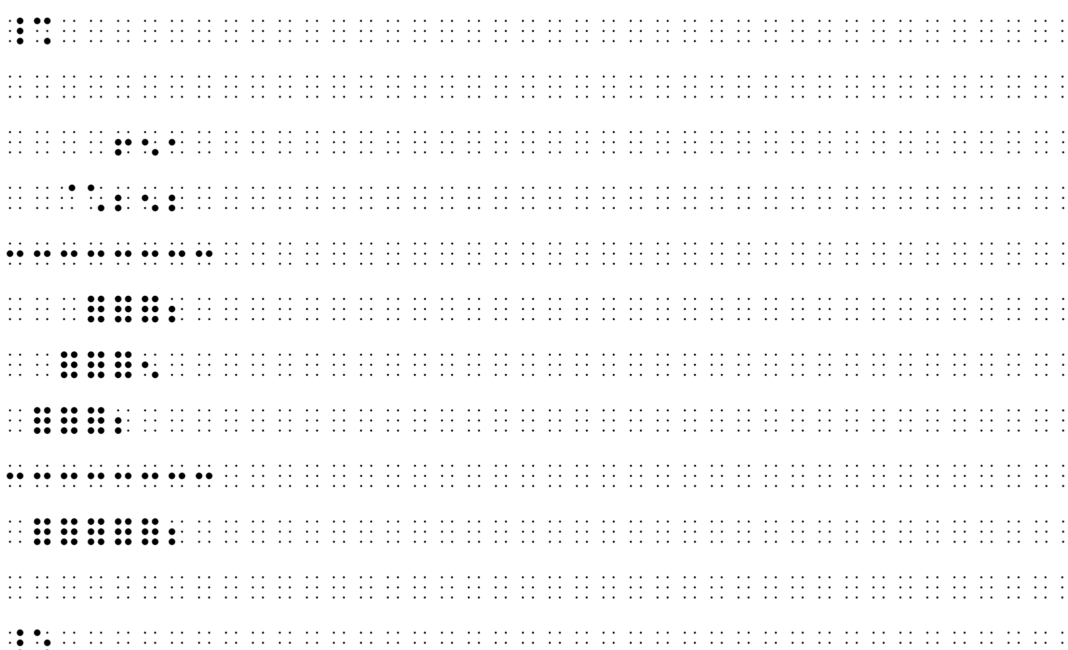
Example 11-21: Omission Dash in Spatial Arrangement

$$\begin{array}{r} 300 \\ \underline{+500} \\ \text{—} \end{array}$$

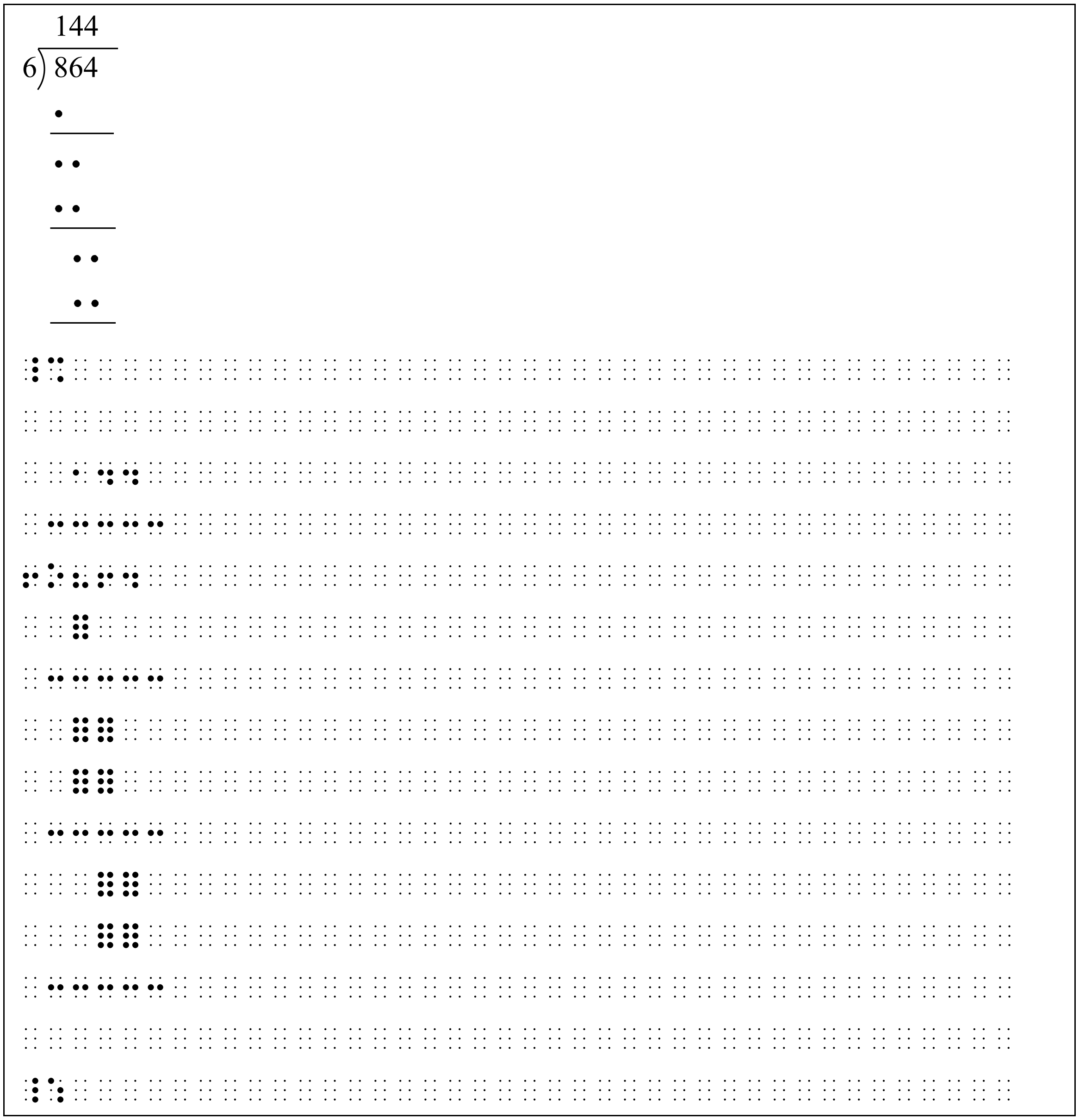


Example 11-22: Dots in Spatial Arrangement to Show Omission

$$\begin{array}{r} 651 \\ \underline{\times 252} \\ \bullet\bullet\bullet 2 \\ \bullet\bullet\bullet 5 \\ \underline{\bullet\bullet\bullet 2} \\ \bullet\bullet\bullet\bullet\bullet 2 \end{array}$$



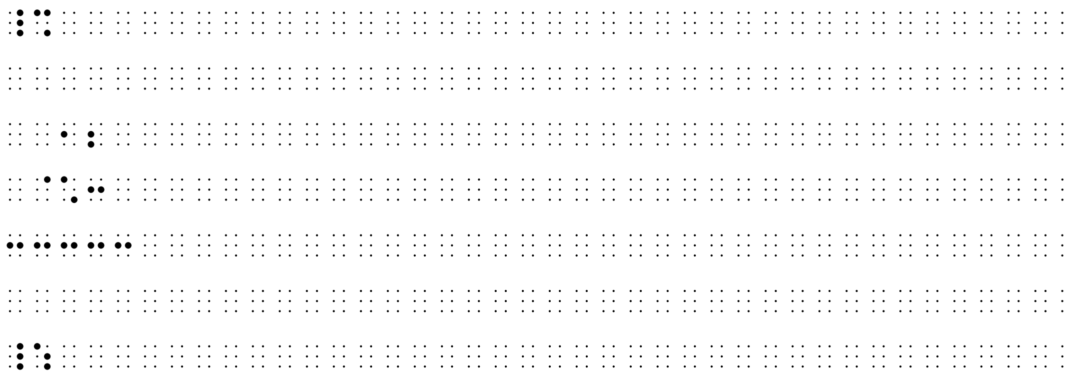
Example 11-23: Dots in Spatial Arrangement to Show Omission



11.1.6 The general omission symbol is not used when there is a blank space in a spatial arrangement. A general omission symbol is not used to represent the blank space after the last separation line in spatial arrangements.

Example 11-24: No Answer Shown in a Spatial Arrangement

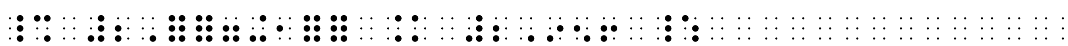
$$\begin{array}{r} 12 \\ \times \quad 3 \\ \hline \end{array}$$



11.1.7 The number of omission symbols to be used must be the same as the number of omission signs in print.

Example 11-25: Number of General Omission Symbols

2,??7 + 1?? = 2,956



Rule 12
Cancellation

Cancellation Indicators

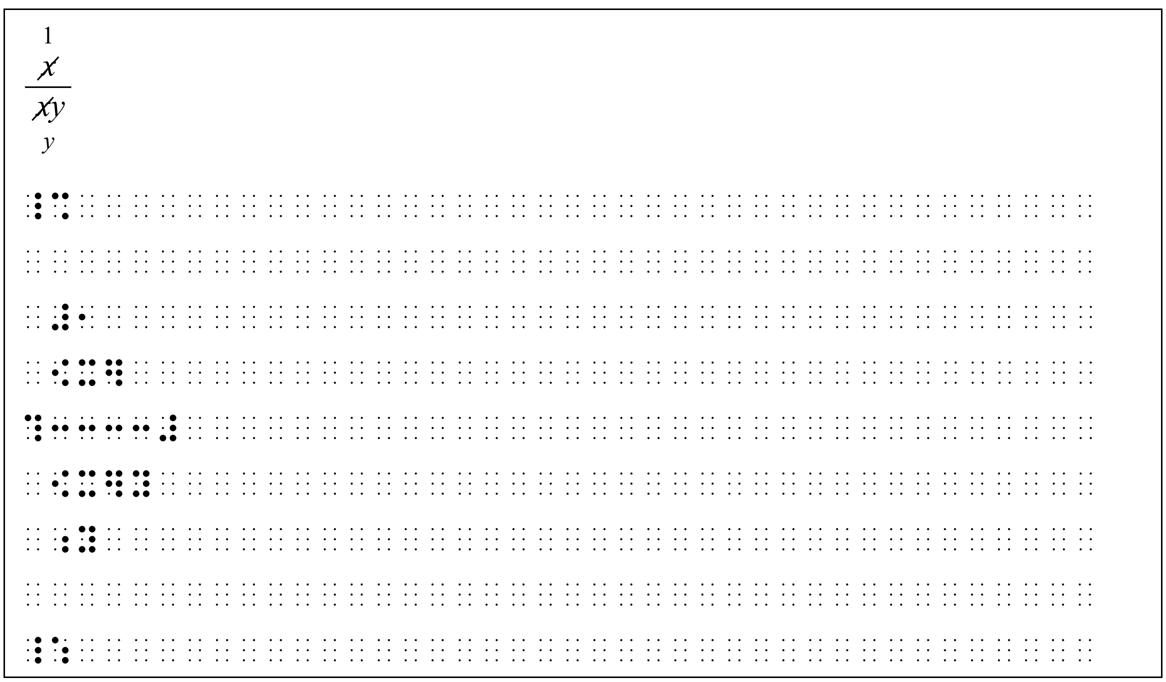
Opening ⠪

Closing ⠻

12.1 Cancellation Indicators

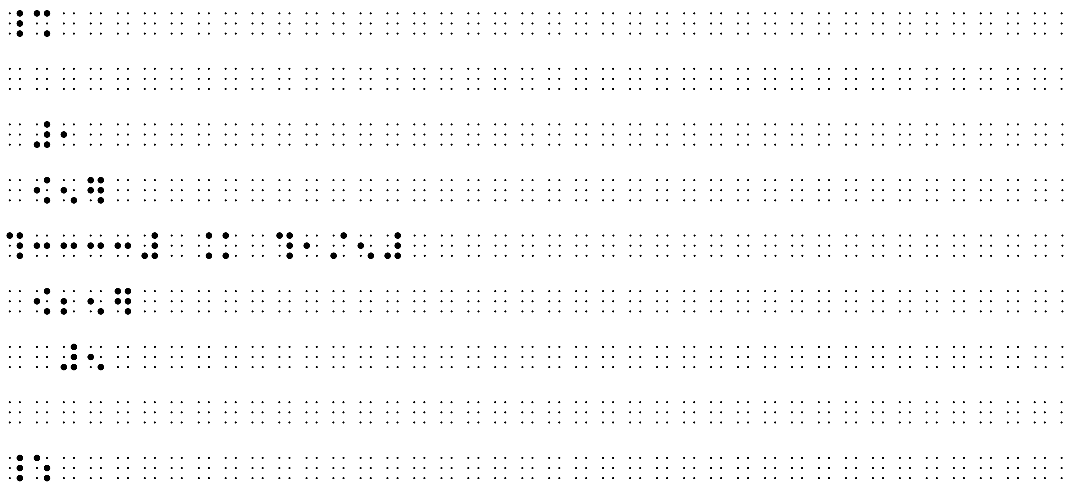
12.1.1 The cancellation indicators must be used to show the extent of a mathematical expression which has been canceled in print. A spatial arrangement is required when cancellation with replacement is represented in braille. Material containing cancellation with no replacement may be transcribed either linearly or spatially. Items which are individually canceled in print must be represented as individually canceled in the transcription.

Example 12-1: Cancellation in a Fraction



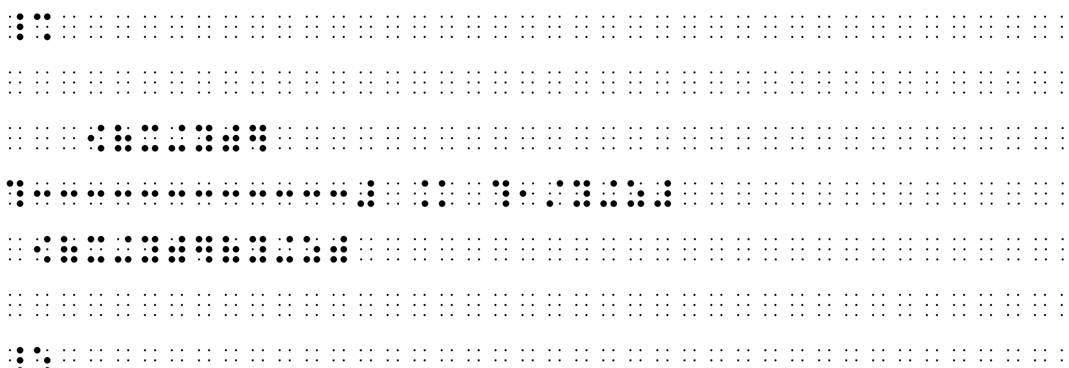
Example 12-2: Cancellation in a Fraction

$$\frac{\overset{1}{\cancel{5}}}{\underset{5}{\cancel{25}}} = \frac{1}{5}$$

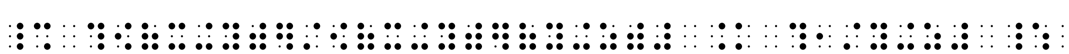


Example 12-3: Cancellation in a Fraction Without Replacement

$$\frac{\cancel{(x+y)}}{\cancel{(x+y)}(y+z)} = \frac{1}{y+z}$$

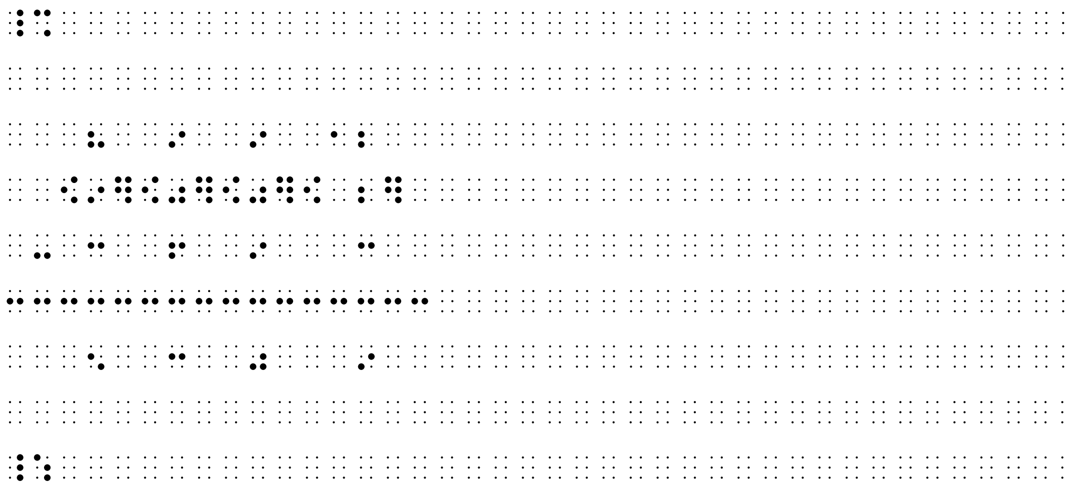


or (linear transcription)



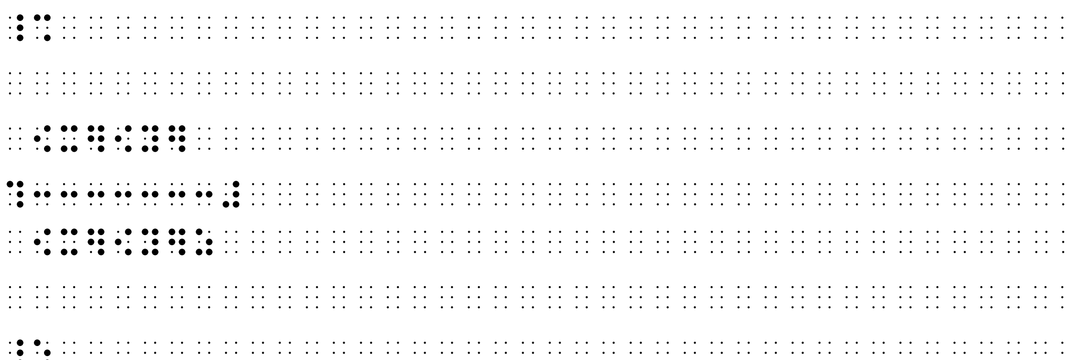
Example 12-4: Cancellation in Subtraction

$$\begin{array}{r} \overset{8}{\cancel{9}}\;\overset{9}{\cancel{0}}\;\overset{9}{\cancel{0}}\;\overset{12}{\cancel{2}} \\ -3\;6\;9\;3 \\ \hline 5\;3\;0\;9 \end{array}$$



Example 12-5: Cancellation Without Replacement

$$\frac{\cancel{x}\cancel{y}}{\cancel{x}\cancel{y}z}$$

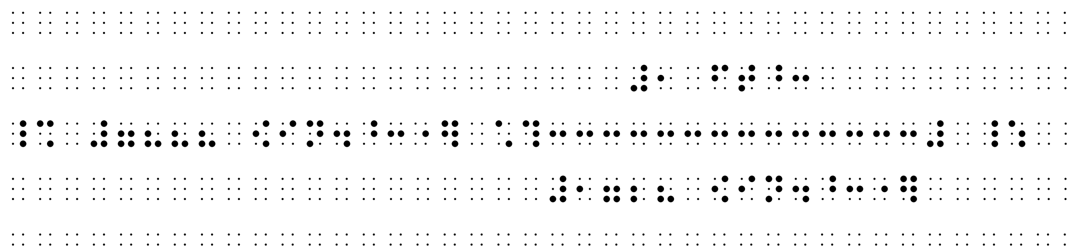


or (linear transcription)

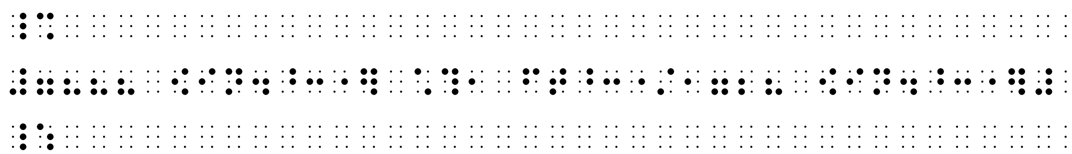
12.1.2 If the abbreviation is preceded by an opening cancellation indicator and followed by a closing cancellation indicator, the required spaces come before the opening cancellation indicator and after the closing cancellation indicator.

Example 12-6: Cancellation of Abbreviations

$$7888\ \cancel{\text{in.}^3} \bullet \frac{1\ \text{ft}^3}{1728\ \cancel{\text{in.}^3}}$$



or (linear transcription)



Rule 13
Fractions

Fraction Indicators		
	Simple	
	Opening	⠹
	Closing	⠼
	Complex	
	Opening	⠠⠹
	Closing	⠠⠼
	Hypercomplex	
	Opening	⠠⠠⠹
	Closing	⠠⠠⠼
	Fractional Part of a Mixed Number	
	Opening	⠸⠹
	Closing	⠸⠼
Fraction Lines		
	Used with Simple-Fraction Indicators	
	Diagonal line or slash	⠸⠌
	Horizontal	⠌
	Used with the Fractional Part of a Mixed Number	
	Diagonal line or slash	⠸⠌
	Horizontal	⠌
	Used with Complex-Fraction Indicators	
	Diagonal line or slash	⠠⠸⠌
	Horizontal	⠠⠌

Used with Hypercomplex-Fraction Indicators

Diagonal line or slash ⠠⠠⠸⠌

Horizontal ⠠⠠⠌

Used with a Spatial Arrangement

Horizontal (varying in length) ⠒⠒⠒⠒⠒

13.1 Simple Fractions

For the purposes of this Code, a *simple fraction* is one whose numerator and denominator contain no fractions except possibly at the superscript or subscript level.

13.2 Use of Simple Fraction Indicators

13.2.1 Simple fraction indicators must be used, except in the case of mixed numbers, to enclose a simple fraction whose numerator and denominator are separated by a horizontal fraction line in print.

Example 13-1: Simple Fraction with Horizontal Fraction Line

$$\frac{1}{3}$$

⠸⠩⠀⠹⠂⠌⠒⠼⠀⠸⠱

Example 13-2: Simple Fraction at the Superscript Level

$$x^{\frac{1}{2}}$$

⠸⠩⠀⠭⠘⠹⠂⠌⠆⠼⠀⠸⠱

Example 13-3: Simple Fraction with Horizontal Fraction Line

$$\frac{a+b}{c}$$

⠸⠩⠀⠹⠁⠬⠃⠌⠉⠼⠀⠸⠱

Example 13-4: Simple Fraction with Superscript Fraction

$$\frac{x^{\frac{1}{2}}}{2}$$

⠸⠩⠀⠹⠭⠘⠹⠂⠌⠆⠼⠐⠌⠆⠼⠀⠸⠱

Example 13-5: Simple Fraction with Horizontal Fraction Line

$$\text{rate} = \frac{\text{distance}}{\text{time}}$$

⠸⠩⠀⠗⠁⠞⠑⠀⠨⠅⠀⠹⠙⠊⠎⠞⠁⠝⠉⠑⠌⠞⠊⠍⠑⠼⠀⠸⠱

13.2.2 Simple fraction indicators are used to enclose a simple fraction whose numerator and denominator are separated by a diagonal line in print, when the expressions on either side of the diagonal line appear at different levels relative to it, or in different type size than is normal for the purpose for which these expressions are used.

Example 13-6: Simple Fraction with Diagonal Fraction Line

$$^{a+b}\!\big/_{c+d}$$

⠸⠩⠀⠹⠁⠬⠃⠸⠌⠉⠬⠙⠼⠀⠸⠱

(in print, the numerator is written near the top of the diagonal line and the denominator is written near the bottom)

Example 13-7: Simple Fraction with Diagonal Fraction Line

$$3\,{\scriptstyle x/y}$$

⠸⠩⠀⠼⠒⠹⠭⠸⠌⠽⠼⠀⠸⠱

(in print the 3, x, and y are at the same level, but the x and y are in smaller type than the 3)

13.3 Non-Use of Simple Fraction Indicators

13.3.1 Simple fraction indicators must not be used to enclose the fractional part of a mixed number.

Example 13-8: Mixed Number with Horizontal Fraction Line

$4\frac{3}{8}$

Example 13-9: Mixed Number with Diagonal Fraction Line

$2\,{}^{3}\!\diagup_{4}\,x$

13.3.2 Simple fraction indicators must not be used to enclose a simple fraction whose numerator and denominator are separated by a diagonal line in print when the expressions on either side of the diagonal line appear at the same level relative to it, or are of the same type size as the surrounding mathematical text.

Example 13-10: Numerator and Denominator at the Same Level

$1/3$

Example 13-11: Superscript Fraction

$x^{1/2}$

(in print, 1 and 2 are at the same level; although the 1 and 2 are in smaller type, they are of normal size for printing superscripts)

Example 13-12: Numerator and Denominator at the Same Level

$x^{\frac{1}{2}}/2$

(in print, the x and 2 are at the same level and are of normal size for printing baseline signs)

Example 13-13: Numerator and Denominator at the Same Level

$x^{1/2}/7$

⠸⠩⠀⠭⠘⠂⠸⠌⠆⠐⠸⠌⠶⠀⠸⠱

(in print, 1 and 2 are at the same level and x and 7 are at the same level; each pair of signs is of normal size for printing at its respective level)

Example 13-14: Numerator and Denominator at the Same Level

$a+b/c+d$

⠸⠩⠀⠁⠬⠃⠸⠌⠉⠬⠙⠀⠸⠱

(in print, all letters are of normal size and at the same level on either side of a diagonal line)

Example 13-15: Numerator and Denominator at the Same Level

$(a+b)/(c+d)$

⠸⠩⠀⠷⠁⠬⠃⠾⠸⠌⠷⠉⠬⠙⠾⠀⠸⠱

(in print, all letters are of normal size and at the same level on either side of a diagonal line)

13.3.3 Sometimes the expressions on either side of the diagonal line are not the terms of a fraction at all. Even when they are, the transcriber cannot always be certain of where the fraction begins or ends. Accordingly, it is better to avoid the use of fraction indicators altogether in these cases and permit the braille reader to make a judgment based on the same information that is available to the sighted reader. When slash means per, divided by, or over, the slash is a fraction line.

Example 13-16: Slash That is Not a Fraction Line

1/31/70

⠼⠁⠸⠌⠼⠉⠁⠸⠌⠼⠛⠚

(the expression represents a date)

Example 13-17: Slash That is Not a Fraction Line

c/o

⠉⠸⠌⠕

(not a fraction; means care of)

13.4 Mixed Numbers

For the purposes of this Code, a mixed number is an expression which begins with a whole number and is followed, usually in smaller type, by a simple fraction whose numerator and denominator are both whole numbers. The fraction line of this simple fraction may be either horizontal or diagonal in print. The mixed-number indicators must be used to enclose the fractional part of a mixed number. An expression is not a mixed number if it contains any letter, even though such an expression is of the same form as a mixed number in every other respect.

Example 13-18: Mixed Number with Horizontal Fraction Line

$4\frac{3}{8}$

⠸⠩⠀⠼⠲⠸⠹⠒⠌⠦⠸⠼⠀⠸⠱

Example 13-19: Mixed Number with Diagonal Fraction Line

$4\,{}^{3}\!/_{8}$

⠸⠩⠀⠼⠲⠸⠹⠒⠸⠌⠦⠸⠼⠀⠸⠱

Example 13-20: Expression That Includes a Letter

$x\frac{3}{8}$

⠸⠩⠀⠭⠹⠒⠌⠦⠼⠀⠸⠱

Example 13-21: Expression That Includes a Letter

$x\,{}^{3}\!/_{8}$

⠸⠩⠀⠭⠹⠒⠸⠌⠦⠼⠀⠸⠱

Example 13-22: Fraction Containing a Letter

$$3\frac{1}{y}$$

⠸⠩⠀⠼⠒⠹⠂⠌⠽⠼⠀⠸⠱

13.5 Complex Fractions

For the purposes of this Code, a complex fraction is one whose numerator, denominator, or both, contains at least one simple fraction. A fraction is not a complex fraction if the only simple fractions it contains are at the superscript or subscript level.

13.6 Use of Complex Fraction Indicators

Complex fraction indicators must be used to enclose a complex fraction.

Example 13-23: Complex Fraction

$$\frac{\frac{3}{8}}{5}$$

⠸⠩⠀⠠⠹⠹⠒⠌⠦⠼⠠⠌⠢⠠⠼⠀⠸⠱

Example 13-24: Complex Fraction

$$\frac{\frac{1}{2}}{2\frac{2}{3}}$$

⠸⠩⠀⠠⠹⠹⠂⠌⠆⠼⠠⠌⠆⠸⠹⠆⠌⠒⠸⠼⠠⠼⠀⠸⠱

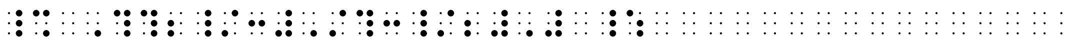
Example 13-25: Complex Fraction

$$\frac{2/3}{3/2}$$

⠸⠩⠀⠠⠹⠆⠸⠌⠒⠠⠌⠒⠸⠌⠆⠠⠼⠀⠸⠱

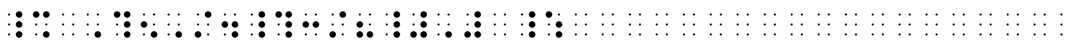
Example 13-26: Complex Fraction

$$\dfrac{{}^{2}\!\diagup_{3}}{{}^{3}\!\diagup_{2}}$$



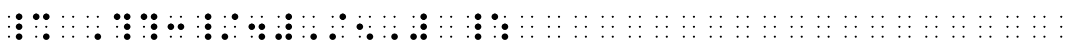
Example 13-27: Complex Fraction

$$\dfrac{5}{4\frac{3}{8}}$$



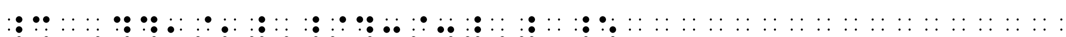
Example 13-28: Complex Fraction

$$\dfrac{{}^{3}\!\diagup_{4}}{5}$$



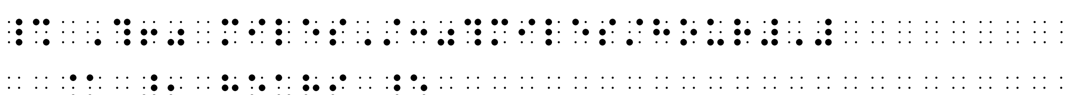
Example 13-29: Complex Fraction

$$\frac{1}{2}\Big/\frac{3}{4}$$



Example 13-30: Complex Fraction with Words

$$\dfrac{60 \text{ miles}}{30\,\dfrac{\text{miles}}{\text{hour}}} = 2 \text{ hours}$$

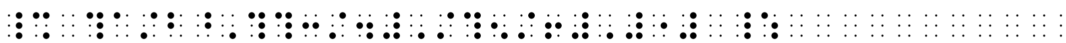


13.7 Hypercomplex Fractions

For the purposes of this Code, a hypercomplex fraction is one whose numerator, denominator, or both, contain at least one complex fraction. A fraction is not a hypercomplex fraction if the only complex fractions it contains are at the superscript or subscript level

Example 13-31: Fraction with Superscript Complex Fraction

$$\frac{a}{b^{\frac{\frac{3}{4}}{\frac{5}{6}}}}$$

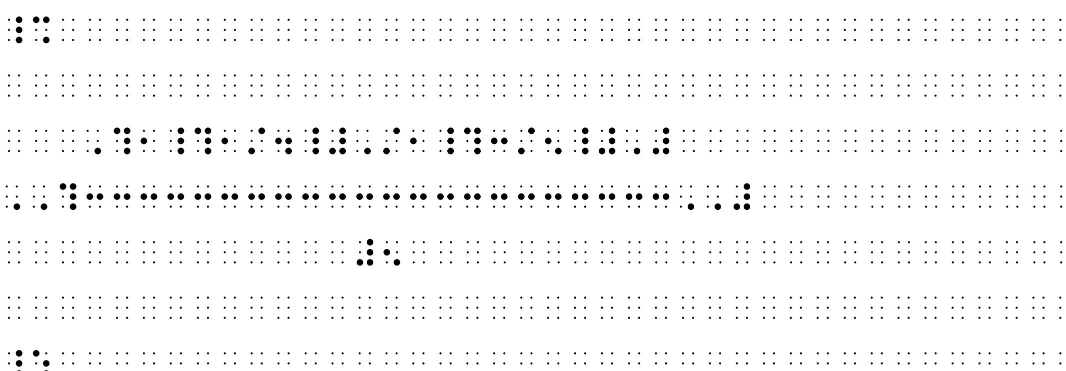


13.8 Use of Hypercomplex Fraction Indicators

13.8.1 Hypercomplex fraction indicators must be used to enclose a hypercomplex fraction. The use of a linear arrangement within a spatial arrangement is preferable to an arrangement which is entirely linear or entirely spatial.

Example 13-32: Hypercomplex Fraction

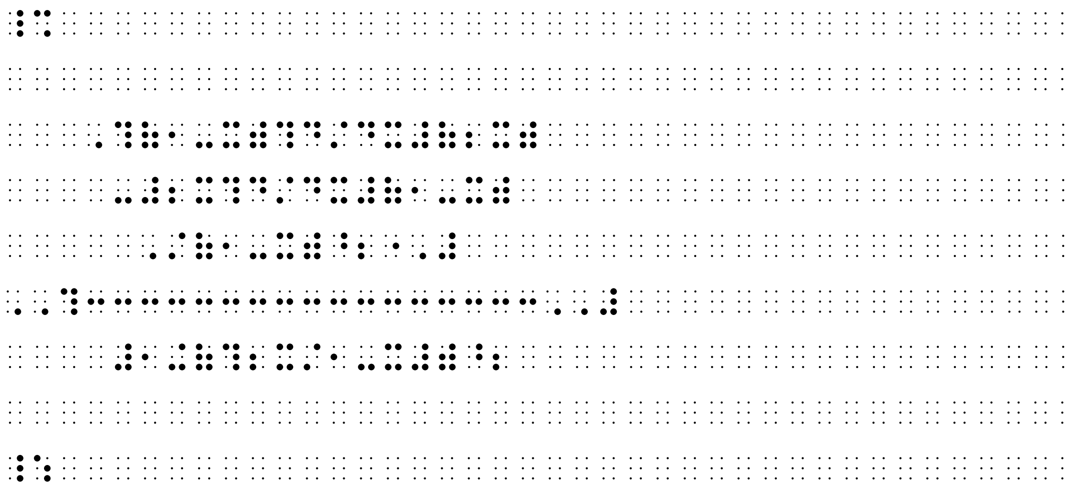
$$\frac{\frac{1\frac{1}{4}}{1\frac{3}{5}}}{5}$$



(preferred method of transcribing a hypercomplex fraction)

Example 13-33: Hypercomplex Fraction

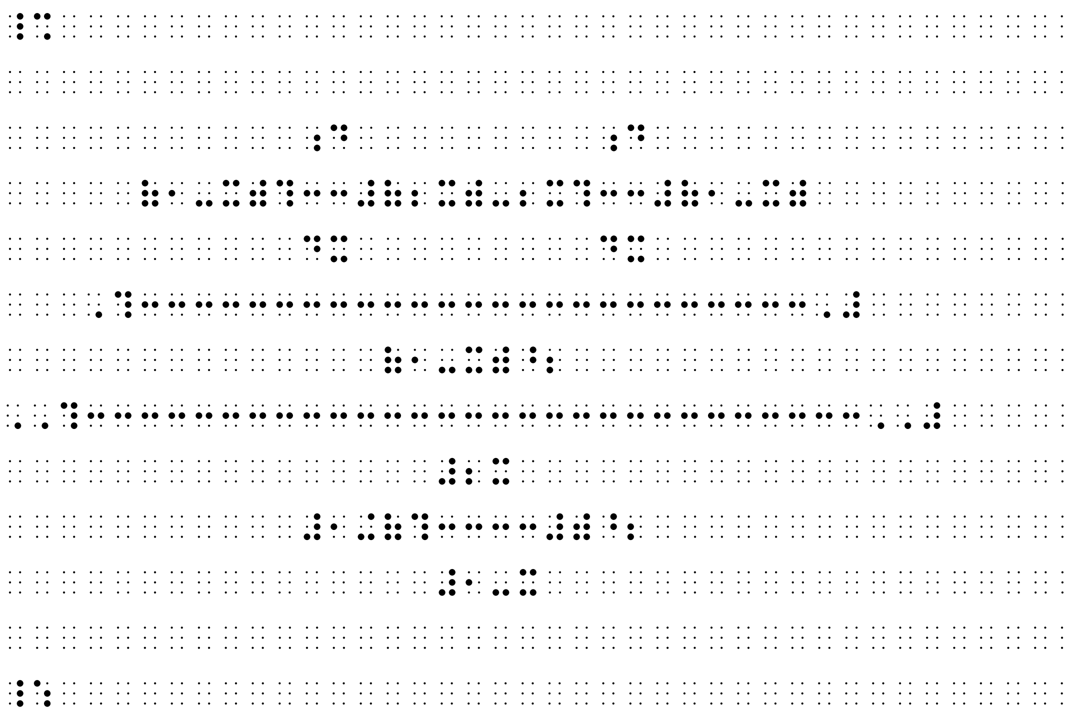
$$\frac{\dfrac{(1-x)\dfrac{d}{dx}(2x)-2x\dfrac{d}{dx}(1-x)}{(1-x)^2}}{1+\left(\dfrac{2x}{1-x}\right)^2}$$



(preferred method of transcribing a hypercomplex fraction)

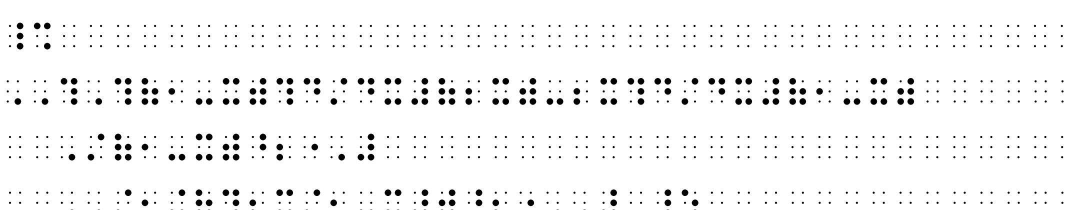
Example 13-34: Fully Spatial Hypercomplex Fraction

$$\frac{\dfrac{(1-x)\dfrac{d}{dx}(2x)-2x\dfrac{d}{dx}(1-x)}{(1-x)^2}}{1+\left(\dfrac{2x}{1-x}\right)^2}$$



Example 13-35: Fully Linear Hypercomplex Fraction

$$\frac{\dfrac{(1-x)\dfrac{d}{dx}(2x)-2x\dfrac{d}{dx}(1-x)}{(1-x)^2}}{1+\left(\dfrac{2x}{1-x}\right)^2}$$



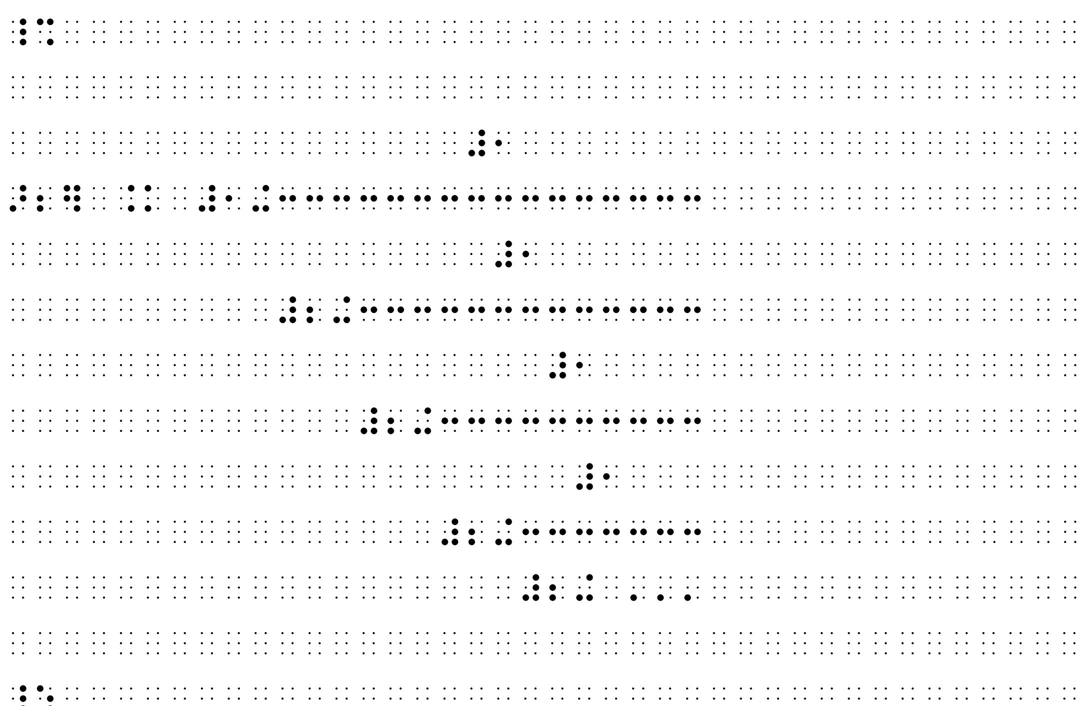
13.8.2 Hypercomplex fractions of higher order are transcribed in the manner suggested by **13.8.1** above. Use dot 6 the proper number of times before the fraction indicators and their matching fraction line.

13.9 Continued Fractions

A continued fraction is one in which each denominator, except possibly the last one, is the sum of a whole number and a fraction. A spatial arrangement must be used for a continued fraction. In this case, each fraction line must have proportionately the length shown in print, and fraction indicators of any kind must not be used. Punctuation, mathematical signs, and other applicable symbols are transcribed on the same line as the principal fraction line.

Example 13-36: Continued Fraction

$$\sqrt{2} = 1 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2 + \ldots}}}}$$



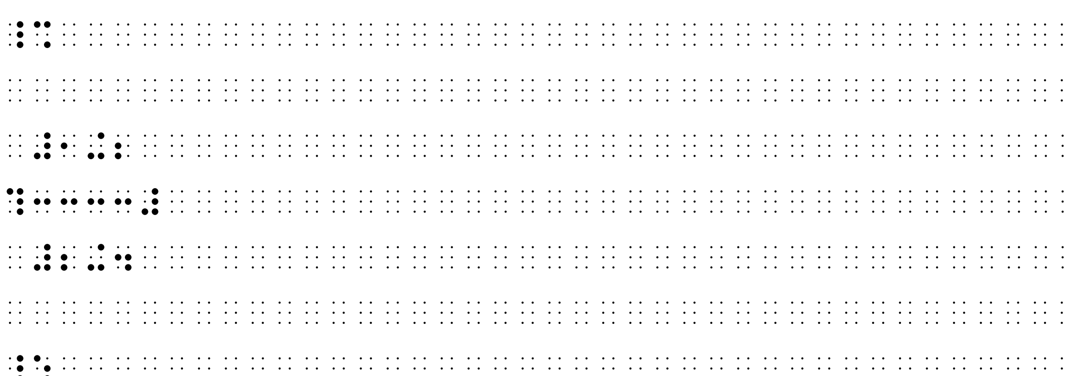
13.10 Spatial Arrangement for Fractions

13.10.1 Except for continued fractions, whenever a fraction is transcribed spatially, all fraction indicators must be shown, and each fraction line must have precisely the length necessary to cover the longest expression to which it applies. The terms of the fraction must be centered on their fraction lines. Punctuation, mathematical signs, and other applicable symbols outside the fraction are transcribed on the same line as the principal fraction line. An expression which is too long to be centered on the fraction line to which it applies may be divided at suitable places in accordance with the rules for runovers. (See **26.2** for division of math expressions.) Each portion of the divided expression must be centered on the fraction line to which the expression, as a whole, applies.

13.10.2 The linear arrangement for fractions is used when not expressly forbidden. However, when fraction notation is first presented to the reader, as in the lower grades, or when there is any other special need, any fraction may be represented spatially.

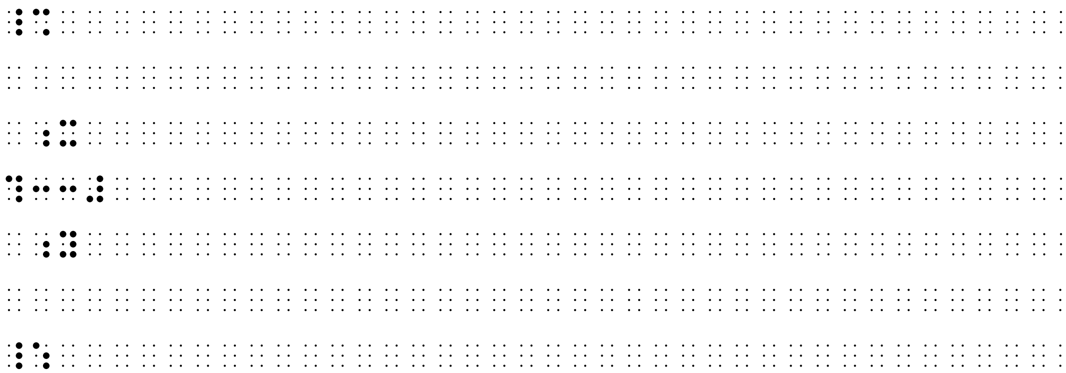
Example 13-37: Spatial Simple Fraction

$$\frac{1+2}{2+4}$$



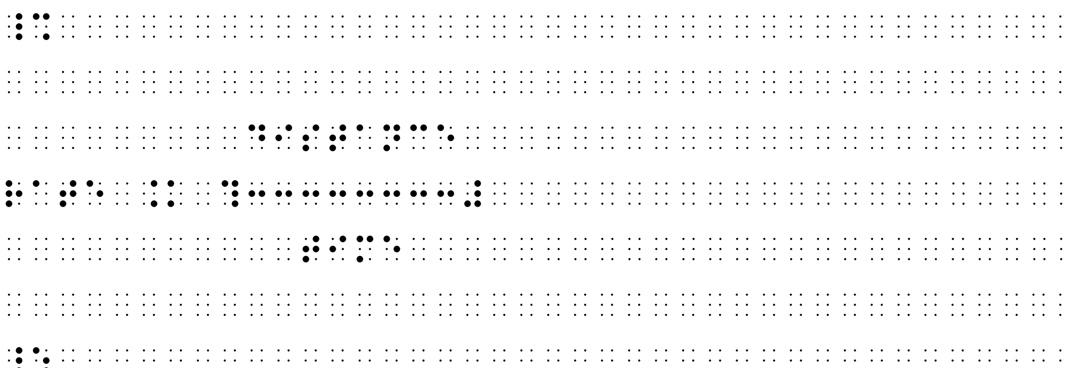
Example 13-38: Spatial Simple Fraction

$$\frac{x}{y}$$



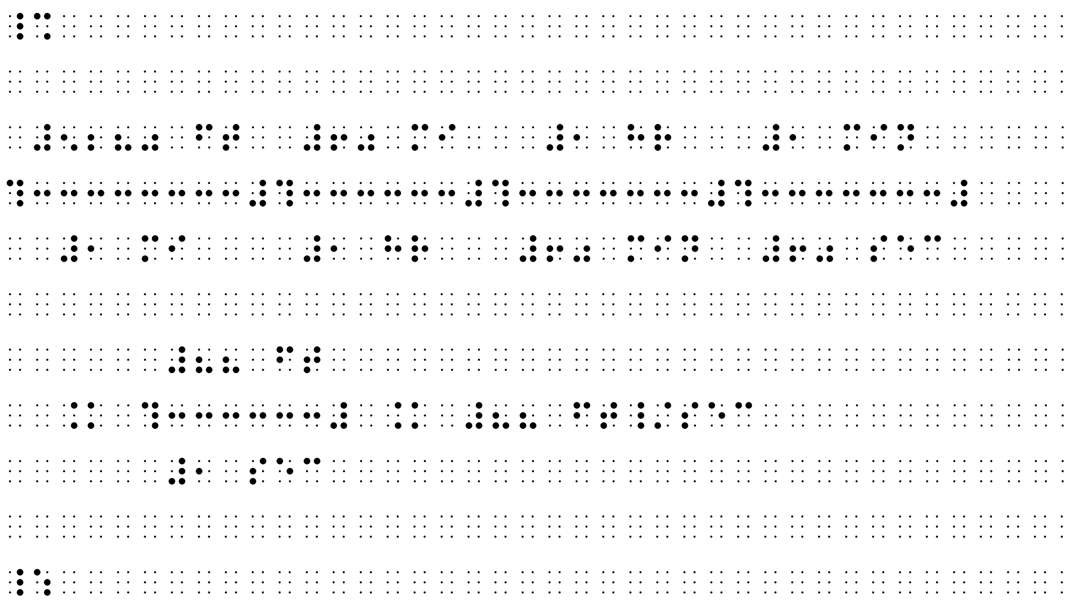
Example 13-39: Spatial Simple Fraction

$$\text{rate} = \frac{\text{distance}}{\text{time}}$$



Example 13-40: Series of Spatial Fractions

$$\frac{5280\text{ ft}}{1\text{ mi}}\;\frac{60\text{ mi}}{1\text{ hr}}\;\frac{1\text{ hr}}{60\text{ min}}\;\frac{1\text{ min}}{60\text{ sec}} = \frac{88\text{ ft}}{1\text{ sec}} = 88\text{ ft/sec}$$



13.10.3 For spatial arrangement of fractions in connection with cancellation see **12.1**.

13.10.4 For spatial arrangement of hypercomplex fractions see **13.8**.

13.10.5 For spatial arrangement of continued fractions see **13.9**.

Rule 14
Superscripts and Subscripts

Baseline Indicator	⠐
Superscript Indicators	
Superscript	⠘
Superscript with Superscript	⠘⠘
Superscript with Subscript	⠘⠰
Superscript with Superscript with Superscript	⠘⠘⠘
Superscript with Superscript with Subscript	⠘⠘⠰
Superscript with Subscript with Superscript	⠘⠰⠘
Superscript with Subscript with Subscript	⠘⠰⠰
Subscript Indicators	
Subscript	⠰
Subscript with Superscript	⠰⠘
Subscript with Subscript	⠰⠰
Subscript with Superscript with Superscript	⠰⠘⠘
Subscript with Superscript with Subscript	⠰⠘⠰
Subscript with Subscript with Superscript	⠰⠰⠘
Subscript with Subscript with Subscript	⠰⠰⠰
Contraction for Comma and Optional Space at Superscript or Subscript Level	⠪

14.1 Nature of Superscripts and Subscripts

It is characteristic of print to employ signs, usually in smaller type, which are elevated or depressed relative to the *baseline*. A mathematical sign which is elevated relative to the baseline is called a *superscript*; one which is depressed relative to the baseline is called a *subscript*. When an entire mathematical expression is at the superscript or subscript level, it is written without an indicator in braille, but its position must be explained to the reader by a transcriber's note.

Note: An expression which occupies a position directly over or directly under the sign to which it applies is called a *modifier* (see **Rule 15**).

Example 14-1: Subscript Number by Itself

In x_2, $_2$ is a subscript.

Example 14-2: Superscript Degree Sign by Itself

The sign for *degree* is °.

14.2 Hierarchy of Superscripts and Subscripts

Superscripts or subscripts may carry superscripts or subscripts of their own; the latter are then referred to as superscripts or subscripts of *second order*, and are thus distinguished from the former, which are called superscripts or subscripts of *first order*. Second order superscripts or subscripts may, in turn, carry superscripts or subscripts of their own, which are then called superscripts or subscripts of *third order*. While it is theoretically possible for a superscript or subscript to be of order higher than the third, this situation rarely arises in practice. See examples **14-21** and **14-22**.

14.3 Level Indicators

A level indicator other than the baseline indicator identifies the symbols which follow it as representing a superscript or subscript. The *baseline* indicator identifies the symbols which follow it as representing signs on the baseline. *The degree of elevation or depression specified by a level indicator is always with respect to the baseline*.

14.4 Orientation by Level Indicator

14.4.1 The effect of a level indicator with one component is to direct the reader's attention upward or downward *from the baseline*.

Example 14-3: Superscript Number

x^2

⠸⠩ ⠭⠘⠆ ⠸⠱

Example 14-4: Superscript Number

y^3

⠸⠩ ⠽⠘⠒ ⠸⠱

Example 14-5: Superscript Asterisk

x^*

⠸⠩ ⠭⠘⠈⠼ ⠸⠱

Example 14-6: Superscript Negative Number

x^{-2}

⠸⠩ ⠭⠘⠤⠆ ⠸⠱

Example 14-7: Subscript Letter

x_a

⠸⠩ ⠭⠰⠁ ⠸⠱

Example 14-8: Subscript Negative Number

x_{-2}

⠸⠩ ⠭⠰⠤⠆ ⠸⠱

14.4.2 The effect of a level indicator with two components may be analyzed as follows:

a. The first print component directs the reader's attention upward or downward *from the baseline* as that component is, in itself, the superscript or subscript.

b. The second print component then directs the reader's attention upward or downward from the previous position as the second component is, in itself, the superscript or subscript to the first component.

Example 14-9: Superscript Letter with Superscript Letter

$n^{x^{y}}$

⠸⠩⠀⠝⠘⠭⠘⠘⠽⠀⠸⠱

Example 14-10: Superscript Letter with Subscript Letter

$x^{n_{a}}$

⠸⠩⠀⠭⠘⠝⠘⠰⠁⠀⠸⠱

Example 14-11: Subscript Letter with Superscript Letter

$x_{n^{a}}$

⠸⠩⠀⠭⠰⠝⠰⠘⠁⠀⠸⠱

Example 14-12: Subscript Letter with Subscript Letter

$n_{x_{y}}$

⠸⠩⠀⠝⠰⠭⠰⠰⠽⠀⠸⠱

14.4.3 The effect of a level indicator with three components may be analyzed as follows:

a. The first two print components direct the reader's attention from the baseline to the position described in b. above.

b. The third print component directs the reader's attention upward or downward from this new position.

Example 14-13: Superscript with Three Components

$n^{x^{y^{z}}}$

⠸⠩⠀⠝⠘⠭⠘⠘⠽⠘⠘⠘⠵⠀⠸⠱

(n carries a superscript x which carries a superscript y which carries a superscript z)

Example 14-14: Superscript with Three Components

$x^{y^{z_{a}}}$

⠸⠩⠀⠭⠘⠽⠘⠘⠵⠘⠘⠰⠁⠀⠸⠱

(x carries a superscript y which carries a superscript z which carries a subscript a)

Example 14-15: Superscript with Three Components

$x^{y_{a^{n}}}$

⠸⠩⠀⠭⠘⠽⠘⠰⠁⠘⠰⠘⠝⠀⠸⠱

(x carries a superscript y which carries a subscript a which carries a superscript n)

Example 14-16: Superscript with Three Components

$n^{x_{a_{j}}}$

⠸⠩⠀⠝⠘⠭⠘⠰⠁⠘⠰⠰⠚⠀⠸⠱

(n carries a superscript x which carries a subscript a which carries a subscript j)

Example 14-17: Subscript with Three Components

$x_{a^{r^{n}}}$

⠸⠩⠀⠭⠰⠁⠰⠘⠗⠰⠘⠘⠝⠀⠸⠱

(x carries a subscript a which carries a superscript r which carries a superscript n)

Example 14-18: Subscript with Three Components

$x_{a^{n_b}}$

⠸⠩ ⠭⠰⠁⠰⠘⠝⠰⠘⠰⠃ ⠸⠱

(x carries a subscript a which carries a superscript n which carries a subscript b)

Example 14-19: Subscript with Three Components

$x_{p_{a^m}}$

⠸⠩ ⠭⠰⠏⠰⠰⠁⠰⠰⠘⠍ ⠸⠱

(x carries a subscript p which carries a subscript a which carries a superscript m)

Example 14-20: Subscript with Three Components

$n_{x_{y_z}}$

⠸⠩ ⠝⠰⠭⠰⠰⠽⠰⠰⠰⠵ ⠸⠱

(n carries a subscript x which carries a subscript y which carries a subscript z)

14.4.4 The effect of a level indicator with more than three components may be analyzed in the same manner suggested for level indicators with two or three components.

Example 14-21: Superscript with Four Components

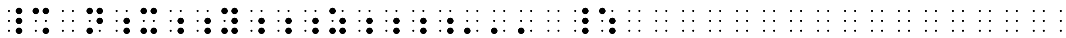
$n^{x^{y^{z^{\iddots}}}}$

⠸⠩ ⠝⠘⠭⠘⠘⠽⠘⠘⠘⠵⠘⠘⠘⠘⠄⠄⠄ ⠸⠱

(the ellipsis indicates the presence of superscripts of increasingly higher order; the dots are printed obliquely)

Example 14-22: Subscript with Four Components

$n_{x_{y_{z_{\ddots}}}}$



(the ellipsis indicates the presence of subscripts of increasingly higher order; the dots are printed obliquely)

14.5 Left Superscripts and Subscripts

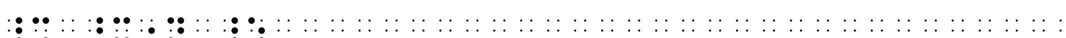
A superscript or subscript may occupy a position to the left, as well as to the right, of the sign to which it applies. The words *left* or *right* are then used with the words *superscript* or *subscript* to make the distinction in position.

14.5.1 A right or left superscript or subscript is represented as such merely by preserving the relative horizontal positions of the superscript or subscript symbol and the symbol to which it applies. Each must be preceded by its appropriate level indicator.

Left superscripts or subscripts of the third or higher order, although rare, are treated in the manner suggested by the examples below.

Example 14-23: Left Superscript

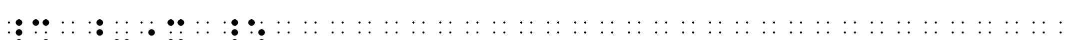
${}^{x}n$



(x is a left superscript to n)

Example 14-24: Left Superscript

${}^{-}x$



(the minus sign is a left superscript to x)

Example 14-25: Left Subscript

${}_{x}n$

⠸⠩⠀⠰⠭⠐⠝⠀⠸⠱

(x is a left subscript to n)

Example 14-26: Left and Right Subscripts

${}_{x}n_{y}$

⠸⠩⠀⠰⠭⠐⠝⠰⠽⠀⠸⠱

(x is a left subscript to n, y is a right subscript to n)

Example 14-27: Right Superscript with Left Superscript

$10^{{}^{-}4}$

⠸⠩⠀⠼⠂⠴⠘⠘⠤⠘⠲⠀⠸⠱

(10 to the minus 4 power; the minus is a left superscript to the 4)

Example 14-28: Left Superscript with Subscript

${}^{n_a}x$

⠸⠩⠀⠘⠝⠘⠰⠁⠐⠭⠀⠸⠱

(n sub a is a left superscript to x)

Example 14-29: Left Superscript with Left Subscript

${}^{{}_{a}n}\mathrm{X}$

⠸⠩⠀⠘⠰⠁⠘⠝⠐⠭⠀⠸⠱

(a is a left subscript to n, the combination is a left superscript to x)

Example 14-30: Left Subscript with Superscript

${}_{n^{a}}\mathrm{X}$

⠸⠩⠀⠰⠝⠰⠘⠁⠐⠭⠀⠸⠱

(a is a right superscript to n, the combination is a left subscript to x)

Example 14-31: Left Subscript with Left Superscript

${}_{{}^{a}n}x$

⠸⠩⠀⠰⠘⠁⠰⠝⠐⠭⠀⠸⠱

(a is a left superscript to n, the combination is a left subscript to x)

Example 14-32: Left Subscript with Subscript

${}_{x_y}n$

⠸⠩⠀⠰⠭⠰⠰⠽⠐⠝⠀⠸⠱

(x sub y is a left subscript to n)

Example 14-33: Left Subscript with Left Subscript

${}_{{}_{y}x}n$

⠸⠩⠀⠰⠰⠽⠰⠭⠐⠝⠀⠸⠱

(y is a left subscript to x, the combination is a left subscript to n)

14.5.2 A multipurpose indicator is inserted between a right superscript/subscript and a left superscript/subscript that follows unspaced in print.

Example 14-34: Right Subscript Followed by Left Superscript

$p_b\ {}^{c}x$

⠸⠩⠀⠏⠰⠃⠐⠘⠉⠐⠭⠀⠸⠱

(p carries a right subscript b; c is a left superscript to x)

Example 14-35: Right Superscript Followed by Left Subscript

$p^b\ {}_{c}x$

⠸⠩⠀⠏⠘⠃⠐⠰⠉⠐⠭⠀⠸⠱

(p carries a right superscript b; c is a left subscript to x)

14.6 Numeric Subscripts

The subscript indicator is not used to indicate a numeric subscript provided that all of the following conditions hold:

a. The corresponding numeric sign must be a *right*, and not a *left*, subscript.

b. The corresponding numeric sign must be a subscript of *first order*, and not of higher order.

c. The sign with which the numeric subscript is associated must be an abbreviated function name or a letter which has a separate identity. In the latter case, this letter must not be any letter which represents a numeral in a non-decimal base. Otherwise, the letter may be from any alphabet and in any typeform, and may be modified by one or more primes, or a superscript. A multi-letter chemical element is treated as if it were one letter.

d. The subscript consists of numeric symbols only, and carries no superscripts or subscripts of its own.

Example 14-36: Right Numeric Subscript to a Letter

x_1

⠸⠩⠀⠭⠂⠀⠸⠱

(x sub 1; subscript indicator not required because all conditions **a-d** hold)

Example 14-37: Right Numeric Subscript to a Letter

x_{11}

⠸⠩⠀⠭⠂⠂⠀⠸⠱

(x sub 11; subscript indicator not required because all conditions **a-d** hold)

Example 14-38: Right Numeric Subscript to a German Letter

$\mathfrak{A}_1$

⠸⠩⠀⠸⠠⠁⠂⠀⠸⠱

(German capitalized ah sub 1; subscript indicator not required because all conditions **a-d** hold)

Example 14-39: Right Numeric Subscript to a Prime

x'_1

⠸⠩⠀⠭⠄⠂⠀⠸⠱

(x prime sub 1; subscript indicator not required because all conditions **a-d** hold)

Example 14-40: Right Subscript with a Subscript

x_{i_1}

⠸⠩⠀⠭⠰⠊⠰⠰⠂⠀⠸⠱

x sub i sub 1; sub subscript indicator is required because condition **b** does not hold)

Example 14-41: Subscript to an Abbreviated Function Name

$\log_2 x$

⠸⠩⠀⠇⠕⠛⠆⠀⠭⠀⠸⠱

(log base 2 of x; subscript indicator not required because all conditions **a-d** hold)

Example 14-42: Right Numeric Subscript to a Number

12_7

⠸⠩⠀⠼⠂⠆⠰⠶⠀⠸⠱

(12 sub 7; subscript indicator is required because condition **c** does not hold)

Example 14-43: Right Numeric Subscript to Punctuation

$(CO_3)_2$

⠸⠩⠀⠷⠠⠉⠠⠕⠒⠾⠰⠆⠀⠸⠱

(the carbonate radical taken twice: subscript indicator is required before the 2 because condition **c** does not hold)

Example 14-44: Right Numeric Subscript to Chemical Symbols

Na_2CO_3

⠸⠩⠀⠠⠝⠁⠆⠠⠉⠠⠕⠒⠀⠸⠱

(sodium carbonate; subscript indicator not required because all conditions **a-d** hold)

Example 14-45: Right Numeric Subscript to a Word

seven_3

⠸⠩⠀⠎⠑⠧⠑⠝⠰⠒⠀⠸⠱

(seven sub 3; subscript indicator is required because condition **c** does not hold)

Example 14-46: Right Subscript with a Superscript

x_{2^n}

⠸⠩⠀⠭⠰⠆⠰⠘⠝⠀⠸⠱

(x carries a subscript 2 which carries a superscript n; subscript indicator is required because condition **d** does not hold)

Example 14-47: Subscript with Prime Sign

$x_{2'}$

⠸⠩⠀⠭⠰⠆⠄⠀⠸⠱

(x sub 2 prime; subscript indicator is required because condition **d** does not hold)

Example 14-48: Right Subscript with Plus Sign

x_{2+k}

⠸⠩⠀⠭⠰⠆⠬⠅⠀⠸⠱

(x carries a subscript of 2 plus k; subscript indicator is required because condition **d** does not hold)

Example 14-49: Right Subscript with a Diagonal Line

$x_{1/2}$

⠸⠩⠀⠭⠰⠂⠸⠌⠆⠀⠸⠱

(x sub one-half; subscript indicator is required because condition **d** does not hold)

Example 14-50: Left Numeric Subscript

${}_3x_1$

⠸⠩⠀⠰⠒⠐⠭⠂⠀⠸⠱

(3 is a left subscript to x, 1 is a right subscript to x; subscript indicator is required before the 3 because condition **a** does not hold)

Example 14-51: Non-Numeric Subscript

a_{x1}

⠸⠩⠀⠁⠰⠭⠂⠀⠸⠱

(a sub x 1; subscript indicator is required because condition **d** does not hold)

Example 14-52: Right Numeric Subscript to a Letter

$x_{10,000}$

⠸⠩⠀⠭⠂⠴⠠⠴⠴⠴⠀⠸⠱

(x sub 10,000; subscript indicator not required because all conditions **a-d** hold)

Example 14-53: Right Decimal Number Subscript to a Letter

$x_{1.2}$

⠸⠩⠀⠭⠂⠨⠆⠀⠸⠱

(x sub 1.2; subscript indicator not required because all conditions **a-d** hold)

Example 14-54: Right Decimal Number Subscript to a Letter

$x_{.6}$

⠸⠩⠀⠭⠨⠖⠀⠸⠱

(x sub .6; subscript indicator not required because all conditions **a-d** hold)

Example 14-55: Right Numeric Subscript to a Greek Letter

$\sum_0^n a_k$

⠸⠩⠀⠨⠠⠎⠴⠘⠝⠐⠁⠰⠅⠀⠸⠱

(the summation from zero to n of a sub k; subscript indicator is not required because all conditions **a-d** hold)

Example 14-56: Letter Representing a Numeral

$3AF_{16}$

⠸⠩⠀⠼⠒⠁⠋⠰⠂⠖⠀⠸⠱

(A and F represent a numeral in base 16; subscript indicator is required because condition **c** does not hold.)

Example 14-57: Right Subscript to an Integral Sign

$\int_0^{\sqrt{1 - x^2}} f(x)\,dx$

⠸⠩⠀⠮⠰⠴⠘⠜⠂⠤⠭⠘⠘⠆⠘⠻⠐⠋⠷⠭⠾⠙⠭⠀⠸⠱

(the integral from 0 to the square root of 1 — x^2 of f of x dx; subscript indicator is required because condition **c** does not hold)

14.7 Comma at Superscript or Subscript Level

A commonly occurring superscript or subscript notation in print is the one in which two consecutive items are separated by a comma or a comma and a space. In this configuration, the symbol (⠪) is used to replace the comma and the optional space used in this way. This contracted form is not used to replace a comma and the optional space which follows it in a configuration which is on the baseline.

Example 14-58: Contracted Comma in a Subscript

$x_{i,\ j,\ k}$

⠸⠩ ⠭⠰⠊⠪⠚⠪⠅ ⠸⠱

(each comma is followed by a space in print)

Example 14-59: Contracted Comma in a Subscript

$x_{(a,b)}$

⠸⠩ ⠭⠰⠷⠁⠪⠃⠾ ⠸⠱

(the comma is not followed by a space in print)

Example 14-60: Contracted Comma in a Subscript

$x_{1,\ 2}$

⠸⠩ ⠭⠰⠂⠪⠆ ⠸⠱

(the comma is followed by a space in print)

Example 14-61: Contracted Comma in a Subscript

$P_{n_{x,\ y}}$

⠸⠩ ⠠⠏⠰⠝⠰⠰⠭⠪⠽ ⠸⠱

(the comma is followed by a space in print)

Example 14-62: Contracted Comma in a Subscript

$x_{n\text{-}1,\ n\text{-}1},\ x_{n\text{-}1,\ n},\ x_{n,\ n\text{-}1}$

⠸⠩ ⠭⠰⠝⠤⠂⠪⠝⠤⠂⠠ ⠭⠰⠝⠤⠂⠪⠝⠠ ⠭⠰⠝⠪⠝⠤⠂ ⠸⠱

(two different kinds of braille commas are used in the example; the contracted comma is not used between items on the baseline)

Example 14-63: Non-Use of Contracted Comma on the Baseline

$(x,\ y)$

⠸⠩ ⠷⠭⠠ ⠽⠾ ⠸⠱

(the contracted comma is not used between items on the baseline)

14.8 Circumstances Determining Changes of Level

The symbols and situations listed below have the following effect in determining changes of level.

14.8.1 A level indicator terminates the effect of a previous level indicator and initiates the level stated by the new indicator. In the case of the baseline level, the previous baseline indicator may only be implied, as in examples **14-64** through **14-66**.

Example 14-64: Use of Baseline Indicator

$x^2 + 1$

⠸⠩⠀⠭⠘⠆⠐⠬⠂⠀⠸⠱

(superscript indicator terminates the previous implied baseline level and initiates the superscript level, the baseline indicator terminates the previous superscript level and initiates the baseline level)

Example 14-65: Use of Baseline Indicator

$x_a + y^2$

⠸⠩⠀⠭⠰⠁⠐⠬⠽⠘⠆⠀⠸⠱

Example 14-66: Use of Baseline Indicator

$$\frac{e^{x^2}}{2}$$

⠸⠩⠀⠹⠑⠘⠭⠘⠘⠆⠐⠌⠆⠼⠀⠸⠱

(the superscript with superscript indicator terminates the effect of the superscript indicator)

14.8.2 The punctuation indicator terminates the effect of any previous level indicator and initiates the baseline level. In addition, the comma, provided it is not a numeric symbol, terminates the effect of any previous indicator and initiates the baseline level. However, the comma, when it is a numeric symbol and the contracted form, preserves the level that is already in effect.

Example 14-67: Level Indicator and Punctuation Indicator

$a^{n + n + n \dots \text{ to } m \text{ } n\text{'s}} = a^{mn}$

⠸⠩⠀⠁⠘⠝⠬⠝⠬⠝⠀⠄⠄⠄⠀⠘⠞⠕⠀⠘⠍⠀⠘⠝⠘⠸⠄⠎⠀⠨⠅⠀⠁⠘⠍⠝⠀⠸⠱

(the superscript indicator preceding the apostrophe preserves the effect of the preceding superscript indicator; otherwise, the punctuation indicator would terminate the effect of the previous level indicator and initiate the baseline level)

Example 14-68: Punctuation Indicator as Level Indicator

The answer is x^2. y^2 is wrong.

⠀⠀⠠⠮⠀⠁⠝⠎⠺⠻⠀⠊⠎⠀⠸⠩⠀⠭⠘⠆⠸⠲⠀⠽⠘⠆⠀⠸⠱⠀⠊⠎⠀⠺⠗⠰⠛⠲

(the period is at the baseline level)

Example 14-69: Comma and Level Indicator

x^2, x^3

⠸⠩⠀⠭⠘⠆⠠⠀⠭⠘⠒⠀⠸⠱

(the comma terminates the effect of the level indicator)

Example 14-70: Internal Comma and Level Indicator

$x^{10,000}$

⠸⠩⠀⠭⠘⠂⠴⠠⠴⠴⠴⠀⠸⠱

(the comma is a numeric symbol and preserves the superscript level)

Example 14-71: Contracted Comma and Level Indicator

$x_{i, j}$

⠸⠩⠀⠭⠰⠊⠪⠚⠀⠸⠱

(the contracted form for a comma and optional space preserves the subscript level that is already in effect)

14.8.3 A space or the transition to a new braille line which is followed by literary text or unrelated mathematical text terminates the effect of any previous level indicator and initiates the baseline level. If a space occurs between the parts of an abbreviation or phrase, the appropriate level indicator must be restated before each part.

Example 14-72: Space and Baseline Level

$2p^2$ is always even.

(the space that follows the superscript terminates the previous superscript level and initiates the baseline level)

Example 14-73: Space and Baseline Level

6.696×10^8 mph

(the space following the superscript terminates the superscript level and initiates the baseline level)

Example 14-74: Space and Baseline Level

$(x^2 \;\; y^2)$

(in context, these items are entries in a matrix and hence unrelated; the space terminates the superscript level on x^2 and initiates the baseline level on y)

Example 14-75: Space and Baseline Level

$\triangle_{\text{reg. polygon}}$

(level indicators are required between each part of this abbreviation to show that they are both at the subscript level)

Example 14-76: Space and Baseline Level

$\triangle_{\text{regular polygon}}$

(level indicators are required between each part of this phrase to show that they are both at the subscript level)

14.8.4 The space which immediately follows a symbol of shape, an abbreviated function name, or a function name that is not abbreviated, provided the latter is in a mathematical context, preserves the level that is already in effect.

If these items carry a superscript or subscript, the space which follows such a superscript or subscript reinstates the level in effect.

Example 14-77: Space Following a Shape

$b_{\triangle\ ABC}$

⠸⠩⠀⠃⠰⠫⠞⠀⠠⠁⠠⠃⠠⠉⠀⠸⠱

(the space preserves the subscript level at which the triangle appears)

Example 14-78: Space Following an Abbreviated Function Name

$e^{\sin x}$

⠸⠩⠀⠑⠘⠎⠊⠝⠀⠭⠀⠸⠱

(the space preserves the superscript level at which sin appears)

Example 14-79: Space Following an Abbreviated Function Name

$\cos^2 x$

⠸⠩⠀⠉⠕⠎⠘⠆⠀⠭⠀⠸⠱

(the space reinstates the baseline level of cos)

Example 14-80: Space Following an Abbreviated Function Name

$e^{\cos^2 x}$

⠸⠩⠀⠑⠘⠉⠕⠎⠘⠘⠆⠀⠭⠀⠸⠱

(the space reinstates the superscript level at which cos appears)

Example 14-81: Space Following Abbreviated Function Name

$e^{\sin x + i \cos x}$

⠸⠩⠀⠑⠘⠎⠊⠝⠀⠭⠬⠊⠉⠕⠎⠀⠭⠀⠸⠱

(each space preserves the superscript level at which the abbreviated function names appear)

Example 14-82: Space Following Abbreviated Function Name

$e^{\sin^2 x + \sin^2 y}$

⠸⠩⠀⠑⠘⠎⠊⠝⠘⠘⠆⠀⠭⠬⠎⠊⠝⠘⠘⠆⠀⠽⠀⠸⠱

(each space reinstates the superscript level at which the abbreviated function names appear)

Example 14-83: Space Following Abbreviated Function Name

$q^{\log_q a}$

⠸⠩⠀⠟⠘⠇⠕⠛⠘⠰⠟⠀⠁⠀⠸⠱

(the space reinstates the superscript level at which log appears)

Example 14-84: Space Following Abbreviated Function Name

$V_{\max (m, n)}$

⠸⠩⠀⠠⠧⠰⠍⠁⠭⠀⠷⠍⠪⠝⠾⠀⠸⠱

(the space preserves the subscript level at which max appears)

14.8.5 The space which occurs in a numeral for the purpose of dividing it into short regular segments preserves the level already in effect.

Example 14-85: Space in Partitioned Number

$e^{3.14159\ 26535}$

⠸⠩⠀⠑⠘⠒⠨⠂⠲⠂⠢⠔⠀⠆⠖⠢⠒⠢⠀⠸⠱

14.8.6 The space which *precedes* an ellipsis or long dash preserves the effect of any level indicator. When no indicator is present the baseline is implied. The space which *follows* the ellipsis or long dash preserves the level that is already in effect. If such a space is followed by unrelated mathematical text, a sign of comparison, or a Nemeth Code switch indicator, this space initiates the baseline level.

Example 14-86: Space and Ellipsis

$x^{1 + 1/2 + 1/3 + \ldots +1/n}$

⠸⠩⠀⠭⠘⠂⠬⠂⠸⠌⠆⠬⠂⠸⠌⠒⠬⠀⠄⠄⠄⠀⠬⠂⠸⠌⠝⠀⠸⠱

(both spaces preserve the superscript level)

Example 14-87: Space and Ellipsis

$s_1 \ldots s_n$

⠸⠩⠀⠎⠂⠀⠄⠄⠄⠀⠎⠰⠝⠀⠸⠱

(both spaces preserve the implied baseline level)

Example 14-88: Space and Dash

$10^{3\,+}$ — is equal to 10^5.

⠀⠀⠸⠩⠀⠼⠂⠴⠘⠒⠬⠀⠤⠤⠤⠤⠀⠸⠱⠀⠊⠎⠀⠑⠟⠥⠁⠇⠀⠞⠕

⠸⠩⠀⠼⠂⠴⠘⠢⠀⠸⠱⠲

(the space before the dash preserves the previous superscript level, and the space followed by the Nemeth Code terminator terminates the effect of the previous superscript level and initiates the baseline level)

Example 14-89: Space and Dash

$10^{3\,+}$ — $= 10^5$

⠸⠩⠀⠼⠂⠴⠘⠒⠬⠀⠤⠤⠤⠤⠀⠨⠅⠀⠼⠂⠴⠘⠢⠀⠸⠱

(the space before the dash preserves the previous superscript level, and the space followed by a comparison sign terminates the effect of the previous superscript level and initiates the baseline level)

Example 14-90: Space and Ellipsis

$x^2 \cdots$ and y^2

⠸⠩⠀⠭⠘⠆⠀⠠⠠⠠⠀⠠⠄⠯⠀⠽⠘⠆⠀⠸⠱

14.8.7 The space, or transition to a new braille line, which is followed by a comparison symbol terminates the effect of a level indicator already in effect and initiates the baseline level. The space *after* a comparison symbol preserves the level that is already in effect.

Example 14-91: Space Preceding a Comparison Sign

$w^2 + x^2 + y^2 + z^{12} \;=\; z^{12} + y^2 + x^2 + w^2$

(the space which is followed by the equals sign terminates the effect of the preceding superscript level and initiates the baseline level, the space after the equals sign preserves the baseline level)

Example 14-92: Space Preceding a Comparison Sign

$2^x < \; 3^x$

(the space which is followed by the less than sign terminates the effect of the preceding superscript level and initiates the baseline level, the space after the less than sign preserves the baseline level)

Example 14-93: Space Preceding a Comparison Sign

$q^{\log_q a} = a$

(the space which is followed by the equals sign terminates the effect of the preceding superscript level and initiates the baseline level, the space after the equals sign preserves the baseline level)

Example 14-94: Space Preceding a Comparison Sign

$\int_{u=a}$

(the subscript indicator before the equals sign keeps this symbol at the subscript level; the space after the equals sign preserves the level that is already in effect)

14.8.8 Any other symbol or situation preserves the level that is already in effect.

14.9 Use of Level Indicators

14.9.1 A level indicator must be used before any braille indicator or grouping symbol whenever this braille indicator or grouping symbol applies to a level other than the one currently in effect.

Example 14-95: Level Indicator and Terminator

$\sqrt{x^2 + y^2}$

⠸⠩⠀⠜⠭⠘⠆⠐⠬⠽⠘⠆⠐⠻⠀⠸⠱

(the termination indicator must be at the same level as the radical symbol)

Example 14-96: Level Indicator and Terminator

$e^{\sqrt{x^2 + y^2}}$

⠸⠩⠀⠑⠘⠜⠭⠘⠘⠆⠘⠬⠽⠘⠘⠆⠘⠻⠀⠸⠱

(the termination indicator must be at the same level as the radical symbol)

Example 14-97: Level Indicator and Fraction Indicator

$\frac{1}{x^2}$

⠸⠩⠀⠹⠂⠌⠭⠘⠆⠐⠼⠀⠸⠱

(the closing simple-fraction indicator must be at the same level as the opening simple-fraction indicator)

Example 14-98: Level Indicator and Fraction Indicators

$$\frac{d\left(\frac{x}{y}\right)}{1+\left(\frac{x}{y}\right)^2}$$

⠸⠩⠀⠠⠹⠙⠷⠹⠭⠌⠽⠼⠾⠠⠌⠂⠬⠷⠹⠭⠌⠽⠼⠾⠘⠆⠐⠠⠼⠀⠸⠱

(the closing complex-fraction indicator must be at the same level as the opening complex-fraction indicator)

Example 14-99: Level Indicator and Modifier

$\overline{x^2}$

(the multipurpose indicator initiating the modification is at the baseline; the modifier must be at the same level)

Example 14-100: Level Indicator and Cancellation Indicators

$x^2\ \cancel{y^2}$

(the opening and closing cancellation indicators must be at the same level)

Example 14-101: Level Indicator and Grouping Sign

$(x^2 + y^2)$

(the right grouping symbol must be on the same level as the left grouping symbol)

Example 14-102: Level Indicator and Grouping Sign

$x^{(m^n)}$

(the right grouping symbol must be on the same level as the left grouping symbol)

14.9.2 The superscript indicator must be used to restate the superscript level when two superscripts are consecutive but one applies to the expression which precedes it and the other applies to the expression which follows it. Similarly, the subscript indicator must be restated when two subscripts are consecutive and one applies to the expression preceding it and the other applies to the expression following it. A dot 5 separates the two level indicators. A superscript or subscript indicator must be restated before a modified expression which is interior to the superscript or subscript expression, provided that the multipurpose indicator is also used.

Example 14-103: Left Superscript Following Right Superscript

$p^{b}\ {}^{c}q$

Example 14-104: Right Subscript Followed by Left Subscript

$P_{b}\ {}_{c}Q$

Example 14-105: Right Numeric Subscript and Left Subscript

$P_{1}\ {}_{2}Q$

Example 14-106: Subscript with Subscript

$A_{\tilde{x}+\tilde{y}}$

(after the plus sign in this example, because the multipurpose indicator would return the following item to the baseline, the subscript level must be restated before the multipurpose indicator to keep y at the subscript level)

14.9.3 The appropriate level indicator must be used before each part of an abbreviation, phrase, or letter which is at a level other than the baseline.

Example 14-107: Words in a Subscript

$\triangle_{\text{regular polygon}}$

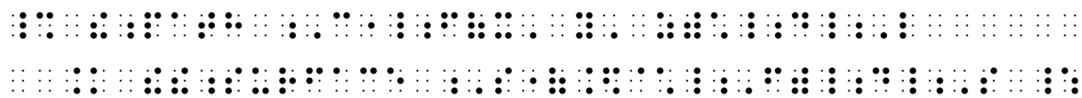
Example 14-108: Superscript with Ellipsis and Words

$a^{n+n+n\ldots\text{ to }m\ n\text{'s}}$

(the letters m and n are not “single letters” because they are not preceded by a space in braille)

Example 14-109: Subscript with a Letter

$$\int_{path\ C} \mathbf{f}(x, y, z) \cdot \mathbf{dL} = \iint_{surface\ S} (\nabla \times \mathbf{F})\mathbf{dS}$$

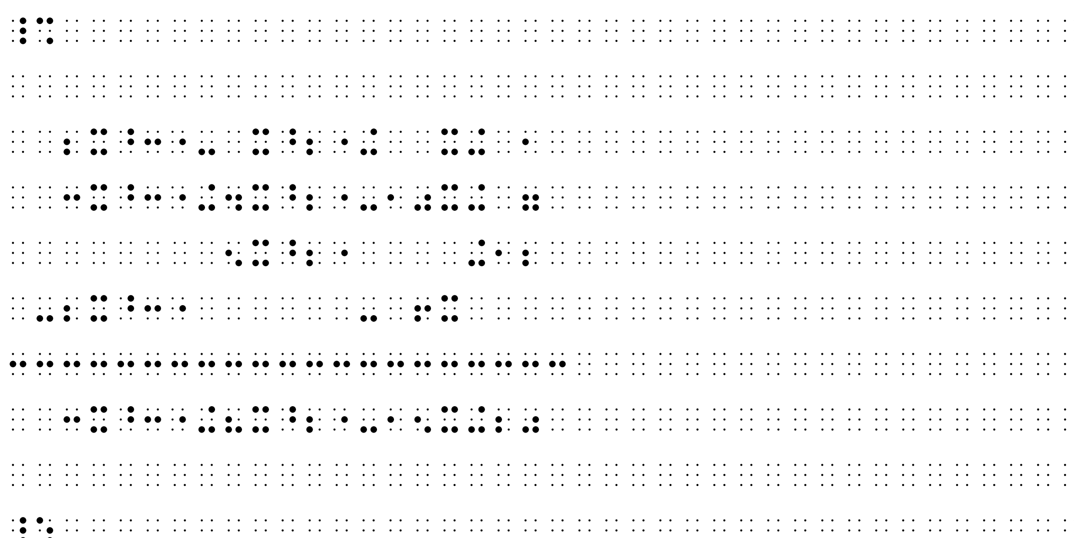


(the letters C and S are not “single letters” because they are not preceded by a space in braille)

14.9.4 Whenever spaces are left for the purpose of achieving alignment, level indicators must be used as though such spaces were not present. The level indicator precedes the space.

Example 14-110: Level Indicators and Spaces with Alignment

$$\begin{array}{rrrr}
2x^3 - & x^2 + & x + & 1 \\
3x^3 + & 4x^2 - & 10x + & 7 \\
 & 5x^2 & & +12 \\
-2x^3 & & -\ \ 6x & \\
\hline
3x^3 + & 8x^2 - & 15x & +20
\end{array}$$



14.9.5 The appropriate level indicator must be used before any symbol or situation in which a change of level is required but the change is not affected by any of the conditions of **14.8**.

Example 14-111: Hyphen and Baseline Indicator

360°-interval

⠸⠩ ⠼⠒⠖⠴⠘⠨⠡⠐⠤⠊⠝⠞⠑⠗⠧⠁⠇ ⠸⠱

(the baseline indicator places the hyphen at the baseline level)

Example 14-112: Level Indicators and Comparison Signs

$t\Big]_{t\,=\,a}^{t\,=\,b} = b - a$

⠸⠩ ⠞⠈⠾⠰⠞ ⠰⠨⠅ ⠁⠘⠞ ⠘⠨⠅ ⠃ ⠨⠅ ⠃⠤⠁ ⠸⠱

(the subscript and superscript indicators before the first two equals symbols keep these at the subscript and superscript levels respectively, while the space before the last equals symbol places it at the baseline level)

Example 14-113: Level Indicator and Comparison Signs

$e^{\sin x = a} > y$

⠸⠩ ⠑⠘⠎⠊⠝ ⠭ ⠘⠨⠅ ⠁ ⠨⠂ ⠽ ⠸⠱

(the superscript indicator before the equals symbol keeps this symbol at the superscript level)

Example 14-114: Level Indicator and Ellipsis

$P_{s_1 \, \ldots \, s_n}$

⠸⠩ ⠠⠏⠰⠎⠰⠰⠂ ⠰⠄⠄⠄ ⠎⠰⠰⠝ ⠸⠱

(the subscript indicator before the ellipsis places the ellipsis at the first-order subscript level)

Example 14-115: Level Indicator and Ellipsis

$P_1^{\alpha_1} \ldots P_r^{\alpha_r}$

⠸⠩ ⠠⠏⠂⠘⠨⠁⠘⠰⠂ ⠐⠄⠄⠄ ⠠⠏⠰⠗⠘⠨⠁⠘⠰⠗ ⠸⠱

(the baseline indicator places the ellipsis at the baseline level)

14.10 Non-Use of Level Indicators

14.10.1 The baseline indicator is not used to return to the baseline from a numeric subscript if the subscript indicator has not been used before the numeric subscript.

Example 14-116: Right Numeric Subscript to a Letter

$(x_1 + 1)$

⠸⠩⠀⠷⠭⠂⠬⠂⠾⠀⠸⠱

(baseline indicator not required before the plus symbol)

Example 14-117: Right Numeric Subscript

$(x_1y_1 + x_2y_2)$

⠸⠩⠀⠷⠭⠂⠽⠂⠬⠭⠆⠽⠆⠾⠀⠸⠱

(baseline indicator not required after any of the numeric subscripts)

14.10.2 The baseline indicator is not used before a right enlarged grouping symbol if the grouping symbol is separated from its preceding material by one or more spaces. The baseline indicator is not used before a right enlarged grouping symbol if the expression which touches the right grouping symbol is continued on the next line.

Example 14-118: Level Indicator and Matrix

$$\left\{ \begin{matrix} u = x^2 \\ v = x^2 + y^2 \end{matrix} \right\}$$

⠸⠩

⠀

⠨⠠⠷⠥⠀⠨⠅⠀⠭⠘⠆⠀⠀⠀⠀⠀⠀⠨⠠⠾

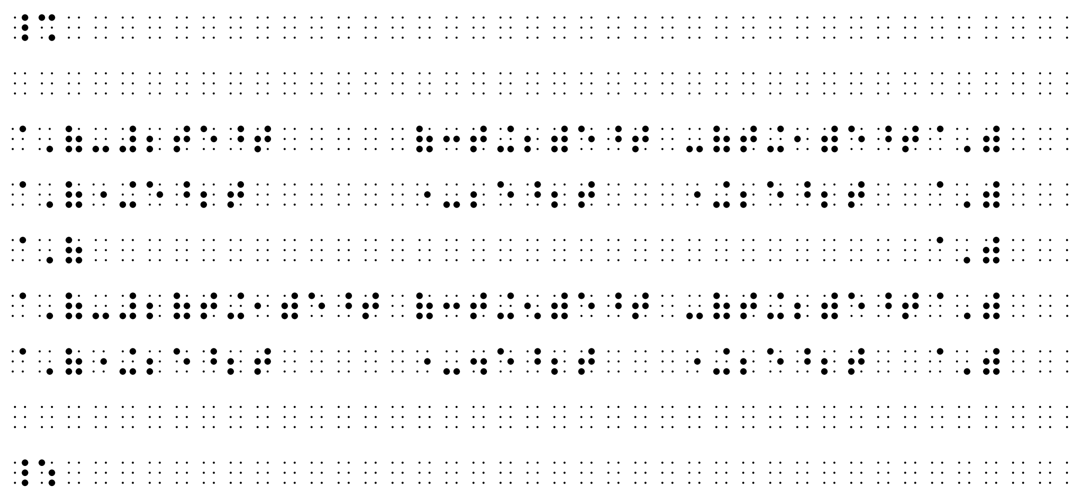
⠨⠠⠷⠧⠀⠨⠅⠀⠭⠘⠆⠐⠬⠽⠘⠆⠐⠨⠠⠾

⠀

⠸⠱

Example 14-119: Level Indicator with Divided Expression

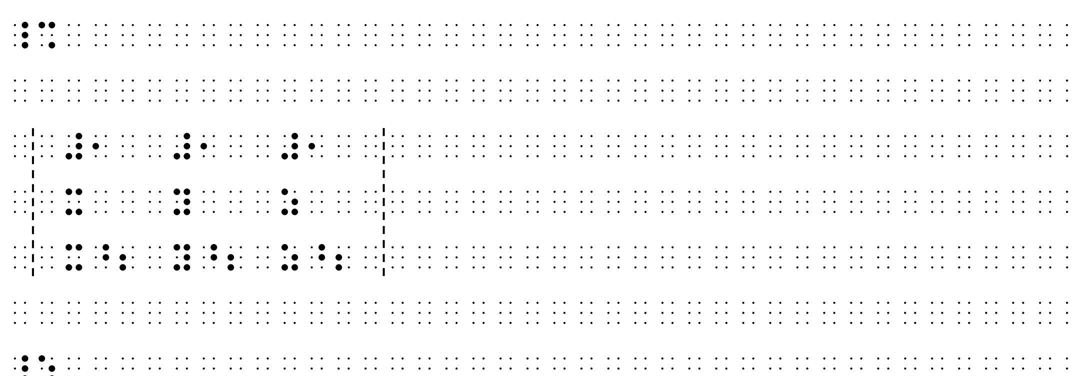
$$\begin{bmatrix} -2te^{t} + e^{2t} & (3t+2)\,e^{t} - 2e^{2t} & -(t+1)\,e^{t} + 2e^{2t} \\ -2\,(t+1)\,e^{t} + 2\,e^{2t} & (3t+5)\,e^{t} - 4e^{2t} & -(t+2)\,e^{t} + 2e^{2t} \end{bmatrix}$$



14.10.3 A level indicator must not be used before any right grouping symbol which is drawn in.

Example 14-120: Level Indicator and Graphics

$$\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix}$$



14.11 Simultaneous and Non-Simultaneous Superscripts and Subscripts

14.11.1 When an expression simultaneously carries a superscript and subscript, the subscript must be indicated first, even if the subscript is numeric and does not require the subscript indicator. If this expression carries one or more primes in addition, see **14.12**.

Example 14-121: Simultaneous Subscript and Superscript

x_a^n

⠸⠩⠀⠭⠰⠁⠘⠝⠀⠸⠱

(x carries simultaneously a subscript of a and a superscript of n)

Example 14-122: Simultaneous Subscript and Superscript

${}_a^n x$

⠸⠩⠀⠰⠁⠘⠝⠐⠭⠀⠸⠱

(x carries simultaneously a left subscript of a and a left superscript of n)

Example 14-123: Simultaneous Subscript and Superscript

x_1^2

⠸⠩⠀⠭⠂⠘⠆⠀⠸⠱

(x carries simultaneously a subscript of 1 and a superscript of 2)

14.11.2 When the same expression carries a superscript and a subscript which are not simultaneous, the relative horizontal positions of the signs must be retained in the transcription, but the baseline indicator must be inserted before making the transition to the other level.

Example 14-124: Non-Simultaneous Superscript and Subscript

$a^{n}{}_{m}$

⠸⠩⠀⠁⠘⠝⠐⠰⠍⠀⠸⠱

(the superscript is closer to the a than is the subscript)

Example 14-125: Non-Simultaneous Subscript and Superscript

$a_m{}^n$

(the subscript is closer to the a than the superscript)

Example 14-126: Non-Simultaneous Superscript and Subscript

${}^{a}{}_{b}x$

(the left subscript is closer to the x than the left superscript)

Example 14-127: Non-Simultaneous Subscript and Superscript

${}_{b}{}^{a}x$

(the left superscript is closer to the x than the left subscript)

Example 14-128: Non-Simultaneous Subscript and Superscript

$x_1{}^2$

(the subscript is closer to the x than the superscript)

Example 14-129: Non-Simultaneous Subscript and Superscript

$x'_a{}^b$

(the subscript is closer to x prime than the superscript)

14.12 Primes in Addition to Superscripts or Subscripts

14.12.1 The prime symbol must never be preceded by the superscript indicator.

Example 14-130: Prime Sign

x'

14.12.2 When an expression carries one or more primes in addition to superscripts or subscripts, the prime symbol or symbols must be indicated first unless such symbols do not occur at the beginning of the superscript or subscript, in which case they must retain the same position as in print.

Example 14-131: Prime Sign and Subscript

x'_a

⠸⠩ ⠭⠄⠰⠁ ⠸⠱

Example 14-132: Prime Sign and Superscript

x'^2

⠸⠩ ⠭⠄⠘⠆ ⠸⠱

Example 14-133: Prime and Simultaneous Super and Subscript

x'^b_a

⠸⠩ ⠭⠄⠰⠁⠘⠃ ⠸⠱

Example 14-134: Double Prime with Superscript and Subscript

x''^3_1

⠸⠩ ⠭⠄⠄⠂⠘⠒ ⠸⠱

Example 14-135: Prime Sign and Superscript Asterisk

x'^*

⠸⠩ ⠭⠄⠘⠈⠼ ⠸⠱

Example 14-136: Superscript Asterisk with Prime Sign

$x^{*\prime}$

⠸⠩ ⠭⠘⠈⠼⠄ ⠸⠱

Example 14-137: Simultaneous Sub and Superscript with Prime

$A^{*\prime}_{ue}$

⠸⠩ ⠠⠁⠰⠥⠑⠘⠈⠼⠄ ⠸⠱

Example 14-138: Prime Sign with Simultaneous Super and Sub

$A_{ue}^{\prime *}$

⠸⠩⠀⠠⠁⠄⠰⠥⠑⠘⠈⠼⠀⠸⠱

14.12.3 For primes in other roles see **23.15**.

14.13 Plurals and Possessives

The punctuation indicator returns the apostrophe-s to the baseline when it follows a superscript or subscript. For plurals or possessives of mathematical expressions in general, see **8.4**.

Example 14-139: Possessive with Superscript

x^2's

⠸⠩⠀⠭⠘⠆⠸⠄⠎⠀⠸⠱

(the plural of x squared)

Example 14-140: Possessive with Subscript

c_i's

⠸⠩⠀⠉⠰⠊⠸⠄⠎⠀⠸⠱

(the plural of c sub i)

Example 14-141: Possessive with Subscript

The c_1's, c_2's, ..., c_n's.

⠀⠀⠠⠮⠀⠸⠩⠀⠉⠂⠸⠄⠎⠠⠀⠉⠆⠸⠄⠎⠠⠀⠄⠄⠄⠠⠀⠉⠰⠝⠸⠄⠎⠀⠸⠱⠲

(the plurals of c sub 1, c sub 2, ..., c sub n)

Rule 15
Modifiers

Modification Indicators		
Directly Over		
First order		⠣
Second order		⠣⠣
Directly Under		
First order		⠩
Second order		⠩⠩
Multipurpose		⠐
Superposition		⠈
Termination		⠻
Modifiers		
Arc		
Concave downward	⌢	⠫⠁
Concave upward	⌣	⠫⠄
Arrow		
Barbed at both ends	↔	⠫⠪⠒⠒⠕
Barbed at left	←	⠫⠪⠒⠒
Barbed at left and dotted at right	←●	⠫⠪⠒⠒⠡
Barbed at right		
Contracted form	→	⠫⠕
Uncontracted form	→	⠫⠒⠒⠕
Dotted at both ends	●—●	⠫⠡⠒⠒⠡
Dotted at left (no barb)	●—	⠫⠡⠒⠒
Dotted at left and barbed at right	●→	⠫⠡⠒⠒⠕
Dotted at right (no barb)	—●	⠫⠒⠒⠡

Hollow dot at both ends	○——○	⠫⠨⠡⠒⠒⠨⠡
Hollow dot at left (no barb)	○——	⠫⠨⠡⠒⠒
Hollow dot at left and barbed at right	○——▶	⠫⠨⠡⠒⠒⠕
Hollow dot at right and barbed at left	◀——○	⠫⠪⠒⠒⠨⠡
Hollow dot at right (no barb)	——○	⠫⠒⠒⠨⠡
Bar		
Horizontal (macron)	—	⠱
Vertical	|	⠳
Caret (circumflex)	∧	⠸⠣
Inverted	∨	⠸⠩
Left-pointing	<	⠰⠣
Right-pointing	>	⠰⠩
Dot	●	⠡
Hollow Dot	∘	⠨⠡
Question Mark	?	⠸⠦
Tilde		
Extended	~~	⠈⠠⠱
Simple	~	⠈⠱
Triangle (equilateral)	△	⠫⠞

15.1 Modifiers

A modifier is a superscript or subscript which occupies, respectively, a position directly over or directly under the sign to which it applies. The modifiers in the list at the beginning of this rule are those most commonly used, but other modifiers must be treated in the same manner.

15.2 Modified Expressions

15.2.1 **The Five-Step Rule for Transcribing Modified Expressions.** The components of a modified expression must appear in the following order:

a. Multipurpose indicator

b. Expression being modified

c. Directly-over indicator or directly-under indicator

d. Modifier

e. Termination indicator

These five components must never be separated from each other by transition to another braille line. The termination indicator terminates only the modified expression; it does not affect the level at which the modified expression occurs.

Example 15-1: AB with Superscribed Bar

$\overline{\mathrm{AB}}$

⠸⠩ ⠐⠠⠁⠠⠃⠣⠱⠻ ⠸⠱

Example 15-2: x Plus y with Superscribed Bar

$\overline{x+y}$

⠸⠩ ⠐⠭⠬⠽⠣⠱⠻ ⠸⠱

Example 15-3: Limit with Subscribed Expression

$\lim_{x \to 0} f(x)$

⠸⠩ ⠐⠇⠊⠍⠩⠭ ⠫⠕ ⠼⠴⠻ ⠋⠷⠭⠾ ⠸⠱

(the limit, as *x* approaches 0, of *f* of *x*)

Example 15-4: x Squared with Superscribed Bar

$\overline{x^2}$

⠸⠩ ⠐⠭⠘⠆⠐⠣⠱⠻ ⠸⠱

Example 15-5: x Prime with Superscribed Bar

$\overline{x'}$

Example 15-6: 1 Subscript to x with Superscribed Bar

$\overline{x_1}$

Example 15-7: n Subscript to x with Superscribed Bar

$\overline{x_n}$

Example 15-8: Termination of Modified Expression

${}_{x}\mathrm{A}_1$

(modified expression $\tilde{x}$ is terminated at the subscript level; a baseline indicator terminates the subscript level and returns A to the baseline)

15.2.2 When the expression being modified is a single digit or a letter, lowercase or capitalized, from any alphabet, and in any typeform, and when the modifier is the horizontal bar directly above such a single digit or letter, the digit or letter, followed by the bar, serves to express the modification. This construction should be regarded as a contracted form of the expression and is used whenever applicable. If the expression being modified includes a superscript, subscript, or prime, the five-step rule of **15.2.1** above is followed. The five-step rule may be used in conjunction with the contracted form.

Example 15-9: x with Superscribed Bar

$\overline{x}$

Example 15-10: x and y with Superscribed Bars

$\overline{x}+\overline{y}$

Example 15-11: x with Superscribed Bar

$\overline{x}y$

Example 15-12: y with Superscribed Bar

$x\overline{y}z$

Example 15-13: x with Superscribed Bar Squared

$\overline{x}^{2}$

Example 15-14: x with Superscribed Bar Primed

$\overline{x}'$

Example 15-15: x with Superscribed Bar and Subscript 1

$\overline{\mathrm{x}}_{1}$

Example 15-16: x with Superscribed Bar and Subscript n

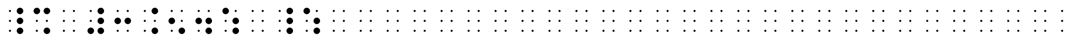
$\overline{\mathrm{x}}_{\mathrm{n}}$

Example 15-17: Boldface Z with Superscribed Bar

$\overline{\mathbf{Z}}$

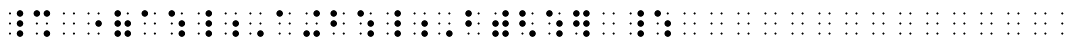
Example 15-18: Number with Superscribed Bar

$3.5\overline{4}$



Example 15-19: Lowercase Letters with Superscribed Bars

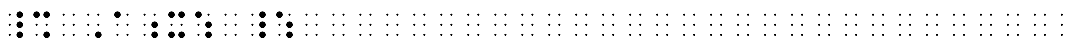
$\left(\overline{\bar{a}\mathbf{A}+\bar{b}\mathbf{B}}\right)$



(a with superscribed bar times boldface capitalized A plus b with superscribed bar times boldface capitalized B. The bar above the whole expression follows the 5-step rule of **15.2.1** above.)

Example 15-20: Subscript x with Superscribed Bar

$A_{\bar{x}}$



(A with a right subscript of x with superscribed bar)

Example 15-21: Subscript Has Letters with Superscribed Bars

$A_{\bar{x}+\bar{y}}$

(A with a right subscript of x with superscribed bar plus y with superscribed bar)

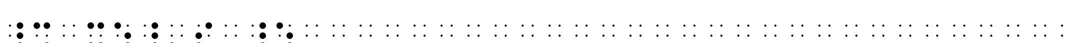
Example 15-22: Superscript Containing x with Superscribed Bar

$e^{a\bar{x}}$

(e with a right superscript of a times x with superscribed bar)

Example 15-23: x with Superscribed Bar and Apostrophe s

$\bar{x}$'s



15.2.3 When the expression being modified is a single digit or a letter, lowercase or capitalized, from any alphabet, and in any typeform, and when the modifier is the horizontal bar directly under such a single digit or letter, the digit or letter, followed by the directly under symbol and the bar, serves to express the modification. This construction should be regarded as a contracted form of the expression and is used whenever applicable. If the modification includes a superscript, subscript, or prime, the five-step rule of **15.2.1** above is followed. The five-step rule may be used in conjunction with the contracted form.

Example 15-24: x with Subscribed Bar

$\underline{x}$

⠸⠩⠀⠭⠩⠱⠀⠸⠱

Example 15-25: Letters with Subscribed Bars

$\underline{x} + \underline{y}$

⠸⠩⠀⠭⠩⠱⠬⠽⠩⠱⠀⠸⠱

Example 15-26: x with Subscribed Bar

$\underline{x}y$

⠸⠩⠀⠭⠩⠱⠽⠀⠸⠱

Example 15-27: y with Subscribed Bar

$x\underline{y}z$

⠸⠩⠀⠭⠽⠩⠱⠵⠀⠸⠱

Example 15-28: x with Subscribed Bar Squared

$\underline{x}^2$

⠸⠩⠀⠭⠩⠱⠘⠆⠀⠸⠱

Example 15-29: x with Subscribed Bar Primed

$\underline{x}'$

⠸⠩⠀⠭⠩⠱⠄⠀⠸⠱

Example 15-30: Subscribed Bar and Subscript 1

$\underline{x}_1$

Example 15-31: Subscribed Bar and Subscript n

$\underline{x}_n$

Example 15-32: Boldface Z with Subscribed Bar

$\underline{\mathbf{Z}}$

Example 15-33: Underlined Numbers

Write the place value of the underlined digit.

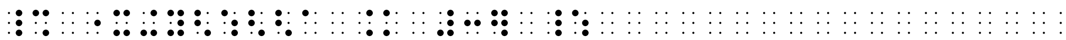
1. 9$\underline{6}$5,132
2. 3.049$\underline{2}$
3. 12,$\underline{7}$52
4. 94,23$\underline{7}$.1

15.3 Modifiers of Higher Order

15.3.1 A modifier of the second order is preceded by the second-order directly-over or directly-under indicator, and a modifier of the third order is preceded by the third-order directly-over or directly-under indicator. The termination indicator, however, is used only once, after the last modifier symbol.

Example 15-34: Modifier of Second Order

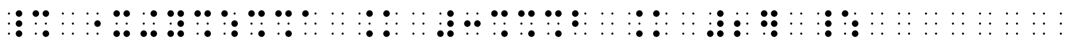
$$\overset{a=3}{\overline{x+y}}$$



(x plus y superscribed by a bar, which in turn is superscribed by a equals 3)

Example 15-35: Modifier of Third Order

$$\underset{\substack{a=3\\ b=2}}{\underline{x+y}}$$



(x plus y subscribed by a bar, which in turn is subscribed by a equals 3, which in turn is subscribed by b equals 2)

15.3.2 A modifier of order higher than the third is treated in the manner suggested in **15.3.1** above.

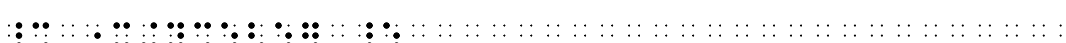
15.3.3 A modifier, to be of order higher than the first, must be associated with the same expression as a modifier of lower order. In Example **15-19** above, the long bar is not a modifier of second order because no modifier of first order is associated with the same expression as the long bar.

15.4 Simultaneous Modifiers

When a mathematical expression is simultaneously modified above and below, the modifier below is indicated first. The termination indicator is used only once, after the last modifier symbol. If the modifiers involved are of order higher than the first, they are treated as described in **15.3**.

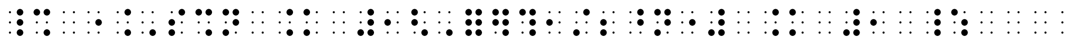
Example 15-36: x Plus y with Subscribed and Superscribed Bars

$$\overline{\underline{x+y}}$$



Example 15-37: Subscribed and Superscribed Expressions

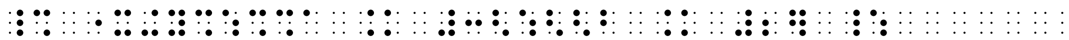
$$\sum_{n=1}^{\infty} \frac{1}{2^n} = 1$$



(the Greek capitalized sigma with subscribed n equals 1 and superscribed infinity sign)

Example 15-38: Subscribed and Superscribed Expressions

$$\underset{a=3}{\underline{\overset{b=2}{\overline{x+y}}}}$$



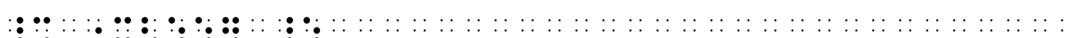
(x plus y subscribed by a bar which is in turn subscribed by a equals 3; superscribed by a bar which is in turn superscribed by b equals 2)

15.5 Parallel Horizontal Bars

Parallel horizontal bars must not be regarded as the equals sign or the identity sign when they occur above or below a mathematical expression other than a comparison sign. Furthermore, the bar which is more remote from the mathematical expression being modified is not regarded as a modifier of second or third order; the double or triple bar is regarded as a single modifier.

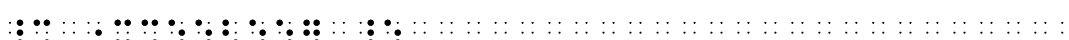
Example 15-39: x with Superscribed Double Bar

$$\overset{=}{x}$$



Example 15-40: Subscribed and Superscribed Double Bars

$$\underset{=}{\overset{=}{x}}$$



Example 15-41: Subscribed Double Bar and Superscribed Bar

$\underline{\underline{\bar{x}}}$

Example 15-42: x with Subscribed Triple Bar

$\underline{\underline{\underline{x}}}$

15.6 Binomial Coefficient

The two expressions which constitute a binomial coefficient are separated by the directly-under indicator. The expression which follows the left parenthesis and precedes the directly-under indicator corresponds to the upper sign in the binomial coefficient; the expression which follows the directly-under indicator and precedes the right parenthesis corresponds to the lower sign of the binomial coefficient.

Example 15-43: Binomial Coefficient

$$\binom{n}{k}$$

Example 15-44: Binomial Coefficient Containing Subscripts

$$\binom{g_j}{a_j}$$

(the binomial coefficient with g sub j as the upper sign and a sub j as the lower sign)

15.7 Modified Expressions in Superscripts and Subscripts

If a modified expression is part or all of a right superscript or subscript, the multipurpose indicator is preceded by the appropriate level indicator. This will automatically be the case if the modified expression occurs at the beginning of the superscript or subscript; but the appropriate level indicator must be restated if the modified expression occurs at an interior position of the superscript or subscript. If the contracted form for a modified expression is used so that the multipurpose indicator does not appear, the appropriate level indicator is not restated.

Example 15-45: A with Modified Subscript Expression

$A_{\tilde{x}}$

⠸⠩⠀⠠⠁⠰⠐⠭⠣⠈⠱⠻⠀⠸⠱

(A carries a subscript of x with superscribed tilde)

Example 15-46: A with Modified Subscript Expression

$A_{\tilde{x}+\tilde{y}}$

⠸⠩⠀⠠⠁⠰⠐⠭⠣⠈⠱⠻⠬⠰⠐⠽⠣⠈⠱⠻⠀⠸⠱

(A carries a subscript of x with superscribed tilde plus y with superscribed tilde; the subscript level after the plus sign is restated before the multipurpose indicator)

Example 15-47: A with Modified Subscript Expression

$A_{\bar{x}+\bar{y}}$

⠸⠩⠀⠠⠁⠰⠭⠱⠬⠽⠱⠀⠸⠱

(A carries a subscript of x with superscribed bar plus y with superscribed bar)

15.8 Plural Modified Expressions

See **8.4**.

15.9 Modification by Superposition

Superposed signs are signs which are printed one upon another so that one sign extends beyond the boundary of the other. The components of a sign modified by superposition are joined by the superposition indicator (⠈)

and are transcribed unspaced, without transition to another braille line. The termination indicator follows the second component. In deciding which is the basic sign (transcribed first) and which is the superposed sign (transcribed second), the following hierarchy is used as a guide:

a. Integral sign

b. Operation signs

c. Bars — horizontal and vertical

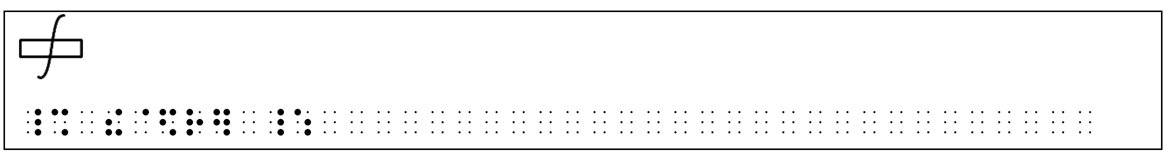
d. Shape signs

e. Comparison signs

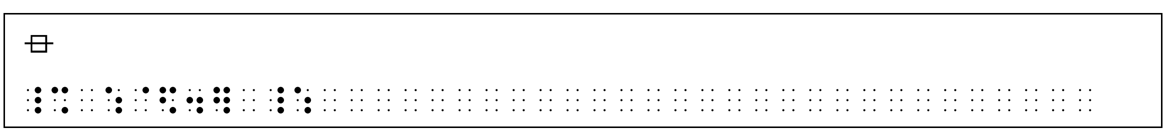
f. Signs not covered above

A sign belonging to a category lower on the list is regarded as superposed on a sign higher on the list, and the superposition transcribed accordingly. If two signs belong to the same category, it is permissible to represent the superposition in either order, provided that the same order is used consistently throughout the entire transcription. (For other examples, see "Comparison Signs Compounded by Superposition" in **Rule 21**.)

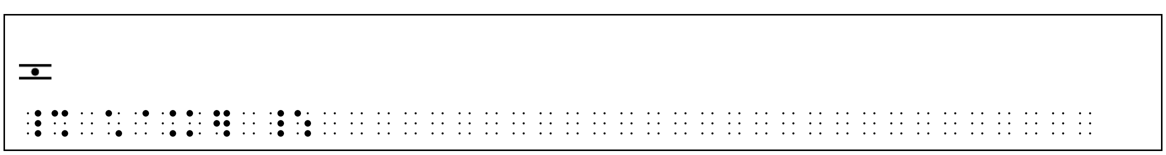
Example 15-48: Integral Sign with Superposed Shape



Example 15-49: Horizontal Bar with Superposed Shape



Example 15-50: Operation Sign with Superposed Equals Sign



Example 15-51: Superposed Comparison Signs

$⋹$

or

Example 15-52: Superposed Signs of Shape

$\measuredangle$

or

15.10 Interior Modifiers with Signs of Shape

See **17.6**.

15.11 Arc

Example 15-53: "A" with Subscribed Arc Concave Downward

$\underset{\frown}{\mathrm{A}}$

Example 15-54: AB with Superscribed Arc Concave Downward

$\overset{\frown}{\mathrm{AB}}$

Example 15-55: "A" with Subscribed Arc Concave Upward

$\underset{\smile}{\mathrm{A}}$

Example 15-56: AB with Subscribed Arc Concave Upward

$\underset{\smile}{\mathrm{AB}}$

15.12 Arrows

Arrows are not regarded as modifiers when they occur directly over or directly under a comparison sign. In that event, they become a component of a sign of comparison compounded vertically.

When a right-pointing arrow with a single shaft of ordinary length is in regular type, has a full barb, and is not part of a more complex construction or compound modifier, it is transcribed in its contracted form. If such an arrow is in non-regular type, does not have a full barb or shaft of ordinary length, is part of a compound modifier, or is itself modified, it is represented in its uncontracted form.

Example 15-57: Superscribed Arrow with Right Barb

$\overrightarrow{AB}$

⠸⠩⠀⠐⠠⠁⠠⠃⠣⠫⠕⠻⠀⠸⠱

Example 15-58: Superscribed Arrow with Left Barb

$\overleftarrow{AB}$

⠸⠩⠀⠐⠠⠁⠠⠃⠣⠫⠪⠒⠒⠻⠀⠸⠱

Example 15-59: Superscribed Arrow Barbed at Both Ends

$\overleftrightarrow{AB}$

⠸⠩⠀⠐⠠⠁⠠⠃⠣⠫⠪⠒⠒⠕⠻⠀⠸⠱

Example 15-60: Superscribed Barbed Arrow with Right End Dot

AB

⠸⠩⠀⠐⠠⠁⠠⠃⠣⠫⠪⠒⠒⠡⠻⠀⠸⠱

Example 15-61: Superscribed Arrow Dotted at Both Ends

AB

⠸⠩⠀⠐⠠⠁⠠⠃⠣⠫⠡⠒⠒⠡⠻⠀⠸⠱

Example 15-62: Superscribed with Hollow Dot at Both Ends

$\overline{AB}$

⠸⠩⠀⠐⠠⠁⠠⠃⠣⠫⠨⠡⠒⠒⠨⠡⠻⠀⠸⠱

Example 15-63: Superscribed Arrow Dotted at Left

$\overline{AB}$

⠸⠩⠀⠐⠠⠁⠠⠃⠣⠫⠡⠒⠒⠻⠀⠸⠱

Example 15-64: Superscribed Arrow with Left Dot, Right Barb

$\overrightarrow{AB}$

⠸⠩⠀⠐⠠⠁⠠⠃⠣⠫⠡⠒⠒⠕⠻⠀⠸⠱

Example 15-65: Superscribed Arrow Dotted at Right

$\overline{AB}$

⠸⠩⠀⠐⠠⠁⠠⠃⠣⠫⠒⠒⠡⠻⠀⠸⠱

Example 15-66: Arrow Shaft with Superscribed Expression

$X \xrightarrow{f \circ g} Y$

⠸⠩⠀⠠⠭⠀⠐⠫⠒⠒⠕⠣⠋⠨⠡⠛⠻⠀⠠⠽⠀⠸⠱

15.13 Bar (Horizontal)

15.13.1 The horizontal bar is often used to indicate the recurrence of one or more digits in a decimal numeral by placing it over the digits which recur. See **15.2.1** for contracted modification.

Example 15-67: Horizontal Bar as Contracted Modification

$.\overline{3}$

⠸⠩⠀⠼⠨⠒⠱⠀⠸⠱

Example 15-68: Decimal Number with Superscribed Bar

$.\overline{7128}$

⠸⠩⠀⠼⠨⠐⠶⠂⠆⠦⠣⠱⠻⠀⠸⠱

Example 15-69: Decimal Number with Superscribed Bar

$3.57\overline{29}$

15.13.2 The horizontal bar must not be regarded as a modifier when it occurs directly over or directly under a comparison sign. In that event, it becomes a component of a sign of comparison compounded vertically (see **21.9**). When the horizontal bar is itself modified by a dot under it or a caret directly over or under it, the combination is a modified sign of comparison (see **21.8**). When the horizontal bar is itself modified by a dot over it, the combination is a sign of operation.

15.13.3 When the horizontal bar occurs over or under the integral sign, over or under the function name for *limit*, or its abbreviated form *lim*, the bar is not treated as a modifier (see **23.12** and **18.3**, respectively).

15.14 Brace or Bracket

See Rule **19.2** regarding transcribing a horizonal brace or bracket as a modifier.

15.15 Caret

Example 15-70: x with Superscribed Caret

$\hat{x}$

Example 15-71: Equals Sign with Superscribed Caret

$\stackrel{\wedge}{=}$

Example 15-72: x with Subscribed Inverted Caret

$\underset{\vee}{x}$

Example 15-73: = with Superscribed Left-Pointing Caret

$\overset{<}{=}$

⠸⠭⠀⠐⠨⠅⠣⠰⠣⠻⠀⠸⠱

Example 15-74: = with Superscribed Right Pointing Caret

$\overset{>}{=}$

⠸⠭⠀⠐⠨⠅⠣⠰⠩⠻⠀⠸⠱

15.16 Dot

15.16.1 The dot is frequently used to indicate the recurrence of one or more digits in a decimal numeral. When used for this purpose, a dot is usually placed in print over each digit of the recurring sequence. In braille only a single dot is used as a modifier.

Example 15-75: Decimal Number with Superscribed Dot

$.\dot{3}$

⠸⠭⠀⠼⠨⠐⠒⠣⠡⠻⠀⠸⠱

Example 15-76: Superscribed Dot Over Each Digit

$.\dot{1}\dot{3}\dot{5}$

⠸⠭⠀⠼⠨⠐⠂⠒⠢⠣⠡⠻⠀⠸⠱

Example 15-77: Superscribed Dot Over One Digit

$.13\dot{5}$

⠸⠭⠀⠼⠨⠂⠒⠐⠢⠣⠡⠻⠀⠸⠱

Example 15-78: Superscribed Dot Over the 3 and 6

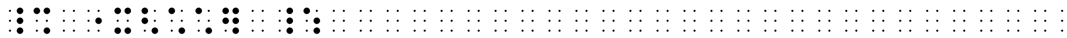
$.1\dot{3}5\dot{6}$

⠸⠭⠀⠼⠨⠂⠐⠒⠣⠡⠻⠢⠐⠖⠣⠡⠻⠀⠸⠱

15.16.2 If more than one dot is placed over or under a single digit or letter, the same number of dots is transcribed.

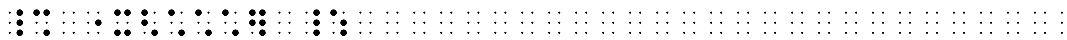
Example 15-79: x with Two Superscribed Dots

$\ddot{x}$



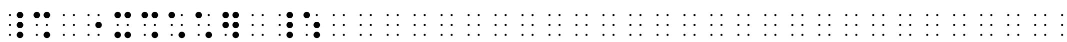
Example 15-80: x with Three Superscribed Dots

$\dddot{x}$



Example 15-81: x with Two Subscribed Dots

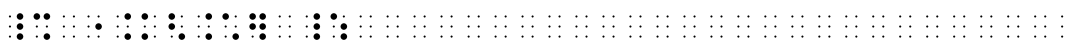
$\underset{..}{x}$



15.17 Hollow Dot

Example 15-82: Equals Sign with Superscribed Hollow Dot

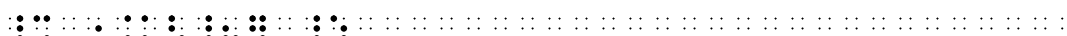
$\overset{\circ}{=}$



15.18 Question Mark

Example 15-83: Equals Sign with Superscribed Question Mark

$\overset{?}{=}$



Example 15-84: Equals Sign with Subscribed Question Mark

$\underset{?}{=}$

15.19 Tilde

The tilde is not regarded as a modifier when it occurs directly over or under a comparison sign. In that event, it becomes a component of a sign of comparison compounded vertically (see **21.9**). When the tilde is itself modified by a dot or a caret directly over or under it, the combination is a modified sign of comparison (see **21.8**).

Rule 16
Radicals

Radical	√	⠜
Radical Indicators		
Index-of-Radical		⠣
Order-of-Radical		
First inner radical		⠨
Second inner radical		⠨⠨
Third inner radical		⠨⠨⠨
Termination		⠻

16.1 Simple Radicals

The most commonly occurring radical is the square root.

16.1.1 When the square root sign has a vinculum (horizontal bar) which specifies the extent to which the radical sign is effective, the transcription of such a radical is accomplished by the following three steps:

a. The radical symbol ⠜

b. The expression to which it applies (radicand)

c. The termination indicator ⠻

The termination indicator is transcribed on the same level as the radical symbol to which it applies.

Example 16-1: Square Root

$\sqrt{2}$

⠸⠩ ⠜⠆⠻ ⠸⠱

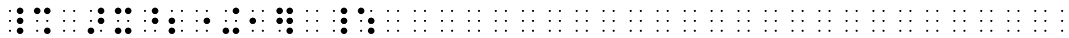
Example 16-2: Square Root

$\sqrt{x+y}$

⠸⠩ ⠜⠭⠬⠽⠻ ⠸⠱

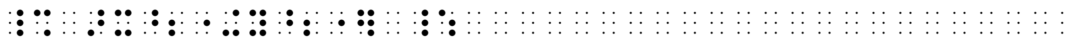
Example 16-3: Square Root

$$\sqrt{x^2+1}$$



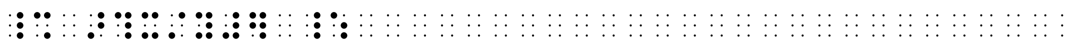
Example 16-4: Square Root

$$\sqrt{x^2+y^2}$$



Example 16-5: Square Root of a Fraction

$$\sqrt{\frac{x}{y}}$$

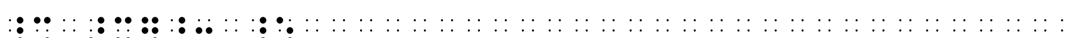


Example 16-6: Square Root

$$3\sqrt{a}$$

Example 16-7: Square Root Cubed

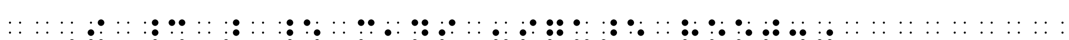
$$\sqrt{x}^3$$



16.1.2 When the square root sign occurs without a radicand, as when attention is being called to a sign in print, or when the extent to which the radical is effective is not indicated in print by the vinculum, the termination indicator must be omitted.

Example 16-8: Square Root Without a Vinculum

The √ means "square root."



Example 16-9: Square Root Without a Vinculum

$\sqrt{}(x+y)$

⠸⠩⠀⠜⠷⠭⠬⠽⠾⠀⠸⠱

(no vinculum in print)

16.2 Index of Radical

Radicals of index other than 2 require a specific index. The transcription of such a radical is accomplished by the following three steps:

a. The index-of-radical indicator ⠣

b. The index of the radical

c. Then proceed according to the three steps in **16.1.1**.

Example 16-10: Index of Radical

$\sqrt[3]{2}$

⠸⠩⠀⠣⠒⠜⠆⠻⠀⠸⠱

Example 16-11: Index of Radical

$3\sqrt[3]{x+y}$

⠸⠩⠀⠼⠒⠣⠒⠜⠭⠬⠽⠻⠀⠸⠱

Example 16-12: Index of Radical

$\sqrt[n]{a}$

⠸⠩⠀⠣⠝⠜⠁⠻⠀⠸⠱

Example 16-13: Index of Radical

$\sqrt[m+n]{p+q}$

⠸⠩⠀⠣⠍⠬⠝⠜⠏⠬⠟⠻⠀⠸⠱

16.3 Nested Radicals

Occasionally, radicals are nested one within the other. The first inner radical is then regarded as having a depth of order 1, the second inner radical as having a depth of order 2, and so on. In such cases, the order-of-radical indicator (⠨) is repeated before both the radical symbol and its associated termination indicator as many times as is necessary to indicate the depth of that radical. If one of the inner radicals is associated with an index, the proper number of order-of-radical indicators is placed before the index-of-radical indicator rather than before the radical symbol itself. The order-of-radical indicators are provided for the purpose of enabling the reader to keep track of the depth of the radical to which it applies.

Example 16-14: Nested Square Root

$$\sqrt{x+\sqrt{x+y}+z}$$

⠸⠩⠀⠜⠭⠬⠨⠜⠭⠬⠽⠨⠻⠬⠵⠻⠀⠸⠱

(the square root of the sum of three terms; the first term is x, the second term is the square root of x plus y; the third term is z)

Example 16-15: Nested Cube Root

$$\sqrt[3]{x^2+\sqrt[3]{x^2+y^2}+y^2}$$

⠸⠩⠀⠣⠒⠜⠭⠘⠆⠐⠬⠨⠣⠒⠜⠭⠘⠆⠐⠬⠽⠘⠆⠐⠨⠻⠬⠽⠘⠆⠐⠻⠀⠸⠱

(the cube root of the sum of x squared, the cube root of x squared plus y squared, and y squared)

Example 16-16: Nested Radicals

$$\sqrt{\sqrt[3]{x}}=\sqrt[3]{\sqrt{x}}$$

⠸⠩⠀⠜⠨⠣⠒⠜⠭⠨⠻⠻⠀⠨⠅⠀⠣⠒⠜⠨⠜⠭⠨⠻⠻⠀⠸⠱

(the square root of the cube root of x equals the cube root of the square root of x)

Example 16-17: Nested Square Roots

$$\sqrt{x+\sqrt{y+\sqrt{z}}}$$

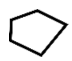
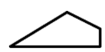
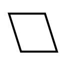
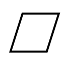
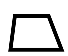
⠸⠩⠀⠜⠭⠬⠨⠜⠽⠬⠨⠨⠜⠵⠨⠨⠻⠨⠻⠻⠀⠸⠱

(a nest of three radicals; the outer radical contains x plus the inner radicals, the first inner radical contains y plus the second inner radical, and the second inner radical contains z)

Rule 17
Shapes

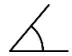
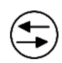
Indicators		
Shape Indicator		⠫
Interior Shape-Modification Indicator		⠸⠫
Keystroke Indicator		⠫⠅
Structural Shape-Modification Indicator		⠨
Filled-In Shape Indicator		⠸
Shaded Shape Indicator		⠨
Termination Indicator		⠻
Basic Shapes		
Angle	∠	⠫⠪
Arc		
Concave downward	⌒	⠫⠁
Concave upward	‿	⠫⠄
Arrow		
Left-pointing	←	⠫⠪⠒⠒
Right-pointing Contracted	→	⠫⠕
Right-pointing Uncontracted	→	⠫⠒⠒⠕
Down-pointing	↓	⠫⠩⠒⠒⠕
Up-pointing	↑	⠫⠣⠒⠒⠕
Circle	○	⠫⠉
Diamond	◇	⠫⠙
Ellipse (oval)	⬭	⠫⠑

(Basic Shapes, cont.)

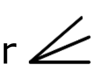
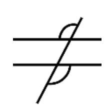
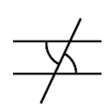
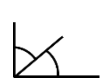
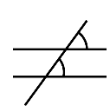
Hexagon		
Irregular		⠫⠓⠭
Regular		⠫⠖
Intersecting Lines		⠫⠊
Is Parallel To	$\parallel$	⠫⠇
Is Not Parallel To	$\nparallel$	⠌⠫⠇
Is Perpendicular To	$\perp$	⠫⠏
Is Not Perpendicular To	$\not\perp$	⠌⠫⠏
Parallelogram		⠫⠛
Pentagon		
Irregular		⠫⠏⠛
Regular		⠫⠢
Quadrilateral		⠫⠟
Rectangle		⠫⠗
Rhombus	 or 	⠫⠓
Square		⠫⠲
Star		⠫⠎
Trapezoid		⠫⠵
Triangle		
Inverted		⠨⠫
Regular (equilateral)		⠫⠞

Shapes with Interior Modification

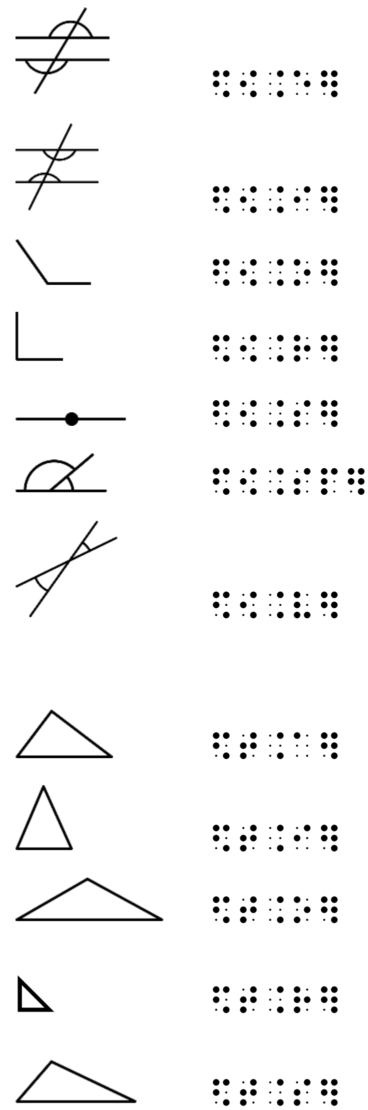
Angle

Angle with interior arc		⠫⠪⠸⠫⠫⠁⠻
Angle with interior clockwise arrow		⠫⠪⠸⠫⠫⠪⠢⠔⠻
Angle with interior counterclockwise arrow		⠫⠪⠸⠫⠫⠢⠔⠕⠻
Circle		
Circle with interior arrow pointing right		⠫⠉⠸⠫⠫⠒⠒⠕⠻
Circle with interior arrow pointing left		⠫⠉⠸⠫⠫⠪⠒⠒⠻
Circle with interior arrow pointing right over interior arrow pointing left		⠫⠉⠸⠫⠫⠒⠒⠕⠫⠪⠒⠒⠻
Circle with interior arrow pointing left over interior arrow pointing right		⠫⠉⠸⠫⠫⠪⠒⠒⠫⠒⠒⠕⠻
Circle with interior arrow pointing up		⠫⠉⠸⠫⠫⠣⠒⠒⠕⠻
Circle with interior arrow pointing down		⠫⠉⠸⠫⠫⠩⠒⠒⠕⠻
Circle with interior arrow pointing up followed by interior arrow pointing down		⠫⠉⠸⠫⠫⠣⠒⠒⠕⠐⠫⠩⠒⠒⠕⠻
Circle with interior arrow pointing down followed by interior arrow pointing up		⠫⠉⠸⠫⠫⠩⠒⠒⠕⠐⠫⠣⠒⠒⠕⠻

(Shapes with Interior Modification, cont.)

Circle with interior cross	⊗	⠫⠉⠸⠫⠈⠡⠻
Circle with interior dot	⊙	⠫⠉⠸⠫⠡⠻
Circle with interior minus sign	⊖	⠫⠉⠸⠫⠤⠻
Circle with interior plus sign	⊕	⠫⠉⠸⠫⠬⠻
Square		
Square with interior diagonals	⊠	⠫⠲⠸⠫⠢⠈⠔⠻
Square with interior dot	⊡	⠫⠲⠸⠫⠡⠻
Square with interior horizontal bar	⊟	⠫⠲⠸⠫⠱⠻
Square with interior vertical bar	◫	⠫⠲⠸⠫⠳⠻
Square with interior northwest-southeast diagonal	⧅	⠫⠲⠸⠫⠢⠻
Square with interior southwest-northeast diagonal	⧄	⠫⠲⠸⠫⠔⠻
Shapes with Structural Modification		
Angles		
Adjacent angles	 or 	⠫⠪⠨⠚⠻
Alternate exterior angles		⠫⠪⠨⠁⠑⠻
Alternate interior angles		⠫⠪⠨⠁⠊⠻
Complementary angles		⠫⠪⠨⠉⠏⠻
Corresponding angles		⠫⠪⠨⠉⠻

(Shapes with Structural Modification, cont.)



Exterior angles

Interior angles

Obtuse angle

Right angle

Straight angle

Supplementary angles

Vertical angles

Triangle

Acute triangle

Isosceles triangle

Obtuse triangle

Right triangle

Scalene triangle

17.1 Basic Shapes

A shape is a sign which is in general a miniature picture or diagram of the object which the sign represents.

A mathematical shape is represented by using a letter, numeral, or a configuration of dots which is suggestive of the shape. The shape indicator must precede the shape symbol. A symbol of shape is used only for the representation of the corresponding sign of shape; it is never used to represent the word or phrase which is the name of such a sign of shape.

Special Considerations:

a. Icons may be used in either UEB or Nemeth context without the insertion of additional switch indicators. Icons must be listed on the Special Symbols page.

b. Rectangles, squares, or other shapes which indicate end-of-proof are represented by the icon ⠈⠫⠟⠑⠙.

c. Bullets are not considered to be shapes and may be transcribed in either UEB or Nemeth context without the use of switch indicators.

Example 17-1: Rectangle Shape

▭

⠸⠩ ⠫⠗ ⠸⠱

Example 17-2: Circle Shape

○

⠸⠩ ⠫⠉ ⠸⠱

Example 17-3: Square Shape

□

⠸⠩ ⠫⠲ ⠸⠱

Example 17-4: Hexagon Shape

⬡

⠸⠩ ⠫⠖ ⠸⠱

Example 17-5: Angle Shape

$\angle$

⠸⠩ ⠫⠪ ⠸⠱

Example 17-6: Right-Pointing Arrow Shape

$\longrightarrow$

⠸⠩ ⠫⠕ ⠸⠱

Example 17-7: Square Shape in Superscript Position

$x^{\square}$

⠸⠩ ⠭⠘⠫⠲ ⠸⠱

Example 17-8: Angle Shapes

$\angle ABC \cong \angle DEF$

⠸⠩ ⠫⠪ ⠠⠁⠠⠃⠠⠉ ⠈⠱⠨⠅ ⠫⠪ ⠠⠙⠠⠑⠠⠋ ⠸⠱

17.2 Other Shapes

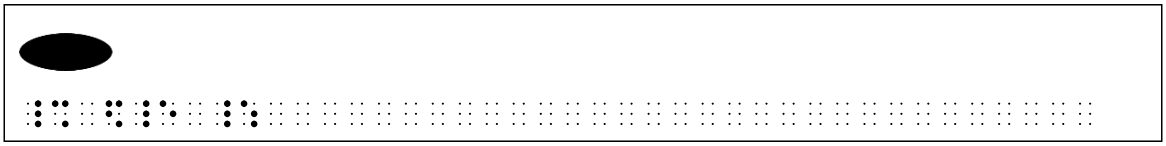
Signs of shape which do not appear in the list of Basic Shapes are represented by the use of one or more letters suggestive of the name of the shape being represented. Care must be exercised not to use an alphabetic symbol to which a meaning is already assigned in the above list. In addition, the transcriber must supply a note of explanation to the reader concerning the name of such a sign of shape and must supply a drawing of the shape if possible. The shape indicator must precede a shape symbol constructed in this way.

17.3 Filled-In and Shaded Shapes

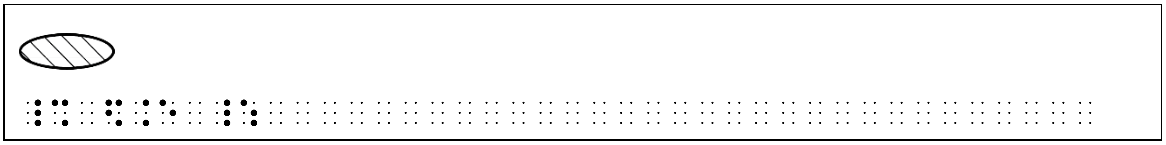
Any of the closed shapes in the above list, if they are filled in or shaded, are represented as such by using (⠸) or (⠨) respectively, preceding the shape symbol. The shape indicator, in turn, precedes whichever indicator has been used.

Note: Filled-in rectangles, squares, or other shapes which indicate end-of-proof are represented by the icon (⠈⠫⠟⠑⠙). This icon may be used in either UEB or Nemeth context without the insertion of additional switch indicators. Icons must be listed on the Special Symbols page.

Example 17-9: Filled-In Ellipse Shape



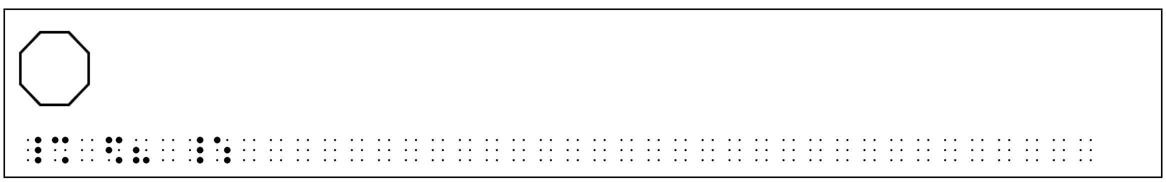
Example 17-10: Shaded Ellipse Shape



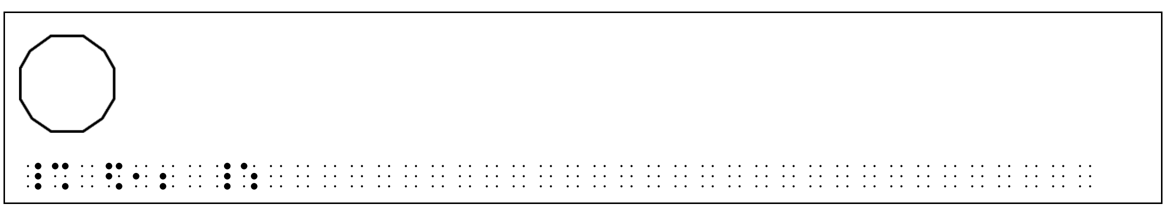
17.4 Polygons

The list of Basic Shapes contains the shapes for regular polygons up to six sides. Any regular polygon with more than six sides is represented in the manner suggested, that is, by using the numeral which specifies the number of sides. An irregular polygon, that is, one which has at least two unequal sides, two unequal angles, or both, is not represented in this way. It is represented as specified in **17.2**.

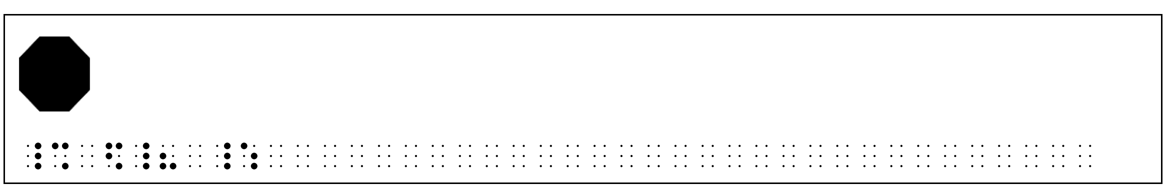
Example 17-11: Regular Octagon Shape



Example 17-12: Regular Polygon with 12 Sides



Example 17-13: Filled-In Regular Octagon



Example 17-14: Irregular Octagon Shape

(a transcriber's note with a drawing is required)

17.5 Shape with Structural Modification

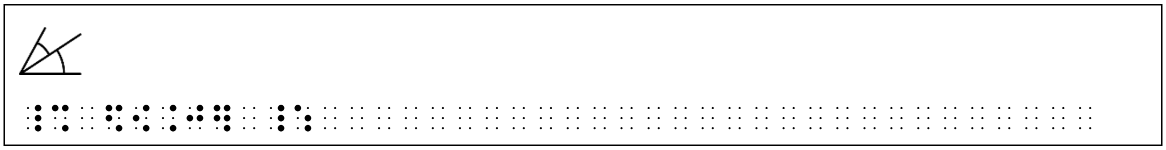
When a sign, which is a special case of a more general situation, is used, (for example, *right angle* is a special case of *angle*), or when two or more signs of shape are combined to form a composite sign with a more detailed structure, (for example, two *angles* are combined to form *adjacent angles*), the shape which is formed in either of these ways is called a *shape with structural modification*.

The modification is indicated by a letter or combination of letters suggestive of the nature of the modification. The symbol used for indicating the modification is preceded by the structural shape-modification indicator and followed by the termination indicator. This combination directly follows the symbol of basic shape which is being modified.

Shapes with structural modification not shown in the list of Shapes with Structural Modification are transcribed in accordance with the principle suggested by those shape symbols. The transcriber must supply a note of explanation to the reader concerning the name of the structural modification and must supply a drawing if possible.

Example 17-15: Isosceles Triangle Shape

Example 17-16: Adjacent Angles Shape



17.6 Shape with Interior Modification

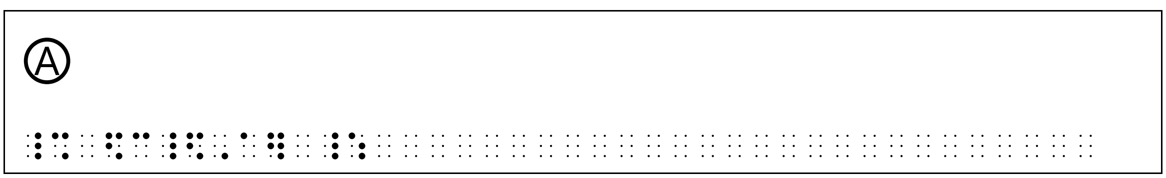
17.6.1 When a letter, operation sign, or other sign is placed inside the basic sign of shape, the shape which is formed in this way is called a *shape with interior modification*.

The modification is indicated by using the symbol which corresponds to the modifying sign. This symbol is preceded by the interior shape-modification indicator and followed by the termination indicator. This combination directly follows the symbol of basic shape which is being modified.

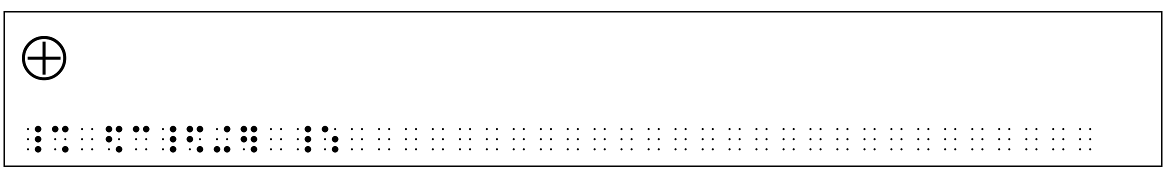
The numeric indicator is used before a numeral or before a decimal point and a numeral following the interior shape-modification indicator.

For material transcribed in UEB, follow *Braille Formats* for words enclosed in shapes.

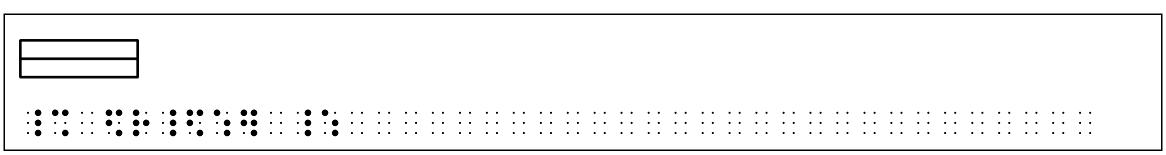
Example 17-17: Circle with Interior Letter A



Example 17-18: Circle with Interior Plus Sign



Example 17-19: Rectangle with Interior Horizontal Bar



Example 17-20: Angle with Interior Degree Measure

⠸⠩⠀⠫⠪⠸⠫⠼⠒⠴⠘⠨⠡⠐⠻⠀⠸⠱

Example 17-21: Non-mathematical Words in Shapes

A fence post is 4 feet tall. The fence post's shadow is 12 feet long.

⠀⠀⠠⠁⠀⠋⠰⠑⠀⠏⠕⠌⠀⠊⠎⠀⠈⠼⠶⠼⠙⠀⠋⠑⠑⠞⠈⠼⠄⠀⠞⠁⠇⠇⠲⠀⠠⠮

⠋⠰⠑⠀⠏⠕⠌⠄⠎⠀⠩⠁⠙⠪⠀⠊⠎⠀⠈⠼⠶⠼⠁⠃⠀⠋⠑⠑⠞⠈⠼⠄⠀⠇⠰⠛⠲

(*Braille Formats* rules are followed for non-mathematical words in shapes. A UEB transcriber-defined typeform, listed on the special symbols page, is used for the circled words)

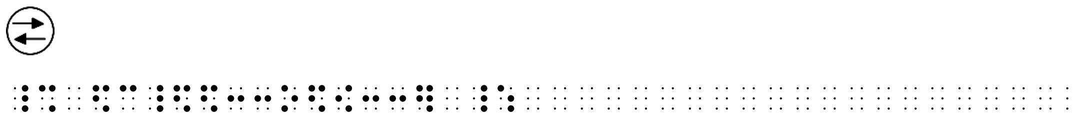
17.6.2 If two or more interior modifiers, arranged horizontally, occur inside the same basic sign of shape, the corresponding symbols are separated by the multipurpose indicator, but the interior shape-modification indicator is used only once, before the first modifying symbol. The entire combination directly follows the basic symbol of shape which is being modified.

Example 17-22: Circle Containing Down and Up Pointing Arrows

⠸⠩⠀⠫⠉⠸⠫⠫⠩⠒⠒⠕⠐⠫⠣⠒⠒⠕⠻⠀⠸⠱

17.6.3 If two or more interior modifiers, arranged vertically, occur inside the same basic sign of shape, the corresponding symbols are transcribed successively, without intervening spaces or indicators, beginning with the symbol which corresponds to the uppermost sign and proceeding in descending order. The first modifying symbol is preceded by the interior shape-modification indicator, and the entire combination directly follows the basic symbol of shape which is being modified. None of the interior signs are regarded as a modifier of any of the others. The technique for representing modified expressions does not apply.

Example 17-23: Circle Containing Right Arrow Above Left Arrow



17.6.4 When a shape with interior modification depicts a labeled calculator or computer key within the instructional text, it is represented with the keystroke indicator.

a. The key label will immediately follow the keystroke indicator. The label is immediately followed, unspaced, by the termination indicator.

b. The shape and color of the key is irrelevant. The actual shape(s) used in a particular text should be specified on the Transcriber's Notes page.

c. It is preferred that the entire series of keystrokes not be divided between braille lines.

d. No single keystroke construction may be divided between braille lines.

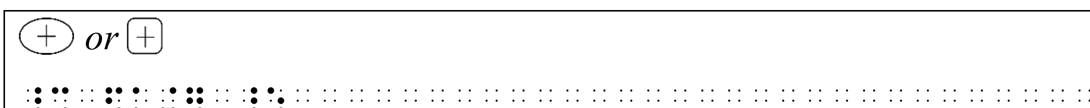
e. The rules for preferred division of mathematical expressions do not apply; do not drop to a new line because the symbol on the key is a sign of comparison.

f. Duplicate the print lines, if possible, when such lines are arranged in a logical sequence.

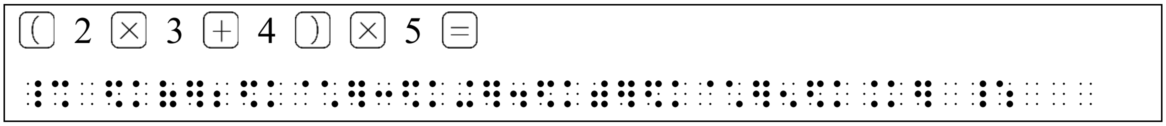
g. No space may be left between keystroke constructions and other similar constructions or mathematical symbols in a sequence of related calculations. Arrows contained in the labels on the keys are not spaced from the material to which they apply.

h. The numeric indicator is not required within the contracted keystroke construction.

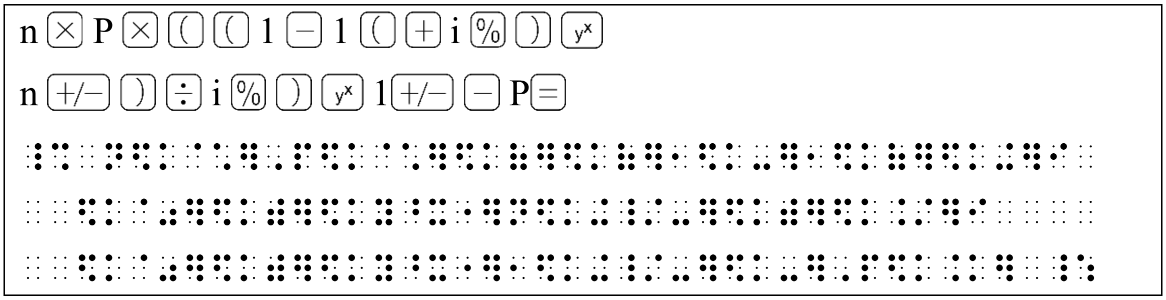
Example 17-24: Keystroke with Interior Plus Sign



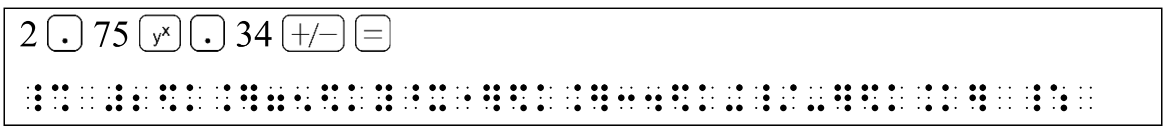
Example 17-25: Numbers in a Keystroke Construction



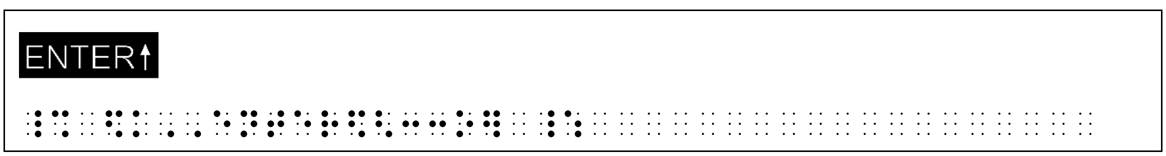
Example 17-26: Numbers in a Keystroke Construction



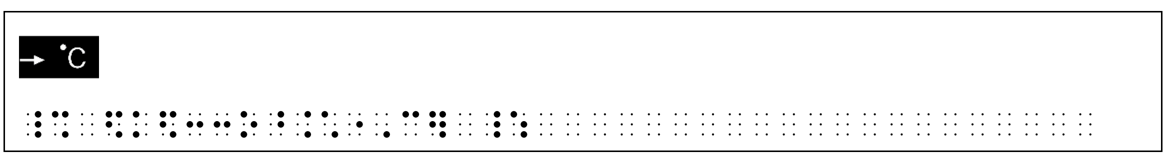
Example 17-27: Numbers in a Keystroke Construction



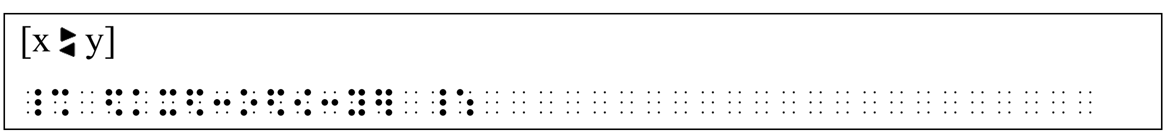
Example 17-28: Keystroke with Capitalized Word and Arrow



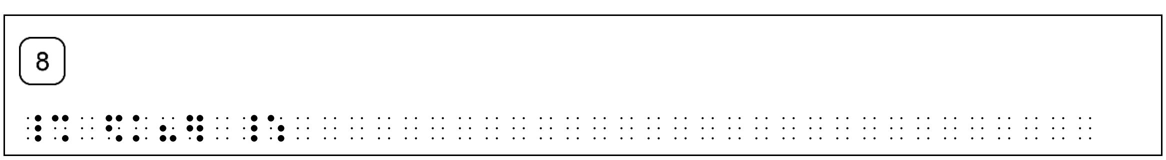
Example 17-29: Keystroke with Arrow and Degree Symbol



Example 17-30: Keystroke Indicated by Brackets



Example 17-31: Keystroke Indicated by a Square



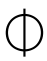
Example 17-32: Radical in a Keystroke

⠸⠩⠀⠫⠅⠜⠭⠻⠻⠀⠸⠱

17.7 Shape Modified by Superposition

When a sign is superposed upon a sign of shape, the shape which is formed in this way is called a *shape modified by superposition*. Superposition is distinguished from interior modification by noting that in superposition one of the signs extends beyond the boundary of the other. (see **15.9**) In the case of interior modification, one of the signs is confined within the boundary of the other (see **17.6**).

Example 17-33: Circle with Superposed Vertical Bar



⠸⠩⠀⠳⠈⠫⠉⠻⠀⠸⠱

(a vertical bar extending beyond the boundary of a circle)

Example 17-34: Circle with Interior Modification

⠸⠩⠀⠫⠉⠸⠫⠳⠻⠀⠸⠱

(the vertical bar extends only as far as the inside boundary of the circle)

17.8 Drawn-In Shapes

It is often better for the reader to have shapes drawn in than to have them represented by the elaborate braille constructions specified in this rule. It is not possible to formulate specific rules concerning which form is used and, therefore, the decision is left to the experience and judgment of the transcriber.

17.9 Plural of a Sign of Shape

The plural or the possessive of a sign of shape is sometimes indicated by placing the letter "s" on the inside of the sign of shape. When this form is employed, the braille transcription is affected simply by placing the lower-case letter "s" after the shape symbol (see **8.4**).

Example 17-35: Plural Triangle Shape

△s

⠸⠩⠀⠫⠞⠎⠀⠸⠱

(in print the "s" is inside the triangle shape)

17.10 Spacing with Symbols of Shape

17.10.1 When a sign of shape is followed by its identification such as a letter, sequence of letters, or numeral, there must be a space between the shape symbol and its identification. In principle, the spacing rule which covers symbols of shape which are identified are the same as those which apply to function names and their abbreviations. See **18.4**.

Example 17-36: Angle Shape with Identifier

∠ 1

⠸⠩⠀⠫⠪⠀⠼⠂⠀⠸⠱

Example 17-37: Triangle Shape with Identifier

△ ABC

⠸⠩⠀⠫⠞⠀⠠⠁⠠⠃⠠⠉⠀⠸⠱

Example 17-38: Circle Shape with Identifier

○ R

⠸⠩⠀⠫⠉⠀⠠⠗⠀⠸⠱

Example 17-39: Triangle and Angle Shapes with Identifiers

In △ ABC, m ∠ A = 90°.

⠀⠀⠠⠔⠀⠸⠩⠀⠫⠞⠀⠠⠁⠠⠃⠠⠉⠠⠀⠍⠫⠪⠀⠠⠁⠀⠨⠅⠀⠼⠔⠴⠘⠨⠡⠀⠸⠱⠲

Example 17-40: Plural Triangle Shape with Identifier

$\triangle$ UVW and XYZ

Example 17-41: Right Angle Shape with Identifier

∟ A

Example 17-42: Angle Shape with Interior Arrow

$\measuredangle$ ABC

Example 17-43: Angle Shapes with Identifiers

$\angle x + \angle y$

Example 17-44: Angle Shapes with Identifiers

$\angle 1 + 2\angle 3$

Example 17-45: Angle Shapes with Identifiers

$$\frac{\Delta ABC}{\Delta EFG}$$

Example 17-46: Angle Shape with Identifier

The measure of $\angle ABC = 45^{\circ}$

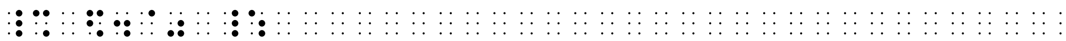
Example 17-47: Angle Shapes with Identifiers

$\angle\ 90^{\circ} + \angle\ 120^{\circ}$

17.10.2 Shape symbols which represent omission are spaced in accordance with the omitted item which they represent.

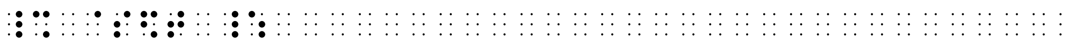
Example 17-48: Square Shape Represents Omitted Number

□%



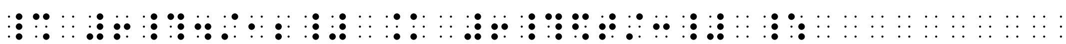
Example 17-49: Triangle Shape Represents Omitted Number

$△



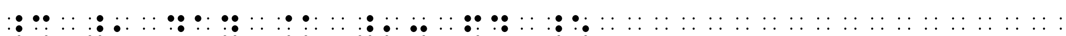
Example 17-50: Triangle Shape Represents Omitted Number

$6\frac{4}{12} = 6\frac{\triangle}{3}$



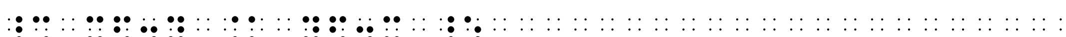
Example 17-51: Diamond Shape Represents Omitted Word

1 day = 24 ◊



Example 17-52: Square Represents Omitted Operation Sign

x□y = y□x

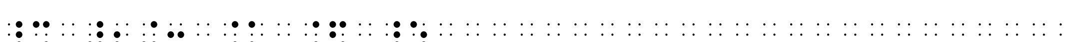


Example 17-53: Triangle Represents Omitted Comparison Sign

2 + 4 △ 7

Example 17-54: Inverted Triangle Shape as a Sign of Omission

2 + 3 = ▽



17.10.3 Symbols of shape which are either comparison symbols or operation symbols are spaced accordingly.

Example 17-55: Arrow is a Sign of Comparison

$f \longrightarrow g$

⠸⠩⠀⠋⠀⠫⠕⠀⠛⠀⠸⠱

Example 17-56: Arrow is a Sign of Comparison

$\lim\limits_{x \to \infty} f(x)$

⠸⠩⠀⠐⠇⠊⠍⠩⠭⠀⠫⠕⠀⠠⠿⠻⠀⠋⠷⠭⠾⠀⠸⠱

Example 17-57: Perpendicular Shape is a Sign of Comparison

$AB \perp CD$

⠸⠩⠀⠠⠁⠠⠃⠀⠫⠏⠀⠠⠉⠠⠙⠀⠸⠱

("is perpendicular to" is a comparison symbol)

Example 17-58: Parallel Negation is a Sign of Comparison

$AB \nparallel CD$

⠸⠩⠀⠠⠁⠠⠃⠀⠌⠫⠇⠀⠠⠉⠠⠙⠀⠸⠱

("is not parallel to" is a comparison symbol)

Example 17-59: Circle with Plus Sign as Omitted Operation Sign

$x \oplus y$

⠸⠩⠀⠭⠫⠉⠸⠫⠬⠻⠽⠀⠸⠱

(the circle with interior plus symbol is a symbol of operation)

Example 17-60: Solid Square as Omitted Operation Sign

$x \blacksquare y$

⠸⠩⠀⠭⠫⠸⠲⠽⠀⠸⠱

17.10.4 A symbol of shape is unspaced from any braille indicator which applies to it.

Example 17-61: Arrow as Modifier

$\overrightarrow{AB}$

⠸⠩⠀⠐⠠⠁⠠⠃⠣⠫⠕⠻⠀⠸⠱

Example 17-62: Diamond Shape and Subscript Indicator

$1101_{\lozenge} + 1000_{\lozenge}$

⠸⠩⠀⠼⠂⠂⠴⠂⠰⠫⠙⠐⠬⠂⠴⠴⠴⠰⠫⠙⠀⠸⠱

17.10.5 No space may be left between keystroke constructions and other similar constructions of mathematical symbols in a sequence of related calculations. See examples in **17.6.4**. Arrows contained in the labels on the keys are not spaced from material to which they apply.

Example 17-63: Unspaced Arrow in a Keystroke Construction

EE ↓

⠸⠩⠀⠫⠅⠠⠠⠑⠑⠫⠩⠒⠒⠕⠻⠀⠸⠱

Rule 18
Function Names and Their Abbreviated Forms

A partial list of function names and their abbreviated forms is given below.

Abbreviated Form	Braille Equivalent	Function Name
amp	⠁⠍⠏	amplitude
antilog	⠁⠝⠞⠊⠇⠕⠛	antilogarithm
arc	⠁⠗⠉	arc
arg	⠁⠗⠛	argument
colog	⠉⠕⠇⠕⠛	cologarithm
cos	⠉⠕⠎	cosine
cosh	⠉⠕⠎⠓	hyperbolic cosine
cot	⠉⠕⠞	cotangent
coth	⠉⠕⠞⠓	hyperbolic cotangent
covers	⠉⠕⠧⠑⠗⠎	coversine
csc	⠉⠎⠉	cosecant
csch	⠉⠎⠉⠓	hyperbolic cosecant
ctn	⠉⠞⠝	cotangent
ctnh	⠉⠞⠝⠓	hyperbolic cotangent
det	⠙⠑⠞	determinant
erf	⠑⠗⠋	error function
exp	⠑⠭⠏	exponential
exsec	⠑⠭⠎⠑⠉	exsecant
grad	⠛⠗⠁⠙	gradient
hav	⠓⠁⠧	haversine
im	⠊⠍	imaginary part
inf	⠊⠝⠋	infimum

Abbreviated Form	Braille Equivalent	Function Name
lim	⠇⠊⠍	limit
$\overline{\text{lim}}$	⠣⠇⠊⠍	upper limit
$\underline{\text{lim}}$	⠩⠇⠊⠍	lower limit
ln	⠇⠝	natural logarithm
log	⠇⠕⠛	logarithm
max	⠍⠁⠭	maximum
min	⠍⠊⠝	minimum
mod	⠍⠕⠙	modulo
re	⠗⠑	real part
sec	⠎⠑⠉	secant
sech	⠎⠑⠉⠓	hyperbolic secant
sin	⠎⠊⠝	sine
sinh	⠎⠊⠝⠓	hyperbolic sine
sup	⠎⠥⠏	supremum
tan	⠞⠁⠝	tangent
tanh	⠞⠁⠝⠓	hyperbolic tangent
vers	⠧⠑⠗⠎	versine

18.1 Transcribe in Nemeth Code

Follow print when transcribing a function name or its abbreviated form. All abbreviated function names are transcribed in Nemeth Code. Function names used in mathematical context that are not abbreviated are also transcribed in Nemeth Code. An English-letter indicator is not used with a letter that follows a function name or its abbreviation.

Example 18-1: Function Name

sine x

⠸⠩⠀⠎⠊⠝⠑⠀⠭⠀⠸⠱

Example 18-2: Abbreviated Function Name

sin x

⠸⠩⠀⠎⠊⠝⠀⠭⠀⠸⠱

Example 18-3: Function Name in Non-Mathematical Context

The Law of Sines: For a plane triangle, ...

⠀⠀⠠⠮⠀⠠⠇⠁⠺⠀⠷⠀⠠⠎⠔⠑⠎⠒⠀⠠⠿⠀⠁⠀⠏⠇⠁⠝⠑⠀⠞⠗⠊⠁⠝⠛⠇⠑⠂
⠲⠲⠲

18.2 Numeric Subscripts with Function Names and Their Abbreviations

See **14.6**.

18.3 Modifiers with Function Names and Their Abbreviated Forms

The bar which occurs over or under the function name "limit" or its abbreviated form "lim" is not treated as a modifier; the combination is transcribed by means of special symbols for *upper limit* ⠣⠇⠊⠍ or ⠣⠇⠊⠍⠊⠞ or *lower limit* ⠩⠇⠊⠍ or ⠩⠇⠊⠍⠊⠞. Other modifiers are transcribed in accordance with the techniques for the representation of modified expressions.

Example 18-4: Modified Upper Limit Function

$$\overline{\lim_{n\to\infty}} f_n(x)$$

⠸⠩⠀⠐⠣⠇⠊⠍⠩⠝⠀⠫⠕⠀⠠⠿⠻⠀⠋⠰⠝⠐⠷⠭⠾⠀⠸⠱

Example 18-5: Modified Lower Limit Function

$$\underline{\lim}_{n\to\infty} f_n(x)$$

⠸⠩⠀⠐⠩⠇⠊⠍⠩⠝⠀⠫⠕⠀⠠⠿⠻⠀⠋⠰⠝⠐⠷⠭⠾⠀⠸⠱

Example 18-6: Modified Abbreviated Function Name

$\lim\limits_{x \to 0} f(x)$

⠸⠩ ⠐⠇⠊⠍⠩⠭ ⠫⠕ ⠼⠴⠻ ⠋⠷⠭⠾ ⠸⠱

18.4 Spacing with Function Names and Their Abbreviated Forms

18.4.1 A space is left after an unmodified function name or its abbreviated form. If the function name or its abbreviated form carries a superscript, subscript, modifier, or other braille indicator, the space follows the superscript, subscript, termination of modifier, or other braille indicator.

Example 18-7: Space After an Abbreviated Function Name

arc AOB

⠸⠩ ⠁⠗⠉ ⠠⠁⠠⠕⠠⠃ ⠸⠱

Example 18-8: Abbreviated Function with Superscript

$\cos^2 x$

⠸⠩ ⠉⠕⠎⠘⠆ ⠭ ⠸⠱

Example 18-9: Superscripted Abbreviated Function

$e^{\sin x}$

⠸⠩ ⠑⠘⠎⠊⠝ ⠭ ⠸⠱

Example 18-10: Abbreviated Function with Subscript

$\log_a x$

⠸⠩ ⠇⠕⠛⠰⠁ ⠭ ⠸⠱

Example 18-11: Modified Abbreviated Function

$\lim\limits_{x \to 0} f(x)$

⠸⠩ ⠐⠇⠊⠍⠩⠭ ⠫⠕ ⠼⠴⠻ ⠋⠷⠭⠾ ⠸⠱

18.4.2 If two or more consecutive function names or their abbreviated forms occur, they may be printed with or without a space between them. The transcription follows print spacing. When there is doubt concerning the presence of a space in print between the function names or their abbreviated forms, a space should be inserted in the transcription.

Example 18-12: Consecutive Unspaced Function Names

arcsin x

⠸⠩⠀⠁⠗⠉⠎⠊⠝⠀⠭⠀⠸⠱

(no space in print between arc and sin)

Example 18-13: Consecutive Spaced Function Names

arc sin x

⠸⠩⠀⠁⠗⠉⠀⠎⠊⠝⠀⠭⠀⠸⠱

(space between arc and sin clearly shown in print)

18.4.3 The expression which follows or precedes the function name or its abbreviated form is spaced in accordance with the other spacing rules of this Code.

Example 18-14: Function Name Followed by a Math Expression

sin x + y

⠸⠩⠀⠎⠊⠝⠀⠭⠬⠽⠀⠸⠱

(in print, there is a space on both sides of the plus sign)

Example 18-15: Function Name Followed by a Fraction

sin π/3

⠸⠩⠀⠎⠊⠝⠀⠨⠏⠸⠌⠒⠀⠸⠱

(in print, there is no space on either side of the diagonal line)

Example 18-16: Function Name Following a Number

sin 30° cos 45° + cos 30° sin 45°

⠸⠩⠀⠎⠊⠝⠀⠼⠒⠴⠘⠨⠡⠐⠉⠕⠎⠀⠼⠲⠢⠘⠨⠡

⠀⠀⠐⠬⠉⠕⠎⠀⠼⠒⠴⠘⠨⠡⠐⠎⠊⠝⠀⠼⠲⠢⠘⠨⠡⠀⠸⠱

(in print, cos 45° and cos 30° are preceded and followed by spaces)

Example 18-17: Function Name Following a Number

$2\sin x + 3\cos y$

(in print, there is no space after the 2 and the 3, and there is a space on both sides of the plus sign)

Example 18-18: Function Name Following a Letter

$\sin a \cos y$

(in print there is a space on both sides of the a and the y)

Example 18-19: Abbreviated Function Following a Minus Sign

$\sqrt{1-\cos^2 x}$

(in print, there is a space on both sides of the minus sign and the x)

Example 18-20: Function Name Following an Operation Sign

$1/\cos - \cos = \tan \cdot \sin$

Example 18-21: Function Name and Fraction or Operation Sign

$\frac{1}{\cos} - \cos = \tan \cdot \sin$

(a fraction indicator does not require a space following it)

18.5 Punctuation with Function Names and Their Abbreviated Forms

A function name is punctuated in either mathematical or literary mode according to its context. An abbreviated function name is a mathematical expression and must be punctuated in mathematical mode.

Example 18-22: Punctuation with Abbreviated Function Names

(1) Some trigonometric functions are sin, cos, and tan.

(2) Some trigonometric functions are sine, cosine, and tangent.

(3) Arc ACB is a major arc.

(4) What is the meaning of logsine?

Rule 19
Signs and Symbols of Grouping

		Normal	Enlarged
Parentheses (round brackets)			
Left	(	⠷	⠠⠷
Right	)	⠾	⠠⠾
Brackets (square brackets)			
Left	[	⠈⠷	⠈⠠⠷
Right	]	⠈⠾	⠈⠠⠾
Boldface Left	**[**	⠸⠈⠷	
Boldface Right	**]**	⠸⠈⠾	
Braces (curly brackets)			
Left	{	⠨⠷	⠨⠠⠷
Right	}	⠨⠾	⠨⠠⠾
Vertical Bar			
Single	|	⠳	⠠⠳
Double	||	⠳⠳	⠠⠳⠠⠳
Boldface Single	**|**	⠸⠳	
Boldface Double	**||**	⠸⠳⠸⠳	
Angle Brackets (angular parentheses)			
Left	<	⠨⠨⠷	⠨⠨⠠⠷
Right	>	⠨⠨⠾	⠨⠨⠠⠾
Barred Brackets			
Left	⟦	⠈⠸⠷	⠈⠸⠠⠷
Right	⟧	⠈⠸⠾	⠈⠸⠠⠾

Barred Braces

Left	⦃		
Right	⦄		

Half Brackets

Upper Left	⌈ or ⌜		
Upper Right	⌉ or ⌝		
Lower Left	⌊ or ⌞		
Lower Right	⌋ or ⌟		

Transcriber-inserted Grouping Symbol

(no print equivalent)

19.1 Symbols of Grouping

19.1.1 Symbols of grouping are transcribed wherever they appear in print.

Example 19-1: Technical Material Enclosed in Grouping Symbols

(s.a.s = s.a.s.)

Example 19-2: Braces Indicating Set Notation

{Wed., Thurs., Fri}

Example 19-3: Braces Indicating Set Notation

{Mary, Sally, Jean}

Example 19-4: Superscript Following Grouping Symbol

(seven)^2 + 1

Example 19-5: Series of Unspaced Letters

(xy-plane)

Example 19-6: Mathematical Equation Using Grouping Symbols

(rate) × (time) = (distance)

Example 19-7: Mathematical Equation Using Grouping Symbols

(divisor)(partial quotient) + (remainder) = (dividend)

Example 19-8: Subtraction of Roman Numerals

(IX–VI)

Example 19-9: Technical Material Within Grouping Symbols

$(x^2 + y^2)(x - y)$

Example 19-10: Parentheses and Square Brackets

$4 - 3[4 - 2(6 - 3)] \div 3$

Example 19-11: Enclosed List in Angle Brackets

The inner product of two vectors is commonly written as ‹*a, b*›.

Example 19-12: Left Square Bracket, Right Parenthesis

$[a, +\infty)$

⠸⠩⠀⠈⠷⠁⠠⠀⠬⠠⠿⠾⠀⠸⠱

19.1.2 Although signs of grouping most commonly occur in pairs, this is not always so. If a left grouping sign occurs without being followed later by the corresponding right grouping sign, or if the right grouping sign occurs without having been preceded by the corresponding left grouping sign, this situation is preserved in the transcription.

Example 19-13: Closing Bracket with Subscript and Superscript

$$\frac{15}{4}\sin 2\,\theta\Big]_0^{\pi}$$

⠸⠩⠀⠹⠂⠢⠌⠲⠼⠎⠊⠝⠀⠼⠆⠨⠹⠈⠾⠰⠴⠘⠨⠏⠀⠸⠱

Example 19-14: Closing Vertical Bar with Subscript

$$\left.\frac{dz}{dt}\right|_{t=0}$$

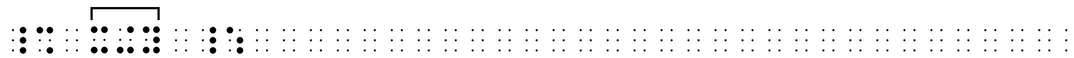
⠸⠩⠀⠹⠙⠵⠌⠙⠞⠼⠳⠰⠞⠀⠰⠨⠅⠀⠼⠴⠀⠸⠱

19.2 Horizontal Grouping Signs

When a horizontal grouping sign occurs over or under a mathematical expression, it is regarded as a modifier. It is recommended that the horizontal grouping symbols be drawn. When they are to be represented in braille, the modified expression is transcribed according to **15.2.1**. The left grouping symbol must be used when the modifier is *directly over* and the right grouping symbol when the modifier is *directly under*. When a horizontal grouping sign is pointing to a label or to explanatory text, it is not a modifier and must be drawn.

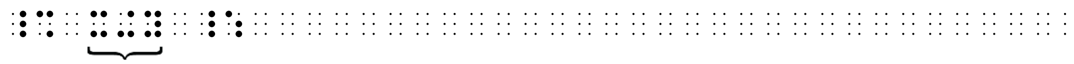
Example 19-15: Drawn Horizontal Bracket Above

$\overline{x+y}$



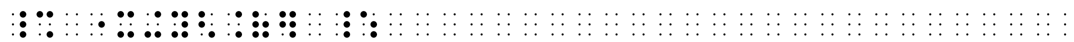
Example 19-16: Drawn Horizontal Brace Below

$\underbrace{x+y}$



Example 19-17: Transcribed Horizontal Brace Above

$\overbrace{x+y}$

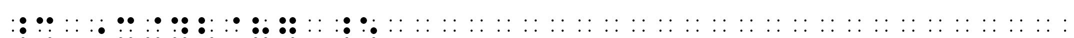


Example 19-18: Transcribed Horizontal Brace Below

$\underbrace{x+y}$

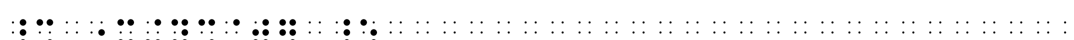
Example 19-19: Transcribed Horizontal Square Bracket Above

$\overline{x+y}$



Example 19-20: Transcribed Horizontal Square Bracket Below

$\underline{x+y}$



19.3 Boldface Brackets

Boldface brackets are often used to designate the *integer function*.

Example 19-21: Bold Brackets

$\mathbf{[}x\mathbf{]}$

19.4 Half-Brackets

The upper half-brackets (left and right) are commonly used to represent the *ceiling function*. The lower half-brackets (left and right) are commonly used to represent the *floor function*. These signs are also used for miscellaneous purposes in many fields of mathematics and science. Half-brackets are transcribed the same regardless of size.

Example 19-22: Upper and Lower Half Brackets

If $x = 3.5$, then $\lfloor x \rfloor = 3$ and $\lceil x \rceil = 4$.

Example 19-23: Upper Half Bracket within Subscript

$A_{n\rceil i}$

(expression using longer half bracket in print)

Example 19-24: Upper and Lower Half Brackets

$\ulcorner a_1\ a_2\ \ldots\ a_n \lrcorner$

19.5 Vertical Bars

19.5.1 Double boldface vertical bars are usually read as *the norm of*.

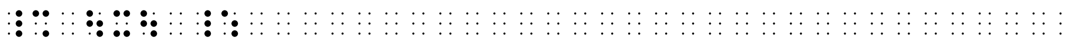
Example 19-25: Double Boldface Vertical Bars

$\boldsymbol{\|} f \boldsymbol{\|}$

19.5.2 Single vertical bars are often read as *the absolute value of*, but may be used for other purposes.

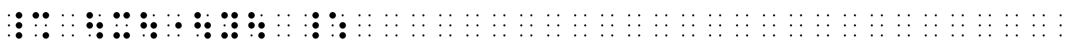
Example 19-26: Absolute Value of x

$|x|$



Example 19-27: Dot 5 Inserted Between Two Absolute Values

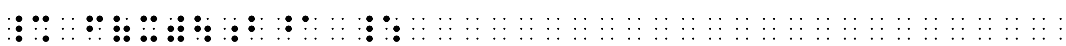
$|x||y|$



For further examples of side by side vertical bars see **24.1.i**.

Example 19-28: Vertical Bar with Subscript and Superscript

$f\left(x\right)\Big|_{\mathrm{b}}^{\mathrm{a}}$



Example 19-29: Vertical Bar with Subscript

$f(x)\Big|_{x=4}$

19.6 Use of Enlarged Grouping Symbols

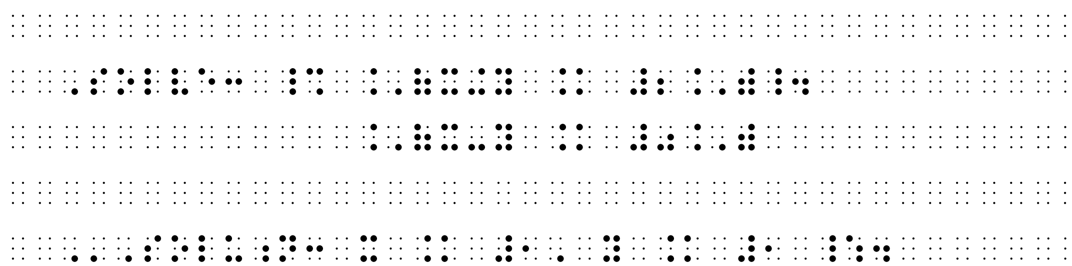
When a system of mathematical expressions is arranged on two or more lines of print, and a sign of grouping is used to unify the system, the corresponding grouping symbol in the transcription is indicated as enlarged by the use of dot 6. Among such systems of mathematical expressions are: systems of equations, determinants, and matrices. Each braille line which contains any part of the transcription of such a system contains the enlarged grouping symbol and these are vertically aligned. If only the left or only the right member of a pair of grouping signs is present in print, only the corresponding grouping symbol is represented in the transcription. When it is advisable for any reason to do so, for example to save space by avoiding runovers, the enlarged grouping symbols may be drawn. When systems are aligned in print, the alignment is retained in braille.

Any identifier, symbol, or punctuation preceding or following the enlarged group is placed on the top line of the arrangement.

Example 19-30: Aligned System in Enlarged Braces

Solve: $\left\{\begin{matrix} \mathrm{x}+\mathrm{y}=2 \\ \mathrm{x}-\mathrm{y}=0 \end{matrix}\right\}.$

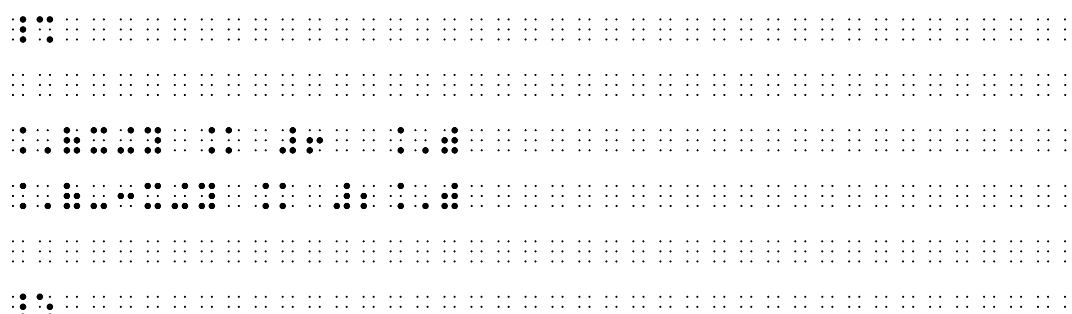
Solution: x = 1, y = 1.



(system of equations aligned and enclosed within braces; in print the period and text are centered on the system)

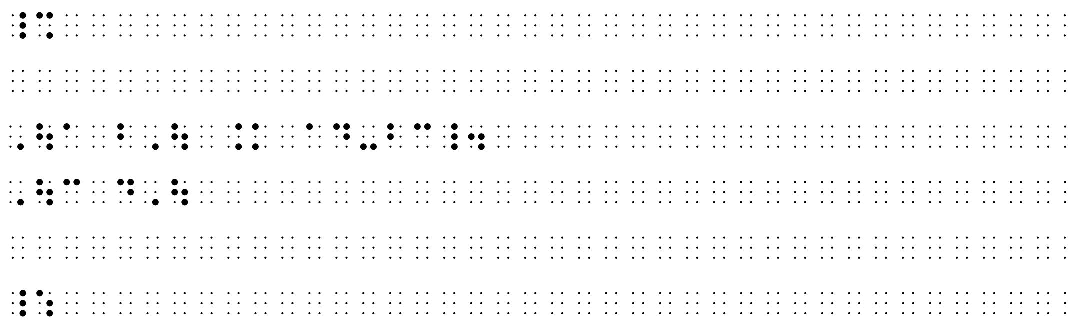
Example 19-31: Unaligned System in Enlarged Braces

$$\left\{\begin{matrix} x+y=6 \\ -3x+y=2 \end{matrix}\right\}$$



Example 19-32: Determinant within Enlarged Vertical Bars

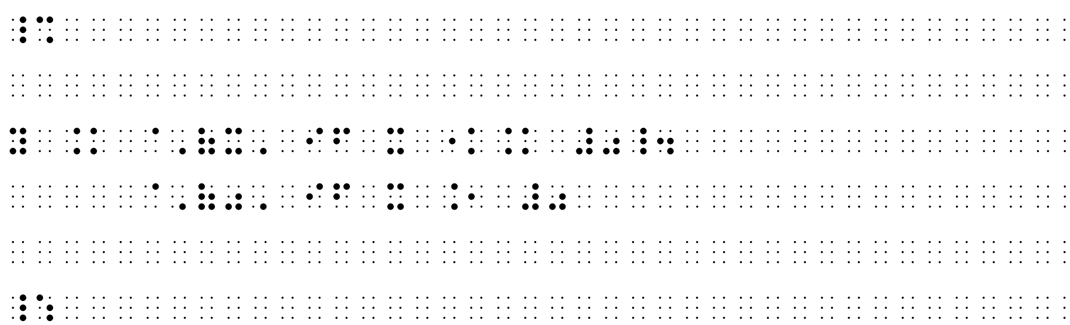
$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc.$$



(two-by-two determinant enclosed within vertical bars; equals sign, ad-bc, and period are centered on the determinant in print)

Example 19-33: Unified System of Equations with Left Bracket

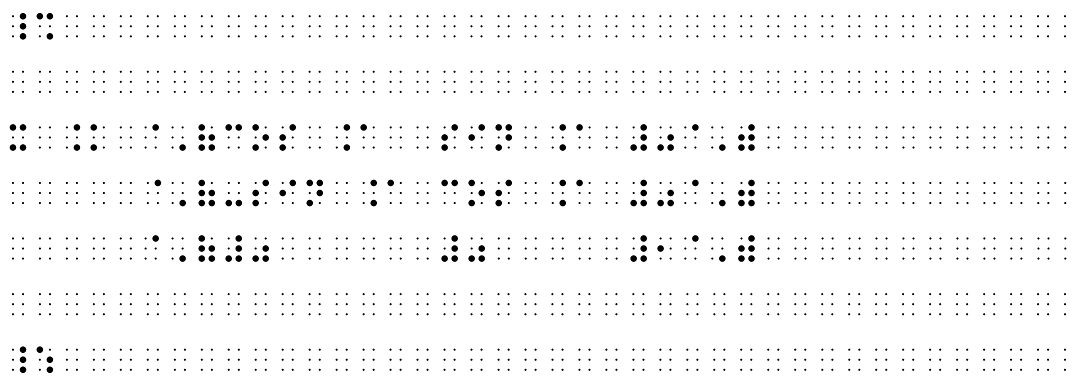
$$y = \left[\begin{array}{l} x, \text{ if } x \leqq 0 \\ 0, \text{ if } x > 0. \end{array} \right.$$



(a two-line system unified on the left by a bracket; the period follows x > 0 in print)

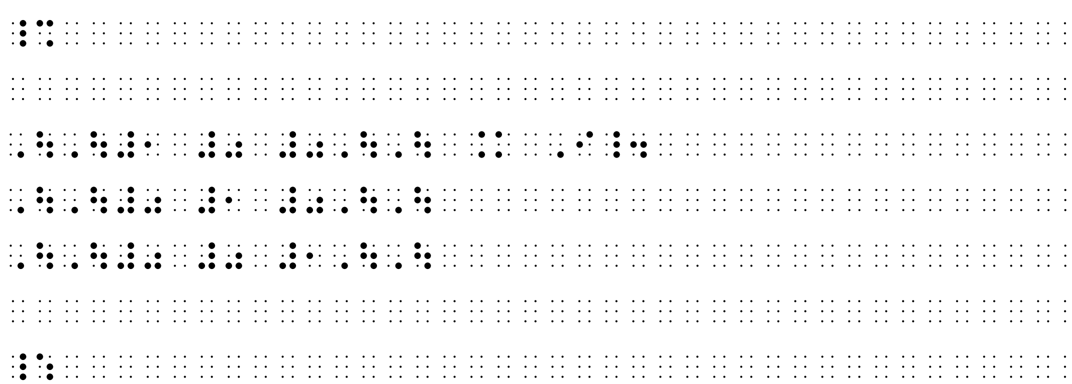
Example 19-34: Three-by-Three Matrix in Enlarged Brackets

$$x = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



Example 19-35: Matrix within Enlarged Double Vertical Bars

$$\begin{Vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{Vmatrix} = \mathrm{I}.$$



(a three-by-three matrix enclosed within double vertical bars; equals sign, I and period are centered in print)

19.7 Non-Use of Enlarged Grouping Symbols

Signs of grouping are not indicated as enlarged in the transcription when the corresponding signs in print are made large such as for the purpose of covering a fraction, binomial coefficient or other material occupying a large amount of vertical space. No signs, except grouping signs, may ever be transcribed as being enlarged.

Example 19-36: Not Enlarged Parentheses

$$\left(\frac{\mathrm{x}+\mathrm{y}}{\mathrm{u}+\mathrm{v}}\right)^{2}$$

Example 19-37: Not Enlarged Brackets

$$a=\left[\frac{b}{c+\frac{d}{e}}\right]$$

Example 19-38: Not Enlarged Parentheses

$$\binom{\mathrm{n}+\mathrm{k}}{\mathrm{k}}$$

Example 19-39: Nested Radical Symbols Not Enlarged

$$\sqrt{\sqrt{\mathrm{x}}}$$

Example 19-40: Integral with a Fraction Is Not Enlarged

$$\int\frac{\mathrm{f}(\mathrm{x})\,\mathrm{dx}}{(\mathrm{x}-\mathrm{t})^{\mathrm{n}}}$$

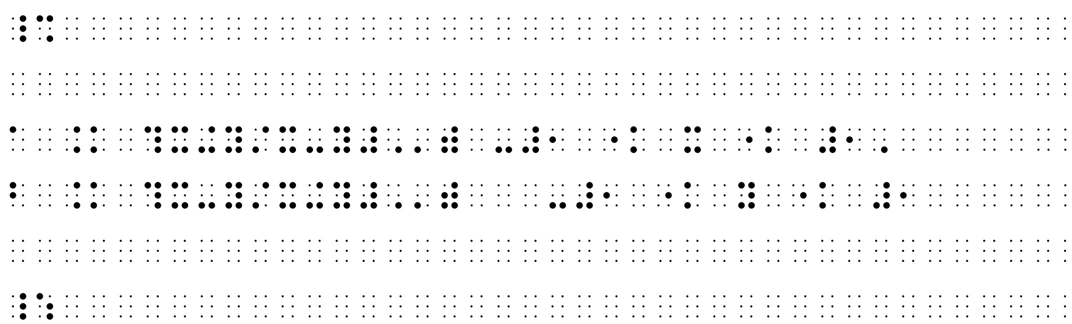
19.8 Transcriber-Inserted Grouping Symbol

When an explanation or comment refers to more than one print line to which no grouping sign as a whole applies, the implied grouping is indicated by using the transcriber-inserted grouping symbol. The explanation is placed to the right in braille regardless of its location in print. There must be at least one clear column of spaces between transcriber-inserted grouping symbols and the associated explanation.

If the explanation requires more braille lines than what is being explained, the transcriber-inserted grouping symbol is extended to cover the explanation, and each runover of the explanation is indented two cells from the column in which the explanation begins. A blank line is required before and after such a grouping.

Example 19-41: Transcriber-Inserted Grouping Symbols

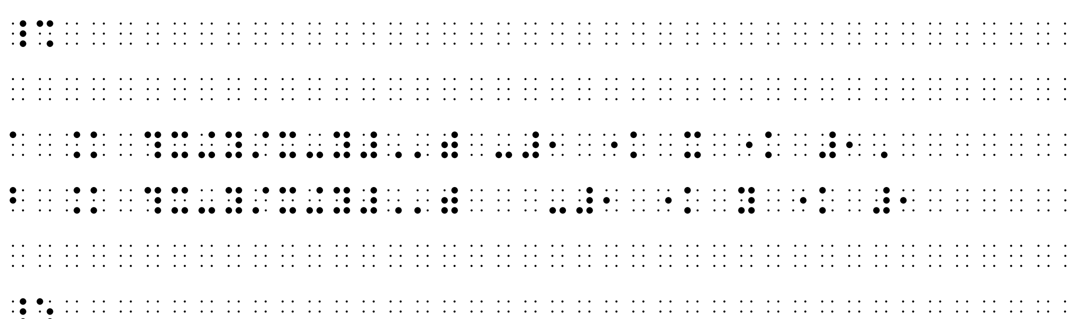
$$\begin{array}{ll} a=\dfrac{x+y}{x-y} & \\ & -1<x<1,\ -1<y<1 \\ b=\dfrac{x-y}{x+y} & \end{array}$$



(in print, the explanation is centered to the right of the two equations to which it applies)

Example 19-42: Transcriber-Inserted Grouping Symbols

$$\begin{array}{ll} & a=\dfrac{x+y}{x-y} \\ -1<x<1,\ -1<y<1 & \\ & b=\dfrac{x-y}{x+y} \end{array}$$



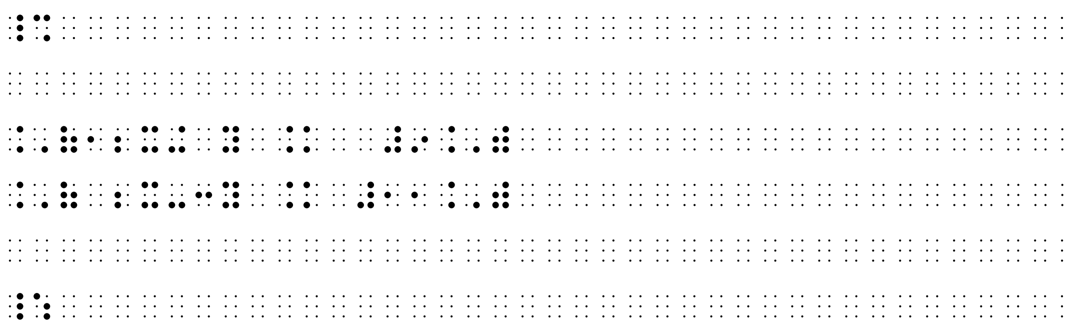
(in print, the explanation is centered to the left of the two equations to which it applies)

19.9 Spacing with Symbols of Grouping

19.9.1 Spaces may be required to be left after a left enlarged grouping symbol or before a right enlarged grouping symbol to preserve vertical alignment of the grouping symbols and/or elements of the system.

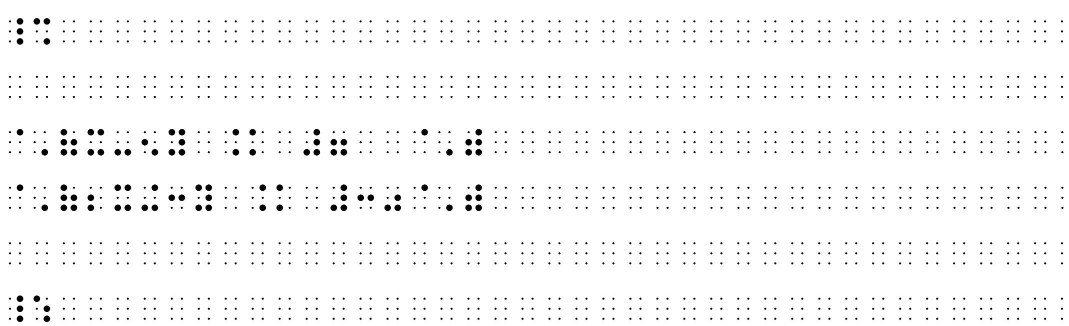
Example 19-43: Aligned System in Enlarged Braces

$$\left\{\begin{array}{r}12x+\ \ y=\ \ 9\\ 2x-3y=11\end{array}\right\}$$



Example 19-44: Unaligned System in Enlarged Brackets

$$\begin{bmatrix} x-5y=7 \\ 2x+3y=30 \end{bmatrix}$$

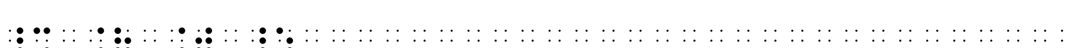


(unified system of two equations not aligned in print)

19.9.2 A space is left between a left and right grouping symbol when there is a blank, not representing omission, between the corresponding signs in print.

Example 19-45: Empty Set

$\{\ \}$



Rule 20
Signs and Symbols of Operation

Ampersand (and, logical product)	&	⠸⠯
Asterisk	$*$	⠈⠼
Backslash (divides, is a factor of)	\	⠸⠡
Circle with Interior Dot	$\odot$	⠫⠉⠸⠫⠡⠻
Circle with Interior Plus	$\oplus$	⠫⠉⠸⠫⠬⠻
Circle with Interior Minus	$\ominus$	⠫⠉⠸⠫⠤⠻
Dagger		
Single	†	⠸⠻
Double	‡	⠸⠸⠻
Division		
Divided By	$\div$	⠨⠌
Division Sign Curving Left	(	⠪
Division Sign Curving Right	)	⠕
Slanted Division Sign	/ or \	⠸
Straight Division Sign	|	⠸
Hollow Dot	$\circ$	⠨⠡
Intersection (cap)	$\cap$	⠨⠩
Logical Product (and, meet)	$\wedge$	⠈⠩
Logical Sum (join, or)	$\vee$	⠈⠬
Minus (or negative sign)		
Regular	$-$	⠤
Boldface	$\boldsymbol{-}$	⠸⠤
Minus followed by Minus	$--$	⠤⠐⠤

Minus Followed by Plus		
Boldface Minus Followed by Boldface Plus	**− +**	⠸⠤⠐⠸⠬
Boldface Minus Followed by Regular Plus	**−** +	⠸⠤⠐⠬
Regular Minus Followed by Regular Plus	− +	⠤⠐⠬
Regular Minus Followed by Boldface Plus	− **+**	⠤⠐⠸⠬
Minus or Plus	∓	⠤⠬
Minus with Dot over (proper difference)	∸	⠨⠤
Multiplication (times)		
Asterisk	∗	⠈⠼
Cross (Cartesian product)	×	⠈⠡
Dot	•	⠡
Number Sign; Crosshatch; Pound Sign	#	⠨⠼
Paragraph Mark	¶	⠈⠠⠏
Plus (or positive sign)		
Regular	+	⠬
Boldface	**+**	⠸⠬
Plus Followed by Minus		
Boldface Plus Followed by Boldface Minus	**+ −**	⠸⠬⠐⠸⠤
Boldface Plus Followed by Regular Minus	**+** −	⠸⠬⠐⠤
Regular Plus Followed by Regular Minus	+ −	⠬⠐⠤
Regular Plus Followed by Boldface Minus	+ **−**	⠬⠐⠸⠤
Plus or Minus	±	⠬⠤
Section Mark	§	⠈⠠⠎
Slash (per, over, divided by)	/	⠸⠌
Square		
Filled-In Square	■	⠫⠸⠲

Hollow Square	□	
Star	☆	
Tilde		
Extended	~~	
Simple	∽	
Union (cup)	$\cup$	
Vertical Bar (is a factor, divides)	$\vert$	

20.1 Spacing with Symbols of Operation

20.1.1 A space is left on either side of an operation symbol under any of the circumstances listed below.

a. Between a comparison symbol and an operation symbol.

Example 20-1: Space Between Equals Sign and Minus Sign

$x = -y$

b. After a function name or an abbreviated function name and before a symbol of operation.

Example 20-2: Abbreviated Function Name and Minus Sign

$\sin - x$

c. Between an ellipsis or dash and a symbol of operation.

Example 20-3: Space Between an Ellipsis and Operation Sign

$1 + 2 + \ldots + n$

Example 20-4: Space Between a Dash and Operation Sign

$10 - \_\_\_ = 8$

d. Between an abbreviation and a symbol of operation other than the fraction line or slash.

Example 20-5: Space Between Abbreviation and Plus Sign

1 yd + 2 yd = 3 yd

⠸⠩ ⠼⠂ ⠽⠙ ⠬⠆ ⠽⠙ ⠨⠅ ⠼⠒ ⠽⠙ ⠸⠱

Example 20-6: Space Between Abbreviation and Plus Sign

3 ft^2 + 3 ft^2 = 6 ft^2

⠸⠩ ⠼⠒ ⠋⠞⠘⠆ ⠬⠒ ⠋⠞⠘⠆ ⠨⠅ ⠼⠖ ⠋⠞⠘⠆ ⠸⠱

(superscript is part of the abbreviation)

e. Where required according to **Rule 23 Miscellaneous Signs and Symbols**.

Example 20-7: Tally Marks and Plus Sign

|||| + ||||

⠸⠩ ⠸⠸⠸⠸ ⠬ ⠸⠸⠸⠸ ⠸⠱

20.1.2 A space is not left on either side of a symbol of operation in any other situation.

Example 20-8: No Space Between Letters and Backslash

a\b

⠸⠩ ⠁⠸⠡⠃ ⠸⠱

Example 20-9: No Space Between Variables and Operation Sign

$x \oplus y$

⠸⠩ ⠭⠫⠉⠸⠫⠬⠻⠽ ⠸⠱

Example 20-10: No Space Between Numbers and Division Sign

$12 \div 3$

⠸⠩ ⠼⠂⠆⠨⠌⠒ ⠸⠱

Example 20-11: No Space Between Letters and Operation Sign

$f \circ g$

Example 20-12: Letters and Function Names and Minus Sign

$\sin x - \sin y$

Example 20-13: Shape as Sign of Omission

$x \square y$

Example 20-14: Shapes as Part of a Mathematical Expression

$\square + \triangle$

Example 20-15: Words and Multiplication Cross

rate × time

Example 20-16: Words and Diagonal Fraction Line

miles/hour

Example 20-17: Words with Operation Signs

quotient × divisor + remainder = dividend

Example 20-18: Operation Signs and Technical Components

$3 \times \text{seven}^2 + 4 \times \text{seven}^1 + 5 \times \text{seven}^0 = 345_{\text{seven}}$

Example 20-19: Vertical Bar as Operation Sign

$(2n + 3) \mid 3$

20.2 Ampersand

When the ampersand is used as a sign of operation, the symbol in the above list must be used.

Example 20-20: Ampersand

The & often denotes logical conjunction.

(specific attention is called to the nature of the sign)

Example 20-21: Ampersand in Mathematical Context

A & B

20.3 Asterisk, Dagger, Double Dagger, Number Sign, Paragraph Mark, Section Mark, and Star

The asterisk, dagger, double dagger, number sign (crosshatch), paragraph mark, section mark, and star are represented by the symbols provided for them in this Code. The numeric indicator is used after an asterisk, crosshatch, section mark, paragraph mark that is followed by a number.

Example 20-22: Asterisk as Operation Sign

$f * g$

Example 20-23: Asterisk as Operation Sign

$3 * 4$

Example 20-24: Asterisk in Superscript Position

x^*

Example 20-25: Crosshatch as Operation Sign

$x \# y$

Example 20-26: Crosshatch as Operation Sign

$2\#3$

Example 20-27: Crosshatch in Superscript Position

$R^{\#}$

Example 20-28: Paragraph Mark as Operation Sign

A ¶ B

Example 20-29: Section Mark as Operation Sign

A § B

Example 20-30: Star as Operation Sign

A ☆ B

20.4 Intersection and Union

These signs are frequently modified directly under, and are consequently printed wide enough to accommodate the modifier. The variable width of these signs is ignored in the transcription. Superscripts or subscripts which are sometimes attached to these signs are treated in the usual manner for handling superscripts and subscripts.

Example 20-31: Intersection

$A \cap B$

⠸⠩⠀⠠⠁⠨⠩⠠⠃⠀⠸⠱

Example 20-32: Intersection with Modification

$\bigcap_{\alpha \in A} x_\alpha$

⠸⠩⠀⠐⠨⠩⠩⠨⠁⠀⠈⠑⠀⠨⠠⠁⠻⠭⠰⠨⠁⠀⠸⠱

Example 20-33: Union

$A \cup B$

⠸⠩⠀⠠⠁⠨⠬⠠⠃⠀⠸⠱

When the intersection sign or the union sign is modified by a superscribed bar, a subscribed bar, or both, the combination is no longer a sign of operation but a sign of comparison compounded vertically (see **Rule 21.9**).

20.5 Logical Product and Logical Sum

Example 20-34: Logical Product

$x \wedge y$

⠸⠩⠀⠭⠈⠩⠽⠀⠸⠱

Example 20-35: Logical Sum

$x \vee y$

⠸⠩⠀⠭⠈⠬⠽⠀⠸⠱

When the signs for *logical product* or *logical sum* are modified by a superscribed bar, a subscribed bar, or both, the combination is no longer a sign of operation but a sign of comparison compounded vertically (see **Rule 21.9**).

20.6 Combinations of Minus and Plus Signs

When the signs for *plus* and *minus* are combined either vertically or horizontally, the combination is regarded as a single sign of operation. Its components must not be divided between braille lines in the transcription.

Example 20-36: Plus and Minus Combined Horizontally

$+2 - +3$

⠸⠩⠀⠬⠆⠤⠐⠬⠒⠀⠸⠱

Example 20-37: Minus and Minus Combined Horizontally

$+4 - -1$

⠸⠩⠀⠬⠲⠤⠐⠤⠂⠀⠸⠱

Example 20-38: Plus and Minus Combined Horizontally

$-3 + -5$

⠸⠩⠀⠤⠼⠒⠬⠐⠤⠢⠀⠸⠱

Example 20-39: Minus and Plus Combined Vertically

$x \mp y$

⠸⠩⠀⠭⠤⠬⠽⠀⠸⠱

Example 20-40: Plus and Minus Combined Vertically

$x \pm y$

⠸⠩⠀⠭⠬⠤⠽⠀⠸⠱

20.7 Multiplication

The common multiplication signs, *cross*, *dot*, or *midline asterisk*, are not used interchangeably in the transcription. Follow print for the correct symbol. The cross is sometimes modified directly under.

Example 20-41: Multiplication Cross

3×10

⠸⠩⠀⠼⠒⠈⠡⠂⠴⠀⠸⠱

Example 20-42: Multiplication Dot

$x \cdot y$

Example 20-43: Multiplication Asterisk

$8 * 16 = x$

Example 20-44: Modified Multiplication Cross

$\underset{\alpha \in A}{\times} A_{\alpha}$

20.8 Slash

When the slash means over, per, or divided by it is a fraction line.

Example 20-45: Fraction Line Slash

The rise/run ratio is 3.

(spoken "rise over run")

Example 20-46: Fraction Line Slash

1 watt = 1 joule/sec.

(spoken "joule per second")

Example 20-47: Fraction Line Slash

volt/amp

Example 20-48: Fraction Line Slash

60 mi/hr

Example 20-49: The Slash is Not a Fraction Line

7/4/76

⠼⠛⠸⠌⠼⠙⠸⠌⠼⠛⠋

(the slash does not mean over, per, or divided by)

20.9 Tilde

This sign of operation is used predominantly in logic with the meaning of *not*.

Example 20-50: Tilde Meaning Not

$\sim p$

⠸⠩⠀⠈⠱⠏⠀⠸⠱

Example 20-51: Tilde Meaning Not

$\sim p \vee \sim q \vee \sim r$

⠸⠩⠀⠈⠱⠏⠈⠬⠈⠱⠟⠈⠬⠈⠱⠗⠀⠸⠱

Example 20-52: Consecutive Tildes

$\sim\sim T \vee R$

⠸⠩⠀⠈⠱⠐⠈⠱⠠⠞⠈⠬⠠⠗⠀⠸⠱

Rule 21
Signs and Symbols of Comparison

Simple Comparison Signs

Arrow		
Left-pointing	$\leftarrow$	⠫⠪⠒⠒
Right-pointing Contracted	$\rightarrow$	⠫⠕
Right-pointing Uncontracted	$\rightarrow$	⠫⠒⠒⠕
Down-pointing	$\downarrow$	⠫⠩⠒⠒⠕
Up-pointing	$\uparrow$	⠫⠣⠒⠒⠕
Two-way Horizontal	$\leftrightarrow$	⠫⠪⠒⠒⠕
Two-way Vertical	$\updownarrow$	⠫⠣⠪⠒⠒⠕
Equals		
Normal (is equal to)	$=$	⠨⠅
Equals Boldface	$\boldsymbol{=}$	⠸⠨⠅
Is Not Equal To	$\neq$ $\nparallel$	⠌⠨⠅
Greater Than		
Normal (is greater than)	$>$	⠨⠂
Greater Than (with curved sides)	$\succ$	⠨⠨⠂
Identity (is congruent to; is identical to)	$\equiv$	⠸⠇
Inclusion (is contained in; is a subset of)	$\subset$	⠸⠐⠅
Less Than		
Normal (is less than)	$<$	⠐⠅
Less Than (with curved sides)	$\prec$	⠨⠐⠅
Membership (is an element of; belongs to)	$\in$	⠈⠑

(Simple Comparison Signs, cont.)

Parallel To (is parallel to)	$\parallel$	⠫⠇
Perpendicular To (is perpendicular to)	$\perp$	⠫⠏
Proportion (as)	$::$	⠰⠆
Ratio (is to)	$:$	⠐⠂
Relation (is related to)	R	⠠⠗
Reverse Inclusion (contains; in logic, implies)	$\supset$	⠸⠨⠂
Reverse Membership (contains the element)	$\ni$	⠈⠢
Tilde		
Simple (is related to; is similar to)	$\sim$	⠈⠱
Extended (is related to)	$\sim\!\sim$	⠈⠠⠱
Variation (varies as)	$\propto$	⠸⠿
Vertical Bar (such that)	$\vert$	⠳

Modified Comparison Signs

Equals Sign

Caret over	$\stackrel{\wedge}{=}$	⠐⠨⠅⠣⠸⠣⠻
Caret under (is projective to, projective correspondence)	$\underset{\wedge}{=}$	⠐⠨⠅⠩⠸⠣⠻
Degree sign over (is equal in degrees to)	$\stackrel{\circ}{=}$	⠐⠨⠅⠣⠨⠡⠻
Dot over (is approximately equal to)	$\doteq$	⠐⠨⠅⠣⠡⠻
Dot over and dot under	$\stackrel{\cdot}{\underset{\cdot}{=}}$	⠐⠨⠅⠩⠡⠣⠡⠻
Equilateral triangle over	$\triangleq$	⠐⠨⠅⠣⠫⠺⠻
Inverted caret over	$\stackrel{\vee}{=}$	⠐⠨⠅⠣⠸⠩⠻
Left-pointing caret over	$\stackrel{<}{=}$	⠐⠨⠅⠣⠰⠣⠻
Question mark over	$\stackrel{?}{=}$	⠐⠨⠅⠣⠸⠦⠻
Right-pointing caret over	$\stackrel{>}{=}$	⠐⠨⠅⠣⠰⠩⠻
Two dots over and two dots under	$\stackrel{\cdot\cdot}{\underset{\cdot\cdot}{=}}$	⠐⠨⠅⠩⠡⠡⠣⠡⠡⠻
Vertical bar over	$\stackrel{\vert}{=}$	⠐⠨⠅⠣⠳⠻

Horizontal Bar

Caret over	$\stackrel{\wedge}{-}$	⠐⠱⠣⠸⠣⠻
Caret under (is perspective to, perspective correspondence)	$\overline{\wedge}$	⠐⠱⠩⠸⠣⠻
Dot under	$\underset{\cdot}{-}$	⠐⠱⠩⠡⠻

Simple Tilde, Dot Under	$\underset{\cdot}{\sim}$	⠐⠈⠱⠩⠡⠻

Comparison Signs Compounded Vertically

Arrow Combinations

Right-pointing over left-pointing	$\substack{\rightarrow \\ \leftarrow}$	⠫⠒⠒⠕⠫⠪⠒⠒
Right-pointing with upper barb over left-pointing with lower barb	$\rightleftharpoons$	⠫⠒⠒⠈⠕⠫⠠⠪⠒⠒
Right-pointing over boldface left-pointing	$\substack{\rightarrow \\ \boldsymbol{\leftarrow}}$	⠫⠒⠒⠕⠫⠸⠪⠒⠒
Left-pointing over boldface right-pointing	$\substack{\leftarrow \\ \boldsymbol{\rightarrow}}$	⠫⠪⠒⠒⠫⠸⠒⠒⠕
Boldface right-pointing over left-pointing	$\substack{\boldsymbol{\rightarrow} \\ \leftarrow}$	⠫⠸⠒⠒⠕⠫⠪⠒⠒
Boldface left-pointing over right-pointing	$\substack{\boldsymbol{\leftarrow} \\ \rightarrow}$	⠫⠸⠪⠒⠒⠫⠒⠒⠕
Boldface right-pointing over boldface left-pointing	$\substack{\boldsymbol{\rightarrow} \\ \boldsymbol{\leftarrow}}$	⠫⠸⠒⠒⠕⠫⠸⠪⠒⠒
Boldface left-pointing over boldface right-pointing	$\substack{\boldsymbol{\leftarrow} \\ \boldsymbol{\rightarrow}}$	⠫⠸⠪⠒⠒⠫⠸⠒⠒⠕
Long right-pointing over short left-pointing	$\substack{\longrightarrow \\ \leftarrow}$	⠫⠒⠒⠒⠕⠫⠪⠒
Short right-pointing over long left-pointing	$\substack{\rightarrow \\ \longleftarrow}$	⠫⠒⠕⠫⠪⠒⠒⠒

(Comparison Signs Compounded Vertically, cont.)

Equivalence (is equivalent to)	$\Bumpeq$	⠈⠣⠠⠣
Greater Than		
Bar over greater than (is equal to or greater than)	$\overline{>}$ or $\geqslant$	⠱⠨⠂
Bar under greater than (is greater than or equal to)	$\geq$ or $\geqslant$	⠨⠂⠱
Equals sign over greater than (is equal to or greater than)	$\overset{=}{>}$ or $\eqslantgtr$	⠨⠅⠨⠂
Equals sign under greater than (is greater than or equal to)	$\geqq$ or $\eqslantgtr$	⠨⠂⠨⠅
Inclusion		
Bar over inclusion (is a subset of)	$\overline{\subset}$	⠱⠸⠐⠅
Bar under inclusion (is a subset of)	$\subseteq$	⠸⠐⠅⠱
Equals sign over inclusion (is a subset of)	$\overset{=}{\subset}$	⠨⠅⠸⠐⠅
Equals sign under inclusion (is a subset of)	$\subseteqq$	⠸⠐⠅⠨⠅
Intersection (cap)		
Bar under intersection	$\underline{\cap}$	⠨⠩⠱
Equals sign under intersection	$\underset{=}{\cap}$	⠨⠩⠨⠅
Less Than		
Bar over less than (is equal to or less than)	$\overline{<}$ or $\leqslant$	⠱⠐⠅
Bar under less than (is less than or equal to)	$\leq$ or $\leqslant$	⠐⠅⠱

(Comparison Signs Compounded Vertically, cont.)

Equals sign over less than (is equal to or less than)	⋜ or ⪙	⠨⠅⠐⠅
Equals sign under less than (is less than or equal to)	≦ or ⪕	⠐⠅⠨⠅
Logical Product (meet)		
Bar over logical product	$\overset{-}{\wedge}$	⠱⠈⠩
Bar over and bar under logical product	$\overset{-}{\underset{-}{\wedge}}$	⠱⠈⠩⠱
Bar over and equals sign under logical product	$\overset{-}{\underset{=}{\wedge}}$	⠱⠈⠩⠨⠅
Bar under logical product	$\underset{-}{\wedge}$	⠈⠩⠱
Equals sign over logical product	$\overset{=}{\wedge}$	⠨⠅⠈⠩
Equals sign over and bar under logical product	$\overset{=}{\underset{-}{\wedge}}$	⠨⠅⠈⠩⠱
Equals sign over and equals sign under logical product	$\overset{=}{\underset{=}{\wedge}}$	⠨⠅⠈⠩⠨⠅
Equals sign under logical product	$\underset{=}{\wedge}$	⠈⠩⠨⠅
Logical Sum (join)		
Bar over logical sum	$\overset{-}{\vee}$	⠱⠈⠬
Bar over and bar under logical sum	$\overset{-}{\underset{-}{\vee}}$	⠱⠈⠬⠱
Bar over and equals sign under logical sum	$\overset{-}{\underset{=}{\vee}}$	⠱⠈⠬⠨⠅
Bar under logical sum	$\underset{-}{\vee}$	⠈⠬⠱
Equals sign over logical sum	$\overset{=}{\vee}$	⠨⠅⠈⠬
Equals sign over and bar under logical sum	$\overset{=}{\underset{-}{\vee}}$	⠨⠅⠈⠬⠱
Equals sign over and equals sign under logical sum	$\overset{=}{\underset{=}{\vee}}$	⠨⠅⠈⠬⠨⠅
Equals sign under logical sum	$\underset{=}{\vee}$	⠈⠬⠨⠅

(Comparison Signs Compounded Vertically, cont.)

Reverse Inclusion		
Bar over reverse inclusion	$\overline{\supset}$	⠱⠸⠨⠂
Bar under reverse inclusion	$\underline{\supset}$	⠸⠨⠂⠱
Equals sign over reverse inclusion	$\overset{=}{\supset}$	⠨⠅⠸⠨⠂
Equals sign under reverse inclusion	$\underset{=}{\supset}$	⠸⠨⠂⠨⠅
Tilde (is related to)		
Bar over double tilde	$\overline{\approx}$	⠱⠈⠱⠈⠱
Bar over single tilde	$\overline{\sim}$	⠱⠈⠱
Bar under double tilde	$\underline{\approx}$	⠈⠱⠈⠱⠱
Bar under single tilde	$\simeq$	⠈⠱⠱
Double tilde	$\approx$	⠈⠱⠈⠱
Equals sign over double tilde	$\overset{=}{\approx}$	⠨⠅⠈⠱⠈⠱
Equals sign over single tilde	$\overset{=}{\sim}$	⠨⠅⠈⠱
Equals sign under double tilde	$\underset{=}{\approx}$	⠈⠱⠈⠱⠨⠅
Equals sign under single tilde	$\cong$	⠈⠱⠨⠅
Union (cup)		
Bar under union	$\underline{\cup}$	⠨⠤⠱
Equals sign under union	$\underset{=}{\cup}$	⠨⠤⠨⠅

Comparison Signs Compounded Horizontally

Arrow Combinations

Up-pointing followed by down-pointing ↑↓ ⠫⠣⠒⠒⠕⠐⠫⠩⠒⠒⠕

Down-pointing followed by up-pointing ↓↑ ⠫⠩⠒⠒⠕⠐⠫⠣⠒⠒⠕

Up-pointing followed by boldface down-pointing ↑**↓** ⠫⠣⠒⠒⠕⠐⠫⠩⠸⠒⠒⠕

Down-pointing followed by boldface up-pointing ↓**↑** ⠫⠩⠒⠒⠕⠐⠫⠣⠸⠒⠒⠕

Boldface up-pointing followed by down-pointing **↑**↓ ⠫⠣⠸⠒⠒⠕⠐⠫⠩⠒⠒⠕

Boldface down-pointing followed by up-pointing **↓**↑ ⠫⠩⠸⠒⠒⠕⠐⠫⠣⠒⠒⠕

Boldface up-pointing followed by boldface down-pointing **↑↓** ⠫⠣⠸⠒⠒⠕⠐⠫⠩⠸⠒⠒⠕

Boldface down-pointing followed by boldface up-pointing **↓↑** ⠫⠩⠸⠒⠒⠕⠐⠫⠣⠸⠒⠒⠕

Greater Than

Followed by less than $> \ <$ ⠨⠂⠐⠐⠅

Followed by equals sign followed by less than $> = <$ ⠨⠂⠐⠨⠅⠐⠐⠅

Less Than

Followed by greater than $< \ >$ ⠐⠅⠐⠨⠂

Followed by equals sign followed by greater than $< = >$ ⠐⠅⠐⠨⠅⠐⠨⠂

Comparison Signs Compounded by Superposition

Dot		
Between bars of equals sign	⩦	⠡⠈⠨⠅⠻
Within inclusion sign	⪽	⠡⠈⠸⠐⠅⠻
Within reverse inclusion sign	⪾	⠡⠈⠸⠨⠂⠻
Equals Sign		
Through inclusion sign	⋹	⠨⠅⠈⠸⠐⠅⠻ or ⠸⠐⠅⠈⠨⠅⠻
Through reverse inclusion	∋	⠨⠅⠈⠸⠨⠂⠻ or ⠸⠨⠂⠈⠨⠅⠻
Greater Than		
Nest of two with straight sides (is large compared with)	≫	⠨⠂⠈⠨⠂⠻
Nest of two with curved sides	⪢	⠨⠨⠂⠈⠨⠨⠂⠻
Horizontal Bar		
Through inclusion sign	⋲	⠱⠈⠸⠐⠅⠻
Through reverse inclusion sign	⋺	⠱⠈⠸⠨⠂⠻
Less Than		
Nest of two with straight sides (is small compared with)	≪	⠐⠅⠈⠐⠅⠻
Nest of two with curved sides	⪡	⠨⠐⠅⠈⠨⠐⠅⠻
Vertical Bar		
Through shaft of right-pointing arrow	⇸	⠳⠈⠫⠒⠒⠕⠻
Through shaft of left-pointing arrow	⇷	⠳⠈⠫⠪⠒⠒⠻

21.1 Negation

Comparison signs may be negated by a vertical stroke or by an oblique stroke in either direction. However the negation is affected in print, the symbol (⠌) is placed unspaced before the comparison symbol being negated.

Example 21-1: Oblique Negated Equals Sign

$\neq$

⠸⠩⠀⠌⠨⠅⠀⠸⠱

(oblique negation sign in print, from lower left to upper right)

Example 21-2: Vertical Negated Equals Sign

⧧

⠸⠩⠀⠌⠨⠅⠀⠸⠱

Example 21-3: Reverse Oblique Negated Membership

$\notin$

⠸⠩⠀⠌⠈⠑⠀⠸⠱

(oblique negation sign in print, from upper left to lower right)

Example 21-4: Oblique Negated Parallel Sign

$\nparallel$

⠸⠩⠀⠌⠫⠇⠀⠸⠱

(oblique negation sign in print, from lower left to upper right)

Example 21-5: Oblique Negated Less Than Sign

$\nless$

⠸⠩⠀⠌⠐⠅⠀⠸⠱

(oblique negation sign in print, from upper left to lower right)

Example 21-6: Vertical Negated Greater Than Equals To

$\ngeq$

⠸⠩⠀⠌⠨⠂⠨⠅⠀⠸⠱

21.2 Arrows

A detailed discussion of the construction of arrows of many types is presented in **Rule 22**. The arrows in the list of simple comparison signs are those which occur with the greatest frequency.

If a right-pointing arrow has a full barb and a single shaft of ordinary length, is in regular type, and occurs by itself, it is represented in its contracted form. If such an arrow is in nonregular type, is itself modified, or occurs as part of a more complex modification, it is represented in its uncontracted form.

Example 21-7: Left-pointing Arrow

$B \leftarrow A$

Example 21-8: Contracted Right-pointing Arrow

$A \rightarrow B$

Example 21-9: Right- and Left-pointing Arrow

$A \leftrightarrow B$

Example 21-10: Modified Right-pointing Arrow

$X \xrightarrow{f \circ g} Y$

21.3 Identity

The triple-bar symbol means is congruent to, is identical to.

Example 21-11: Is Congruent To

$f(x) \equiv 0$

Example 21-12: Is Congruent To

$2 \equiv 5(\text{mod } 3)$

21.4 Membership

This sign is generally used when speaking about sets and the elements of which they are composed. This sign must not be mistaken for the Greek lower-case epsilon.

Example 21-13: Is An Element Of

$x \in A$

21.5 Relation

When a letter or other sign is used between two expressions to show that they are related, the letter or sign used in this way is regarded as a comparison sign. As such, it is subject to all the rules governing comparison signs and symbols. The letter R is frequently used in this situation.

Example 21-14: R to Show Relation

$a \; R \; b$

Example 21-15: Theta to Show Relation

$a \; \theta \; b$

21.6 Tilde

When the tilde, simple or extended, occurs with a dot or caret directly over or directly under it, the combination is a *modified sign of comparison* (see **21.8**). When it occurs directly over or directly under another simple comparison sign, the combination is a *comparison sign compounded vertically* (see **21.9**).

Example 21-16: Regular Tilde

x ~ y

⠸⠩⠀⠭⠀⠈⠱⠀⠽⠀⠸⠱

Example 21-17: Extended Tilde

x ∼∼ y

⠸⠩⠀⠭⠀⠈⠠⠱⠀⠽⠀⠸⠱

21.7 Vertical Bar

A vertical bar meaning "such that" is a sign of comparison. Because the vertical bar is used in several other ways in mathematics (e.g., grouping sign, operation sign), it is helpful to know that when the vertical bar means "such that" it is usually part of an expression within braces used for set notation, or in association with one of the quantifiers.

Example 21-18: Vertical Bar Meaning Such That

Choose $x \mid x = y^2$.

⠀⠀⠠⠡⠕⠕⠎⠑⠀⠸⠩⠀⠭⠀⠳⠀⠭⠀⠨⠅⠀⠽⠘⠆⠀⠸⠱⠲

(x such that x equals y-squared)

Example 21-19: Vertical Bar Meaning Such That

$\{x \mid \; |x| < 10\}$

⠸⠩⠀⠨⠷⠭⠀⠳⠀⠳⠭⠳⠀⠐⠅⠀⠼⠂⠴⠨⠾⠀⠸⠱

(the set of all x such that the absolute value of x is less than 10)

Example 21-20: Vertical Bar Meaning Such That

$\exists|_x \mid x = -x$

⠸⠩⠀⠈⠿⠳⠰⠭⠀⠳⠀⠭⠀⠨⠅⠀⠤⠭⠀⠸⠱

(there exists within x, x such that x = −x)

21.8 Modified Comparison Signs

Modified comparison signs are constructed in accordance with the rules for the representation of modified expressions (see **Rule 15**). Modified signs of comparison other than those in the list above are constructed in accordance with the same principles.

Example 21-21: Question Mark Over Equals Sign

$\overset{?}{=}$

⠸⠩⠀⠐⠨⠅⠣⠸⠦⠻⠀⠸⠱

21.9 Comparison Signs Compounded Vertically

A vertical arrangement of simple comparison signs is represented horizontally in braille. The symbols are transcribed, unspaced, in order from top to bottom. Comparison signs compounded vertically not shown in the list are transcribed in accordance with the above principles.

Example 21-22: Arrows Compounded Vertically

$\rightleftarrows$

⠸⠩⠀⠫⠒⠒⠕⠫⠪⠒⠒⠀⠸⠱

(right-pointing arrow over left-pointing arrow)

21.10 Intersection, Union, Logical Product, and Logical Sum

The unmodified intersection, union, logical product, and logical sum signs are operation signs. If modified they are comparison signs (see **20.4** and **20.5**, respectively).

Example 21-23: Bar Under Union Symbol is a Comparison Sign

$\underline{\cup}$

⠸⠩⠀⠨⠬⠱⠀⠸⠱

21.11 Comparison Signs Compounded Horizontally

A horizontal succession of comparison signs is represented by placing the multipurpose indicator between the unspaced corresponding comparison symbols. Comparison signs compounded horizontally which are not shown in the list are transcribed in accordance with the above principle.

21.12 Comparison Signs Compounded by Superposition

Comparison signs compounded by superposition in the above list are constructed in accordance with the rules for representing superposition (see **15.9**). Comparison signs compounded by superposition other than those in the above list are constructed in accordance with the same principles.

Example 21-24: Dot Within Inclusion Sign

⪽

21.13 Spacing with Symbols of Comparison

A space must be left on either side of a comparison symbol. However, a space is not left between the comparison symbol and any punctuation symbol, grouping symbol, or indicator which applies to it.

Example 21-25: Square as Comparison Sign

$x \square y$

Example 21-26: Equals sign

$x = y$

Example 21-27: Equals Sign in a Modifier

$$\sum_{n=0}^{\infty} a_n$$

Example 21-28: Equals Signs

(a, b) = (c, d) if and only if a = c and b = d.

Example 21-29: Greater Than Sign

$x > y$

⠸⠩ ⠭ ⠨⠂ ⠽ ⠸⠱

Example 21-30: Inclusion

$X \subset Y$

⠸⠩ ⠠⠭ ⠸⠐⠅ ⠠⠽ ⠸⠱

Example 21-31: Inclusion in Modifier

$$\bigcup_{A \subset F} A$$

⠸⠩ ⠐⠨⠬⠩⠠⠁ ⠸⠐⠅ ⠠⠋⠻⠠⠁ ⠸⠱

Example 21-32: Less Than in Modifier

$$\sum_{i<j} a_{ij}$$

⠸⠩ ⠐⠨⠠⠎⠩⠊ ⠐⠅ ⠚⠻⠁⠰⠊⠚ ⠸⠱

Example 21-33: Reverse Membership

$A \ni x$

⠸⠩ ⠠⠁ ⠈⠢ ⠭ ⠸⠱

Example 21-34: Ratio and Proportion

$1 : 2 :: 3 : 6$

⠸⠩ ⠼⠂ ⠐⠂ ⠼⠆ ⠰⠆ ⠼⠒ ⠐⠂ ⠼⠖ ⠸⠱

Example 21-35: Ratio and Proportion

$a + b : b :: c + d : d$

⠸⠩ ⠁⠬⠃ ⠐⠂ ⠃ ⠰⠆ ⠉⠬⠙ ⠐⠂ ⠙ ⠸⠱

Example 21-36: Variation

$x \propto y$

⠸⠩ ⠭ ⠸⠿ ⠽ ⠸⠱

Example 21-37: Such That and Less Than

{all x | each x < 6}

Example 21-38: Such That; Less Than or Equal To

The unit interval = {x | 0 ≤ x ≤ 1}.

Example 21-39: Less Than Within Quotation Marks

The symbol for *less than* is "<".

Example 21-40: Less Than, Equal to, Greater Than

(<, =, >)

Example 21-41: Equals Sign in Modifiers

$$\int_{x=a}^{x=b} f(x)\,dx$$

Rule 22
Arrows

Arrow Components

Arrow Direction Indicators

Depresses nearer arrowhead by 45 degrees	⠰
Elevates nearer arrowhead by 45 degrees	⠘
Makes nearer arrowhead point up	⠣
Makes nearer arrowhead point down	⠩

Arrow Shafts

Curved	⌒ or ◡	⠢⠔
Dashed	— —	⠒⠀⠒
Dotted	• • •	⠂⠂⠂
Long double	═══	⠶⠶⠶
Long single	───	⠒⠒⠒
Ordinary double	══	⠶⠶
Ordinary single	──	⠒⠒
Short double	═	⠶
Short single	─	⠒
Wavy	∼	⠔⠒⠢

Arrow Types

Boldface	⠸
Regular (no indicator)	

Arrowheads

Barbed left full	ᐸ	⠪
Solid left	◄	⠪
Barbed left lower	↽	⠠⠪
Barbed left upper	↼	⠈⠪

Barbed right full	⠕
Solid right	⠕
Barbed right lower	⠠⠕
Barbed right upper	⠈⠕
Blunted left full	⠿
Blunted left lower	⠠⠿
Blunted left upper	⠈⠿
Blunted right full	⠿
Blunted right lower	⠠⠿
Blunted right upper	⠈⠿
Curved left full	⠯
Curved left lower	⠠⠯
Curved left upper	⠈⠯
Curved right full	⠽
Curved right lower	⠠⠽
Curved right upper	⠈⠽
Straight left full	⠳
Straight left lower	⠠⠳
Straight left upper	⠈⠳
Straight right full	⠳
Straight right lower	⠠⠳
Straight right upper	⠈⠳

22.1 Contracted Form of Right-Pointing Arrow

When a right-pointing arrow in regular type, with a single shaft of ordinary length and a full barb, occurs by itself, it is represented in its contracted form (⠫⠕). If such an arrow is in non-regular type, is modified, or occurs as part of a more complex modification, it is represented in its uncontracted form (⠫⠒⠒⠕).

22.2 Arrow Components

There is a large class of signs in the form of arrows which differ from each other in several ways. In the above list there is presented an assortment of arrow components from which such signs are constructed. The entire construction is a simple comparison symbol.

Nemeth arrows are not used for non-mathematical purposes, such as pointing.

$$\begin{array}{rcl} 7.46 & \longleftarrow & \text{2 decimal places} \\ \times\ 3.2 & \longleftarrow & \text{1 decimal place} \\ \hline 1492 & & 2+1=3 \\ 2238 & & \\ \hline 23.872 & \longleftarrow & \text{3 decimal places} \end{array}$$

22.3 Six Steps for Construction of Arrows

Arrow components are transcribed in the following order:

a. The shape indicator

b. The arrow direction, if it must be indicated

c. The arrow type, if it must be indicated

d. The left arrowhead, if any

e. The arrow shaft, if required

f. The right arrowhead, if any

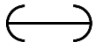
Example 22-1: Boldface Two-way Vertical Arrow

$\boldsymbol{\updownarrow}$

⠸⠩ ⠫⠣⠸⠪⠒⠒⠕ ⠸⠱

(arrow, two-way vertical, boldface, barbed arrowheads at both ends)

Example 22-2: Two-way Arrow with Curved Arrowheads



⠸⠩⠀⠫⠯⠒⠒⠽⠀⠸⠱

(arrow, two-way horizontal, regular type, curved arrowheads at both ends)

Example 22-3: Spear with Blunted Arrowhead and Double Shaft

⠸⠩⠀⠫⠘⠿⠶⠶⠀⠸⠱

(spear, northwest, blunted arrowhead, double shaft)

22.4 Arrow Directions

It is possible to represent eight arrow directions by making proper use of the direction indicators.

22.4.1 The two horizontal directions, *right* and *left*, require no indicator.

Example 22-4: Contracted Right-pointing Arrow

→

⠸⠩⠀⠫⠕⠀⠸⠱

Example 22-5: Uncontracted Right-pointing Arrow

→

⠸⠩⠀⠫⠒⠒⠕⠀⠸⠱

Example 22-6: Left-pointing Arrow

←

⠸⠩⠀⠫⠪⠒⠒⠀⠸⠱

Example 22-7: Two-way Horizontal Arrow

↔

⠸⠩⠀⠫⠪⠒⠒⠕⠀⠸⠱

22.4.2 The two vertical directions, *up* and *down*, require the directly-over indicator or the directly-under indicator, respectively. The directly-over indicator makes the arrowhead point up; the directly-under indicator makes the arrowhead point down. If a vertical arrow is printed with one arrowhead, it is transcribed by using the appropriate symbol for a *right* arrowhead, and not a left one. If a vertical arrow is printed with two arrowheads, only the directly-over indicator is transcribed, before the first arrowhead.

Example 22-8: Arrow Pointing Up

↑

⠸⠩⠀⠫⠣⠒⠒⠕⠀⠸⠱

Example 22-9: Arrow Pointing Down

↓

⠸⠩⠀⠫⠩⠒⠒⠕⠀⠸⠱

Example 22-10: Vertical Two-way Arrow

↕

⠸⠩⠀⠫⠣⠪⠒⠒⠕⠀⠸⠱

22.4.3 The four oblique directions require the superscript indicator or the subscript indicator. The superscript indicator "elevates the nearer arrowhead (if there are two) by 45 degrees from the horizontal position"; the subscript indicator "depresses the nearer arrowhead (if there are two) by 45 degrees from the horizontal position".

Example 22-11: Arrow Pointing Up to the Northeast

↗

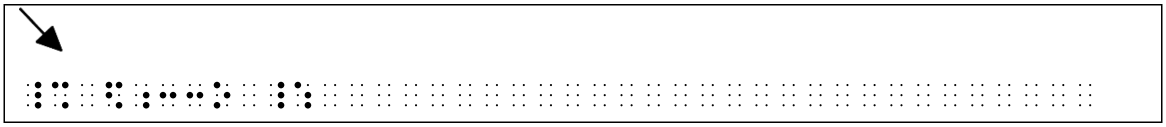
⠸⠩⠀⠫⠘⠒⠒⠕⠀⠸⠱

Example 22-12: Arrow Pointing Up to the Northwest

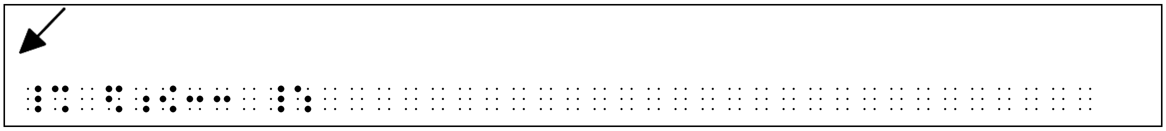
↖

⠸⠩⠀⠫⠘⠪⠒⠒⠀⠸⠱

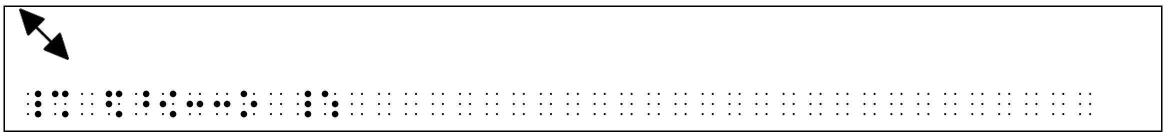
Example 22-13: Arrow Pointing Down to the Southeast



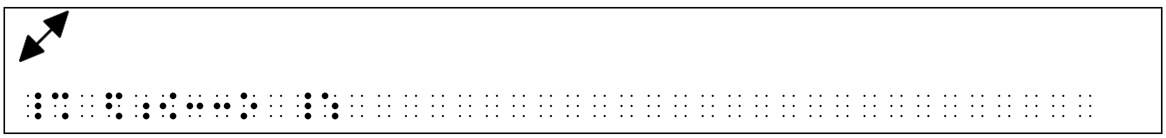
Example 22-14: Arrow Pointing Down to the Southwest



Example 22-15: Two-way Arrow Pointing Northwest-Southeast



Example 22-16: Two-way Arrow Pointing Southwest-Northeast

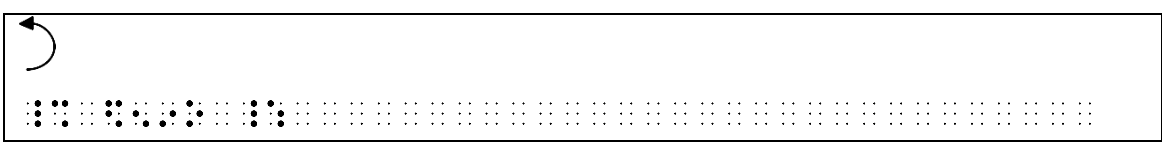


22.5 Arrow Shafts

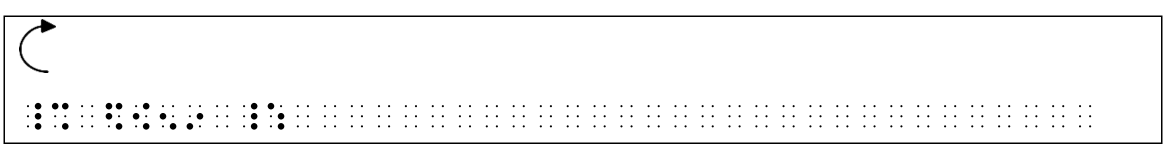
An arrow shaft may be curved, dashed, dotted, straight or wavy, single or double, long or short.

22.5.1 If an arrow shaft is curved, the direction of curvature is indicated by a left arrowhead or a right arrowhead. A curved arrow shaft followed by a right arrowhead represents a counterclockwise arrow; a curved arrow shaft preceded by a left arrowhead represents a clockwise arrow.

Example 22-17: Counterclockwise Arrow



Example 22-18: Clockwise Arrow



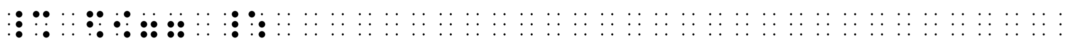
22.5.2 Most arrow shafts are single. An arrow with a double arrow shaft is sometimes called a *spear*.

Example 22-19: Right-pointing Spear

⟹

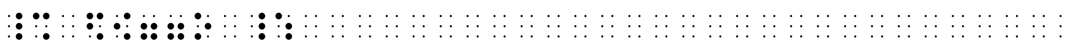
Example 22-20: Left-pointing Spear

⟸



Example 22-21: Two-way Horizontal Spear

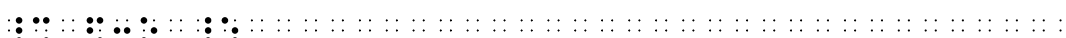
⟺



22.5.3 Where the length of an arrow shaft has significance, the length is indicated by the number of repetitions of the braille arrow shaft symbol. The list distinguishes three lengths, but other lengths may be indicated by repeating the braille arrow-shaft symbol a suitable number of times.

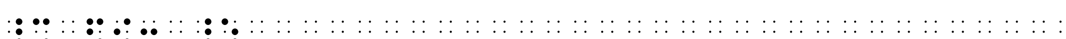
Example 22-22: Short Right-pointing Arrow

⭢



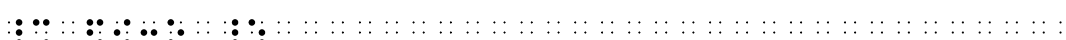
Example 22-23: Short Left-pointing Arrow

⭠



Example 22-24: Short Horizontal Two-way Arrow

⭤



Example 22-25: Long Right-pointing Arrow

→

⠸⠩⠀⠫⠒⠒⠒⠕⠀⠸⠱

Example 22-26: Long Left-pointing Arrow

←

⠸⠩⠀⠫⠪⠒⠒⠒⠀⠸⠱

Example 22-27: Long Horizontal Two-way Arrow

↔

⠸⠩⠀⠫⠪⠒⠒⠒⠕⠀⠸⠱

22.6 Boldface Arrows

Most arrows are printed in regular type. If an arrow is printed in boldface type, the boldface type indicator (⠸) is inserted following the shape indicator.

Example 22-28: Boldface Right-pointing Arrow

→

⠸⠩⠀⠫⠸⠒⠒⠕⠀⠸⠱

Example 22-29: Boldface Left-pointing Arrow

←

⠸⠩⠀⠫⠸⠪⠒⠒⠀⠸⠱

Example 22-30: Boldface Horizontal Two-way Arrow

↔

⠸⠩⠀⠫⠸⠪⠒⠒⠕⠀⠸⠱

22.7 Arrowheads

22.7.1 Most arrowheads are barbed. However, arrowheads also occur as blunted, curved, or straight. They may occur at the left end, right end, or at both ends, of the arrow shaft.

Example 22-31: Right-pointing Arrow with Blunted Arrowhead

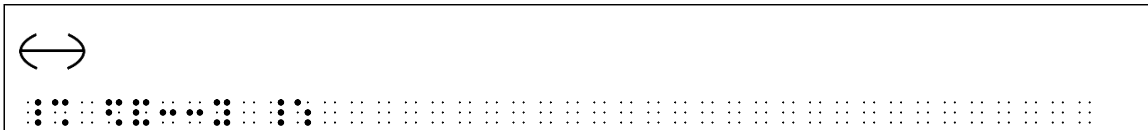
Example 22-32: Left-pointing Arrow with Blunted Arrowhead

Example 22-33: Two-way Arrow with Blunted Arrowheads

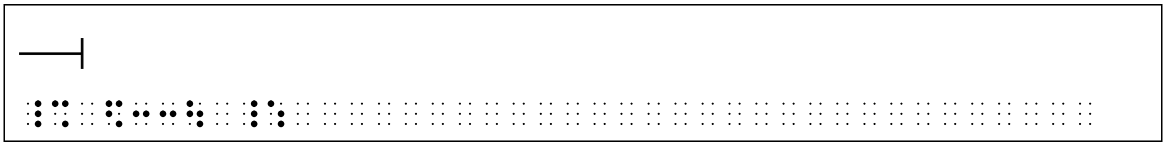
Example 22-34: Right-pointing Arrow with Curved Arrowhead

Example 22-35: Left-pointing Arrow with Curved Arrowhead

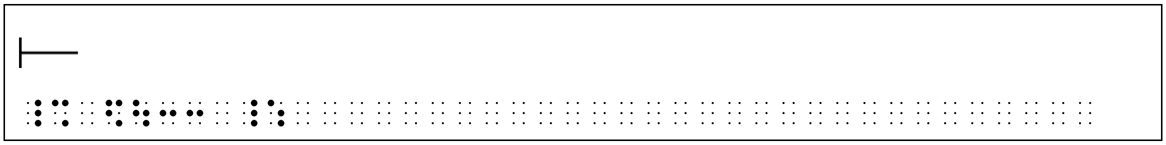
Example 22-36: Two-way Arrow with Curved Arrowheads



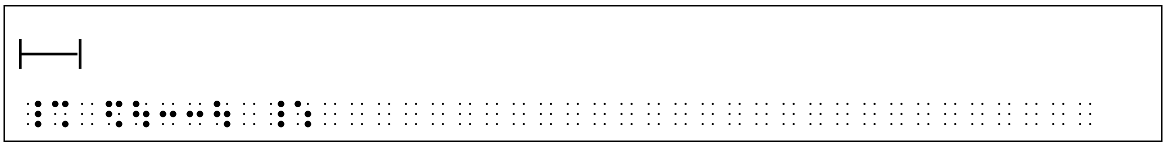
Example 22-37: Right-pointing Arrow with Straight Arrowhead



Example 22-38: Left-pointing Arrow with Straight Arrowhead

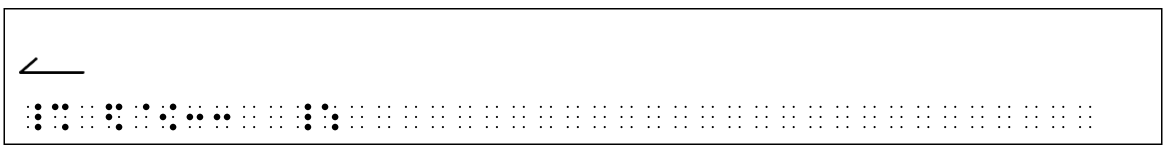


Example 22-39: Two-way Arrow with Straight Arrowheads

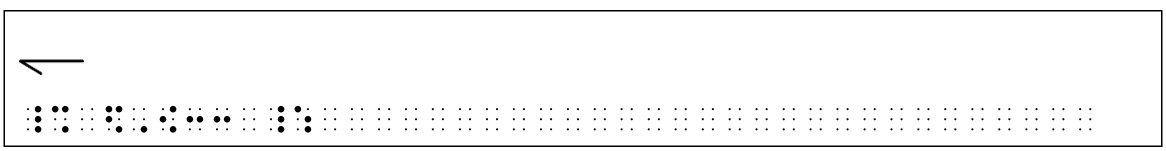


22.7.2 An arrowhead with its upper half only, or its lower half only, may also be present. Any combination of arrowheads – barbed, blunted, curved, straight, left or right, full, lower half, or upper half – may occur.

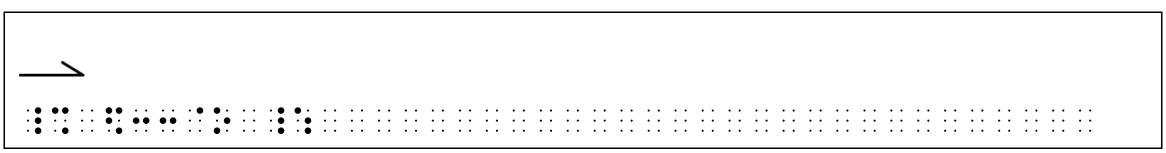
Example 22-40: Arrow with Left Upper Barb



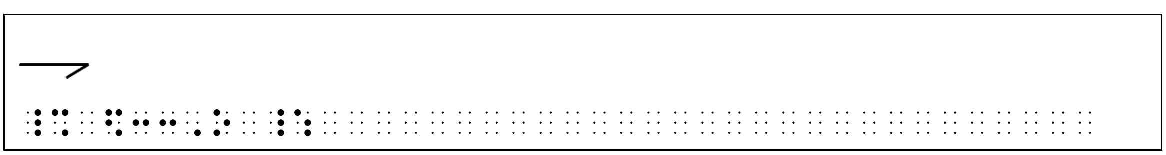
Example 22-41: Arrow with Left Lower Barb



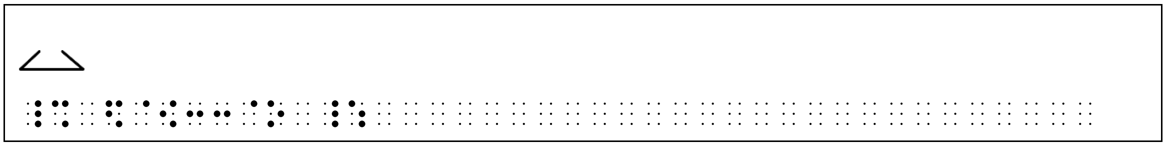
Example 22-42: Arrow with Right Upper Barb



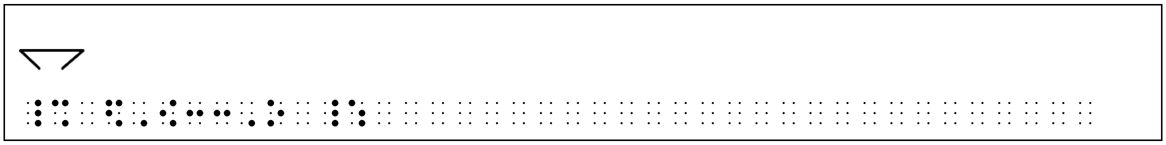
Example 22-43: Arrow with Right Lower Barb



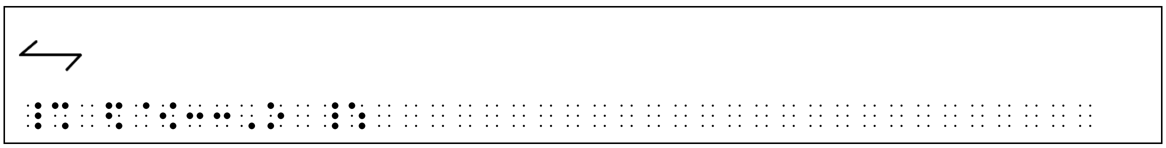
Example 22-44: Two-Way Arrow with Upper Barbs Only



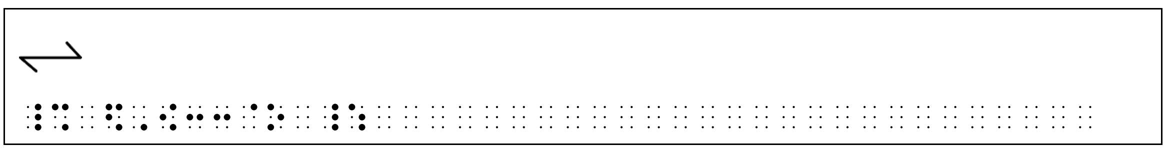
Example 22-45: Two-Way Arrow with Lower Barbs Only



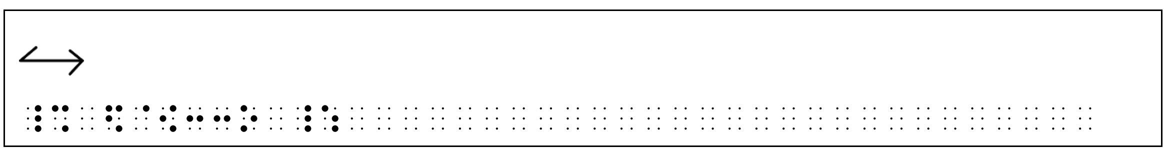
Example 22-46: Arrow with Left Upper and Right Lower Barbs



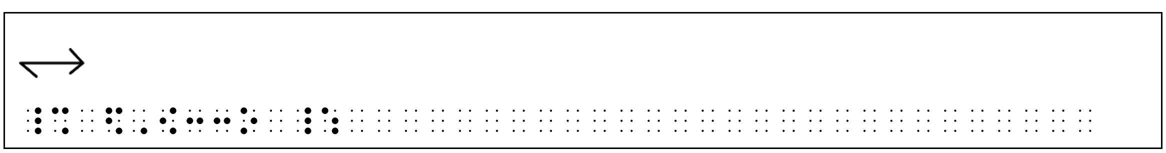
Example 22-47: Arrow with Left Lower and Right Upper Barbs



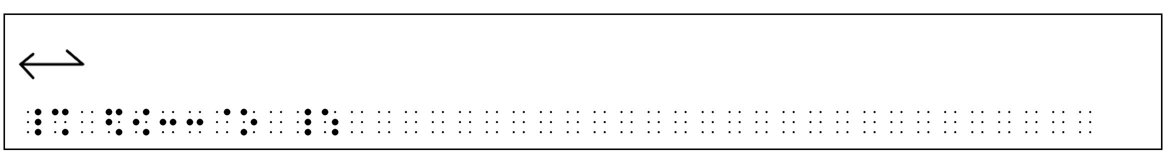
Example 22-48: Arrow with Left Upper Barb and Full Right Barb



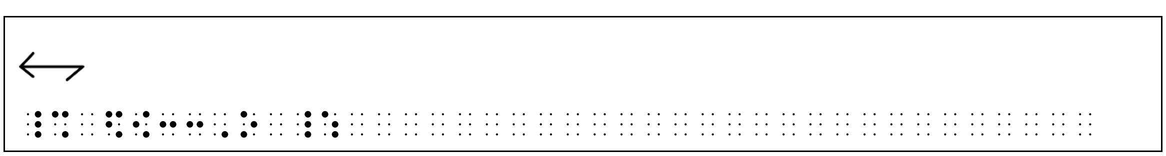
Example 22-49: Arrow with Left Lower Barb and Full Right Barb



Example 22-50: Arrow with Full Left Barb and Right Upper Barb



Example 22-51: Arrow with Full Left Barb and Right Lower Barb



Example 22-52: Two-Way Arrow with Full Left and Right Barbs

$\longleftrightarrow$

⠸⠩ ⠫⠪⠒⠒⠕ ⠸⠱

Rule 23
Miscellaneous Signs and Symbols

Angstrom Unit	Å	⠈⠠⠁
At	@	⠈⠁
Caret (circumflex, hat)	^	⠸⠣
Crossed h (h-bar)	ℏ	⠈⠓
Crossed Lambda (Lambda-bar)	ƛ	⠈⠨⠇
Crossed R (R-bar)	℞	⠈⠠⠗
Del (nabla, gradient)	∇ or ▽	⠨⠫
Ditto Mark	"	⠠⠄
Empty Set		
Represented by Zero with Oblique Bar Through It	∅	⠸⠴
Represented by Facing Braces	{ }	⠨⠷⠀⠨⠾
End of Proof (e.g.)	▌ or □	⠈⠫⠟⠑⠙
Factorial	!	⠯
Hollow Dot	∘	⠨⠡
Infinity	∞	⠠⠿
Integral		
Single	∫	⠮
Double	∬	⠮⠮
Triple	∭	⠮⠮⠮
Lower	$\underline{\int}$	⠩⠮
Upper	$\overline{\int}$	⠣⠮
Integral with Superposed Circle	∮	⠮⠈⠫⠉⠻
Integral with Superposed Infinity	∱	⠮⠈⠠⠿⠻
Integral with Superposed Rectangle	⨒	⠮⠈⠫⠗⠻

Integral with Superposed Square	⨖	⠮⠈⠫⠲⠻
Monetary Units		
Cent	¢	⠈⠉
Dollar	$	⠈⠎
Euro	€	⠈⠑
Franc	₣	⠈⠋
Naira	₦	⠈⠝
Pound sterling	£	⠈⠇
Won	₩	⠈⠺
Yen or Yuan	¥	⠈⠽
Partial Derivative (round d)	∂	⠈⠙
Per Mille	‰	⠈⠴⠴
Percent	%	⠈⠴
Prime	′	⠄
Quantifiers		
Existential Quantifier		
There exists, for some	∃	⠈⠿
There exists uniquely for exactly one	∃| ∃!	⠈⠿⠳
Universal Quantifier (for all, for each, for every)	∀	⠈⠯
Since (because)	∵	⠈⠌
Tally (not a vertical bar)	|	⠸
Therefore		
Normal	∴	⠠⠡
Negated (it does not follow that)	/∴	⠌⠠⠡
Vertical Bar (not a tally mark)	|	⠳

23.1 Angstrom Unit

The angstrom unit is punctuated mathematically and spaced as an abbreviation.

Example 23-1: Angstrom

$1/10{,}000\ \mu = 1\text{Å}$

23.2 At

Except for punctuation, indicators, or symbols of grouping which apply to it, a space is left on both sides of the *at* symbol.

Example 23-2: At Symbol

3 boxes @ 27¢

23.3 Caret

A caret is unspaced from the symbols to which it applies.

Example 23-3: Caret

$.35_{\wedge}73$

23.4 Crossed h, Crossed Lambda, Crossed R

These symbols are unspaced from each other and from other mathematical symbols and symbols of grouping unless rules which govern these other symbols require a space.

Example 23-4: Crossed R

℞ (P_1P_2, P_3P_4)

Example 23-5: Crossed Lambda

$$ƛ = \frac{\lambda}{2\pi}$$

⠸⠩⠀⠈⠨⠇⠀⠨⠅⠀⠹⠨⠇⠌⠆⠨⠏⠼⠀⠸⠱

23.5 Del

Del symbols are unspaced from each other and from other mathematical symbols and symbols of grouping unless rules which govern these other symbols require a space.

Example 23-6: Del Symbols

$$\nabla u + \nabla v$$

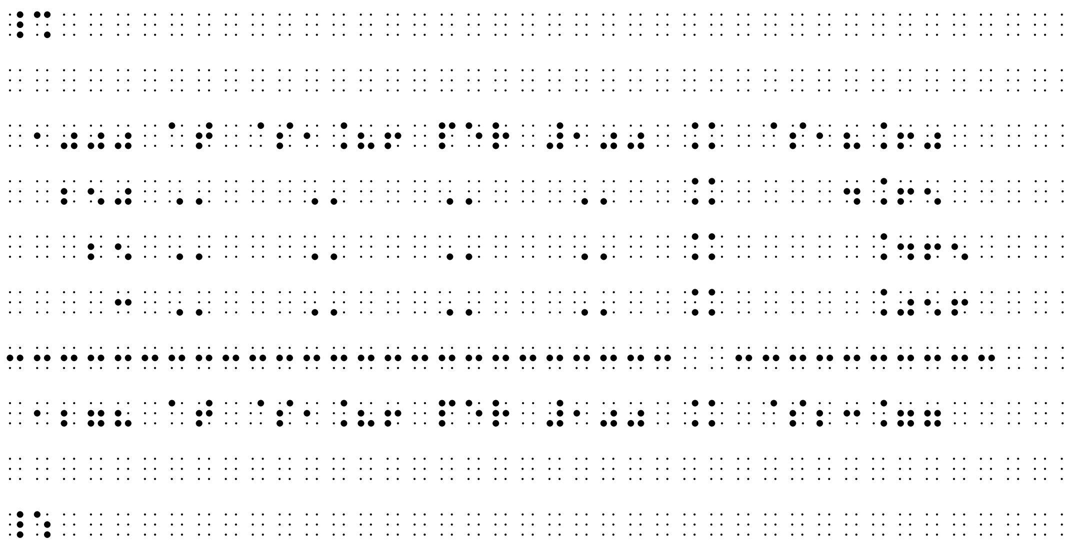
⠸⠩⠀⠨⠫⠥⠬⠨⠫⠧⠀⠸⠱

23.6 Ditto Mark

Ditto marks are centered below the material which they duplicate. A space is left on both sides of a ditto mark, except for punctuation indicators or symbols of grouping which apply to it.

Example 23-7: Ditto Marks

1000	at	\$1.86	per	100	=	\$18.60
250	"	"	"	"	=	4.65
25	"	"	"	"	=	.465
3	"	"	"	"	=	.056
1278	at	\$1.86	per	100	=	\$23.77



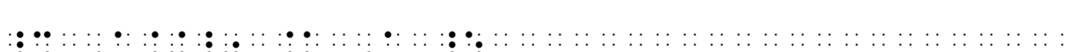
23.7 Empty Set (null set, void set)

The transcriber must not mistake a zero with an oblique bar (∅), meaning *empty set*, for the lowercase Greek letter phi (ϕ) to which it is similar. When the empty set is represented by the zero with an oblique bar in print, the corresponding braille symbol (⠸⠴) is used and is unspaced from other mathematical symbols and symbols of grouping unless rules which govern these other symbols require a space.

When facing braces are used to represent the empty set in print, the corresponding braille symbols (⠨⠷⠀⠨⠾) are used and spaced as grouping symbols.

Example 23-8: Empty Set

$A \cup \emptyset = A$



Example 23-9: Facing Braces Indicating Empty Set

{even integers} ∩ {odd integers} = { }

23.8 End of Proof

End of proof or end of example may be shown in print as a vertical bar, a rectangle, or other shapes. These are all represented in braille by the end of proof icon (⠈⠫⠟⠑⠙). The end of proof icon is preceded by an empty cell. This icon is created with the UEB transcriber-defined shape indicator (⠈⠫) and may be used in either Nemeth Code or UEB without switching.

Example 23-10: End of Proof Indicated by a Solid Rectangle

PROOF From $ab = ac$ we have $a(b\text{-}c) = 0$. Since $a \neq 0$,

we must have $b - c = 0$. ▮

23.9 Factorial

The factorial symbol is unspaced from the quantity to which it applies.

Example 23-11: Factorial

n!

Example 23-12: Factorial

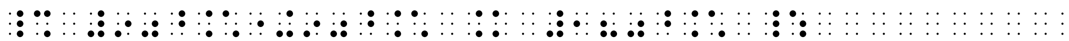
(n - k)!

23.10 Hollow Dot

When the hollow dot is used with the meaning *degrees*, its position at the superscript level is indicated in the transcription.

Example 23-13: Degree Symbol

$$90^\circ + 90^\circ = 180^\circ$$

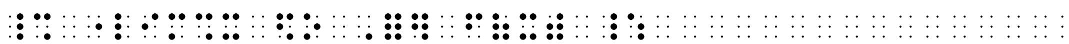


23.11 Infinity

This symbol is unspaced from other mathematical symbols and symbols of grouping unless rules which govern these other symbols require a space.

Example 23-14: Infinity Symbol in a Modifier

$$\lim_{x \to \infty} f(x)$$



Example 23-15: Infinity Symbol in a Modifier

$$\int_0^\infty f(x)\, dx$$

23.12 Integral

The bar over the integral sign, or the bar under the integral sign, is transcribed as shown in the above list. The technique for the representation of modified expressions is not used in these cases; other modifiers, however, are transcribed in accordance with the technique for the representation of modified expressions (see **Rule 15**). The integral, modified or unmodified, is unspaced from the symbol to which it applies.

Example 23-16: Integral with Subscript and Superscript

$$\int_a^b f(x)\,dx$$

Example 23-17: Integral with Modifiers Above and Below

$$\int\limits_o^\infty f(x)\,dx$$

Example 23-18: Integral with Modifier Bar Above

$$\overline{\int}_{a}^{b} f(x)\,dx = 0$$

Example 23-19: Integral with Modifier Bar Below

$$\underline{\int}_{a}^{b} f(x)\,dx = 0$$

Example 23-20: Integral with Modifier Below and Above

$$\int\limits_{x=a}^{x=b} f(t)\,dt$$

Example 23-21: Double Integral Symbol with Modifier Below

$$\iint\limits_R f(x, y)\,dA$$

Example 23-22: Triple Integral Symbol with Modifier Below

$$\iiint\limits_R f(x, y, z)\,dV$$

23.13 Monetary Units

A switch to Nemeth Code is required, but the symbols used are the same as the UEB symbols for monetary units. Monetary symbols are unspaced from the material to which they apply. A numeric indicator is not inserted between a monetary symbol and the number which immediately follows it.

Example 23-23: Cent Symbol

10¢

⠸⠩ ⠼⠂⠴⠈⠉ ⠸⠱

Example 23-24: Cent Symbol with a Letter

x¢

⠸⠩ ⠭⠈⠉ ⠸⠱

Example 23-25: Dollar Symbol

$2.98

⠸⠩ ⠈⠎⠆⠨⠔⠦ ⠸⠱

Example 23-26: Dollar Symbol with a Letter

$x

⠸⠩ ⠈⠎⠭ ⠸⠱

Example 23-27: Pounds Sterling Symbol

£5

⠸⠩ ⠈⠇⠢ ⠸⠱

Example 23-28: Pounds Sterling Symbol with a Letter

£x

⠸⠩ ⠈⠇⠭ ⠸⠱

Example 23-29: Euro Symbol

€3

⠸⠩ ⠈⠑⠒ ⠸⠱

23.14 Partial Derivative

Partial derivative symbols are unspaced from other mathematical symbols and symbols of grouping unless rules which govern these other symbols require a space.

Example 23-30: Partial Derivative

$$\frac{\partial}{\partial x} f(x, y)$$

⠸⠩ ⠹⠈⠙⠌⠈⠙⠭⠼⠋⠷⠭⠠ ⠽⠾ ⠸⠱

23.15 Percent, Per Mille

The symbols are unspaced from the material to which they apply.

Example 23-31: Percent Symbol

7%

⠸⠩ ⠼⠶⠈⠴ ⠸⠱

Example 23-32: Percent Symbol Following a Letter

x%

⠸⠩ ⠭⠈⠴ ⠸⠱

Example 23-33: Per Mille Symbol

10‰

⠸⠩ ⠼⠂⠴⠈⠴⠴ ⠸⠱

23.16 Prime

The single and double primes are often used to denote *feet* and *inches*, respectively. They are also used to denote *minutes* and *seconds*, respectively, whether of time or of angle. Prime symbols are unspaced from each other, and from the quantity to which they apply.

Example 23-34: Prime

x'

Example 23-35: Double Prime

x''

Example 23-36: Triple Prime

x'''

Example 23-37: Prime with Superscript

x'^2

Example 23-38: Prime with Subscript

x'_i

Example 23-39: Prime with Subscript

x'_1

Example 23-40: Parentheses with Prime

$(u + v)' = u' + v'$

Example 23-41: Prime with Superscribed Bar

$\bar{x}'$

Example 23-42 Prime/Double Prime Meaning Feet and Inches

$5'8''$

Example 23-43: Prime Used for Minutes and Seconds

$20^{\circ}30'10''$

23.17 Quantifiers

The existential and universal quantifiers are unspaced from the quantities to which they apply. Sometimes in mathematical language, an exclamation point is used instead of a vertical bar to mean "there exists uniquely one". In braille, both signs are transcribed as a vertical bar.

Example 23-44: Quantifier with x

$\exists x, x < \frac{1}{n}$

Example 23-45: Quantifier; Uniquely One

$\exists\big|_x \, | x = -x$

(there exists, within x, x such that x = − x)

Example 23-46: Quantifier; For All

$\forall x \in A$

Example 23-47: Quantifiers with Subscripts; For All

$\forall_x \forall_y - \frac{y-x}{x+y} = \frac{x-y}{x+y}$

Example 23-48: Exclamation Point: "there exists uniquely one"

$\exists! n \in \mathbb{N}\ (n - 2 = 4)$

⠸⠩ ⠈⠿⠳⠝ ⠈⠑ ⠠⠸⠠⠝ ⠷⠝⠤⠆ ⠨⠅ ⠼⠲⠾ ⠸⠱

(there is exactly one natural number n such that n-2 = 4)

23.18 Since, Therefore

Except for punctuation, indicators, and grouping symbols, the symbol for *since* and symbols for *therefore*, in its normal or negated form, are spaced from the material to which they apply.

Example 23-49: Since

$\because AB = AC$

⠸⠩ ⠈⠌ ⠠⠁⠠⠃ ⠨⠅ ⠠⠁⠠⠉ ⠸⠱

Example 23-50: Therefore

$\therefore AB = AC$

⠸⠩ ⠠⠡ ⠠⠁⠠⠃ ⠨⠅ ⠠⠁⠠⠉ ⠸⠱

23.19 Tally Marks

Tally marks are grouped in braille as they are grouped in print. The cross tally which sometimes appears in print is treated as just another tally mark. Groups of tally marks are separated by a single space from each other and, except for punctuation, indicators, and grouping symbols, from surrounding material. Transition to another braille line takes the place of this required space. Transition to another line of braille is never made from one tally mark to another within the same group.

Example 23-51: Three Tally Marks

| | |

⠸⠩ ⠸⠸⠸ ⠸⠱

Example 23-52: Nine Tally Marks in Groups of Five

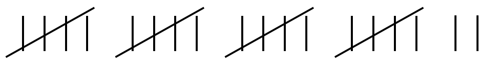
⠸⠩⠀⠸⠸⠸⠸⠸⠀⠸⠸⠸⠸⠀⠸⠱

Example 23-53: Tally Marks in Groups of Six

|||||| |||||| |||

⠸⠩⠀⠸⠸⠸⠸⠸⠸⠀⠸⠸⠸⠸⠸⠸⠀⠸⠸⠸⠀⠸⠱

Example 23-54: Twenty-two Tally Marks in Groups of Five



⠸⠩⠀⠸⠸⠸⠸⠸⠀⠸⠸⠸⠸⠸⠀⠸⠸⠸⠸⠸⠀⠸⠸⠸⠸⠸⠀⠸⠸⠀⠸⠱

23.20 Vertical Bar

The vertical bar is used in several ways in mathematics (e.g., absolute value when enclosing a math expression, comparison sign meaning such that, grouping sign, operation sign).

Example 23-55: Used for Such That; Absolute Value

$\{x \mid | x | < 10\}$

⠸⠩⠀⠨⠷⠭⠀⠳⠀⠳⠭⠳⠀⠐⠅⠀⠼⠂⠴⠨⠾⠀⠸⠱

(spoken math: the set of all x such that the absolute value of x is less than 10)

Example 23-56: Vertical Bar Used for Operation Symbol

$a \mid b$

⠸⠩⠀⠁⠳⠃⠀⠸⠱

Example 23-57: Used in Conditional Probability Function

$P(\text{purple} \mid \text{yellow})$

⠸⠩⠀⠠⠏⠷⠏⠥⠗⠏⠇⠑⠳⠽⠑⠇⠇⠕⠺⠾⠀⠸⠱

(probability of "purple" given "yellow")

Rule 24
Multipurpose Indicator

Multipurpose Indicator ⠐

24.1 Use of the Multipurpose Indicator

The multipurpose indicator is used in the situations below for the specific purposes described and, when used in these situations, it is not regarded as the baseline indicator. (For baseline indicator, also represented by a dot 5, see **Rule 14**.)

a. The multipurpose indicator is used before a modified expression as an indication to the reader of impending modification. See **Rule 15** for additional information and examples.

b. The multipurpose indicator is used between a letter and a succeeding numeric symbol to indicate that the corresponding numeral is not a subscript to the corresponding letter. **Note:** When the letter represents a numeral in a numeration system to a base other than 10, it is regarded as a numeral and, accordingly, the multipurpose indicator is not used.

Example 24-1: Letter Followed by a Number on the Baseline

x5

⠸⠩ ⠭⠐⠢ ⠸⠱

Example 24-2: Letter Followed by Decimal Number on Baseline

x.6

⠸⠩ ⠭⠐⠨⠖ ⠸⠱

Example 24-3: Sigma Followed by a Number on the Baseline

Σ2

⠸⠩ ⠨⠠⠎⠐⠆ ⠸⠱

Example 24-4: Letter as a Numeral

$t1e4$

⠸⠩⠀⠼⠞⠂⠑⠲⠀⠸⠱

(a base-12 numeral)

c. The multipurpose indicator is used between a numeric subscript and a numeral, if the latter is on the baseline.

Example 24-5: Numeric Subscript Followed by Baseline Number

$c_0 10^2 + c_1 10 + c_2$

⠸⠩⠀⠉⠴⠐⠂⠴⠘⠆⠐⠬⠉⠂⠐⠂⠴⠬⠉⠆⠀⠸⠱

Example 24-6: Numeric Subscript Followed by Baseline Number

$2n_1 5^{-3/2} - n_2 5^{-1/2}$

⠸⠩⠀⠼⠆⠝⠂⠐⠢⠘⠤⠒⠸⠌⠆⠐⠤⠝⠆⠐⠢⠘⠤⠂⠸⠌⠆⠀⠸⠱

d. A multipurpose indicator is used between a right superscript/subscript and a left superscript/subscript that follows unspaced in print.

Example 24-7: Right Superscript Followed by Left Superscript

$p^b\ {}^c x$

⠸⠩⠀⠏⠘⠃⠐⠘⠉⠐⠭⠀⠸⠱

(p carries a right superscript b; c is a left superscript to x)

Example 24-8: Right Superscript Followed by Left Subscript

$p^b{}_c x$

⠸⠩⠀⠏⠘⠃⠐⠰⠉⠐⠭⠀⠸⠱

e. The multipurpose indicator is used between two symbols of operation to indicate that the corresponding signs of operation are printed horizontally and not vertically. This does not apply to fraction lines that precede or follow a symbol of operation.

Example 24-9: Plus Sign Followed by Minus Sign

$-3 + -5$

Example 24-10: Fraction Line Followed by Minus Sign

$$\frac{4}{-2}$$

f. The multipurpose indicator is used between two symbols of comparison to indicate that the corresponding signs of comparison are printed horizontally and not vertically (see **Rule 21**).

Example 24-11: Less Than Followed by Equals Sign

In Excel, D2 <= 5 means D2 is less than or equal to 5.

g. The multipurpose indicator is used after the decimal point symbol to indicate that the symbol which follows it is not numeric unless that symbol is the comma or the punctuation indicator.

Example 24-12: Decimal Point Followed by a Letter

$0.a_1a_2 \ldots$

Example 24-13: Decimal Point Followed by a Greek Letter

$0.\alpha_1\alpha_2 \ldots$

Example 24-14: Decimal Point Followed by Question Mark

$\frac{1}{3} = .?$

⠸⠩⠀⠹⠂⠌⠒⠼⠀⠨⠅⠀⠨⠐⠿⠀⠸⠱

(the general omission symbol represents a question mark in print)

Example 24-15: Decimal Point Followed by a Plus Sign

$3. + .4 = 3.4$

⠸⠩⠀⠼⠒⠨⠐⠬⠨⠲⠀⠨⠅⠀⠼⠒⠨⠲⠀⠸⠱

Example 24-16: Decimal Point Followed by a Parenthesis

(3.)

⠸⠩⠀⠷⠒⠨⠐⠾⠀⠸⠱

Example 24-17: Decimal Point Followed by a Fraction Line

$\frac{1.}{2.}$

⠸⠩⠀⠹⠂⠨⠐⠌⠆⠨⠐⠼⠀⠸⠱

h. The multipurpose indicator is used between a tally mark and the punctuation indicator.

Example 24-18: Tally Mark Followed by a Punctuation Indicator

$\cancel{||||}\,|$, $||||$. …

⠸⠩⠀⠸⠸⠸⠸⠸⠠⠀⠸⠸⠸⠸⠐⠸⠲⠀⠄⠄⠄⠀⠸⠱

(in print, the first group of tallies has a cross tally)

i. The multipurpose indicator is used between two vertical bars of which the first is a right grouping symbol and the second is a left grouping symbol. It is also used between two vertical bars which are grouping symbols of which one is shorter and/or thicker than the other.

Example 24-19: Adjacent Vertical Bars

$|x||y|$

Example 24-20: Adjacent Double Vertical Bars

$\|x\|\,\|y\|$

Example 24-21: Vertical Bars of Varying Lengths

$\||x|\|$

Example 24-22: Adjacent Vertical Bars of Varying Lengths

$|x|\Big|_{x=0}$

j. The multipurpose indicator is used between an operation symbol when it is represented by a symbol for a regular polygon and a numeral which follows.

Example 24-23: Square Shape Followed by a Number

$9 \;\square\; 14 = 23$

Example 24-24: Solid Square Shape Followed by a Number

$9_7 \;\blacksquare\; 13_7$

k. The multipurpose indicator is used between two symbols for the tilde to indicate that they are written horizontally, one after the other.

Example 24-25: Two Tildes Written Horizontally

~ ~T

⠸⠩⠀⠈⠱⠐⠈⠱⠠⠞⠀⠸⠱

Rule 25
Spatial Arrangements

Curved Division Signs		
Curving right	)	⠕
Curving left	(	⠪
Directional Ellipsis		
Diagonal (lower left to upper right)	⋰	⠘⠄⠄
Diagonal (upper left to lower right)	⋱	⠰⠄⠄
Horizontal	⋯	⠄⠄⠄
Vertical	⋮	⠩⠄⠄
Regrouping Indicators		
For numbers above the arrangement (varying in length)		⠣⠶⠶
For numbers below the arrangement (varying in length)		⠩⠶⠶
Separation Line (varying in length)	______	⠒⠒⠒
Slanted Division Sign	/ or \	⠸
Spatial fraction line (varying in length)	______	⠹⠒⠒⠒⠼
Square Root Sign	√	⠜
Straight Division Sign	|	⠸
Vertical Line in Division Arrangement (varying in length)	|	⠸

25.1 Definition

Material occupying more than one print line and having a vertical relationship is a spatial arrangement. In braille, spatial material is preceded and followed by a blank line. Transition to a new braille page before beginning or after ending a spatial arrangement takes the place of the required blank line. When a running head is used, a line is skipped between the running head and a spatial arrangement. See **26.6.1**.

25.2 Code Switching with Spatial Problems

Code switch indicators are placed outside of the spatial material in order not to interfere with alignments. See **26.6.4** regarding layout of embedded spatial material.

An opening Nemeth Code indicator that precedes a spatial problem is placed on the same line as the end of the text above the problem if it fits. If there is not room on that line, the opening Nemeth Code indicator is placed on the next line in cell 1. The required blank line follows the opening Nemeth Code indicator. If there is no identifier or text preceding the spatial problem, the opening indicator is placed in cell 1 on a line by itself followed by the requisite blank line. When Nemeth Code is closed after a spatial problem, the terminator is placed in cell 1 on a line by itself and is preceded by the required blank line.

25.3 Addition and Subtraction

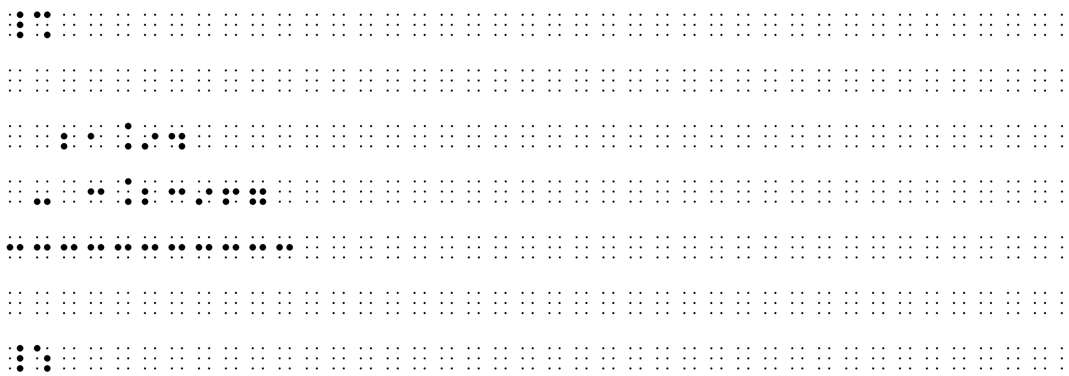
25.3.1 The numeric indicator is not used in work arranged in columns and aligned for addition or subtraction.

25.3.2 In a spatial arrangement for addition or subtraction, the numeric symbols, fractions, abbreviations, interior signs of operation or comparison are vertically aligned with digits under digits, commas under commas, decimal points under decimal points, fractions under fractions, abbreviations under abbreviations, signs of operation under signs of operation, and signs of comparison under signs of comparison. If these are deliberately misaligned in print as in an exercise requiring the student to make a suitable correction, this misalignment must be preserved in the transcription.

25.3.3 Plus, minus, or monetary symbols are placed one column of cells to the left of the left-most column of numeric symbols or indicators which appear in the part of the arrangement above the separation line.

Example 25-1: Placement of Operation Signs

$$\begin{array}{r} 21.94 \\ -\ \ 3.23967 \\ \hline \end{array}$$



25.3.4 The separation line which appears in addition or subtraction must be made one cell longer at either end than the over-all width of the arrangement.

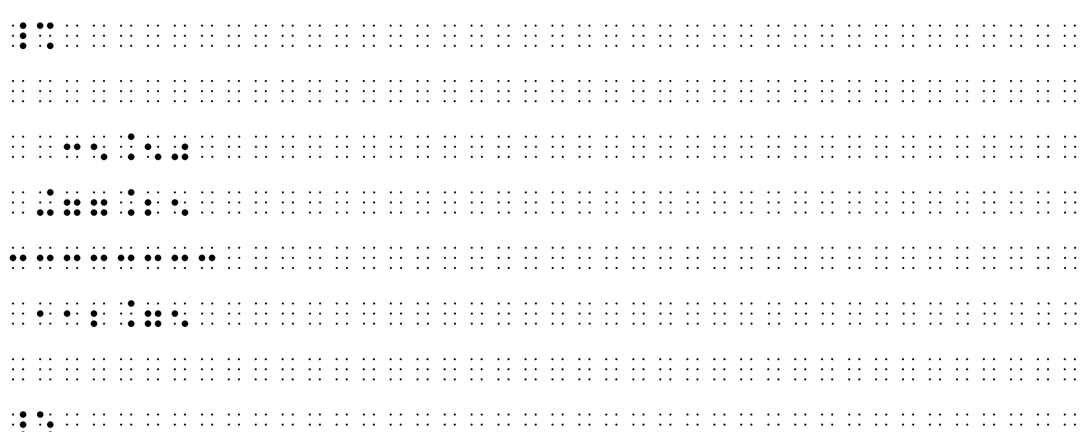
Example 25-2: Addition Problem with Separation Line

$$\begin{array}{r} 508 \\ 8876 \\ 259 \\ +427 \\ \hline 10070 \end{array}$$

(in print, the plus sign is next to the first digit of the last addend in the problem)

Example 25-3: Addition Problem with Separation Line

$$\begin{array}{lr} & 35.50 \\ + & 77.25 \\ \hline & 112.75 \end{array}$$



(in print, the plus sign is farther to the left than any term in the problem or the answer)

Example 25-4: Subtraction Problem with Separation Line

$$\begin{array}{r} 3.704 \\ -\ .915 \\ \hline 2.789 \end{array}$$

```
⠸⠩

⠀⠀⠒⠨⠶⠴⠲
⠀⠤⠀⠨⠔⠂⠢
⠒⠒⠒⠒⠒⠒⠒⠒
⠀⠀⠆⠨⠶⠦⠔

⠸⠱
```

(in print, part of the minus sign falls under the 3, the rest extends farther to the left)

Example 25-5: Addition of Fractions with Separation Line

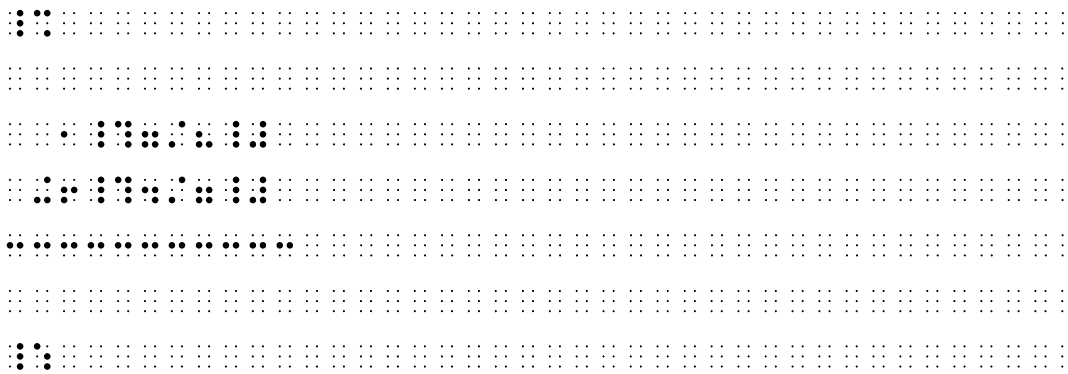
$$\begin{array}{r} \frac{5}{8} \\ +\ \frac{3}{4} \\ \hline \end{array}$$

```
⠸⠩

⠀⠀⠹⠢⠌⠦⠼
⠀⠬⠹⠒⠌⠲⠼
⠒⠒⠒⠒⠒⠒⠒⠒

⠸⠱
```

(in print, the plus sign is farther to the left than any term in the problem)

Example 25-6: Addition of Mixed Numbers with Separation Line

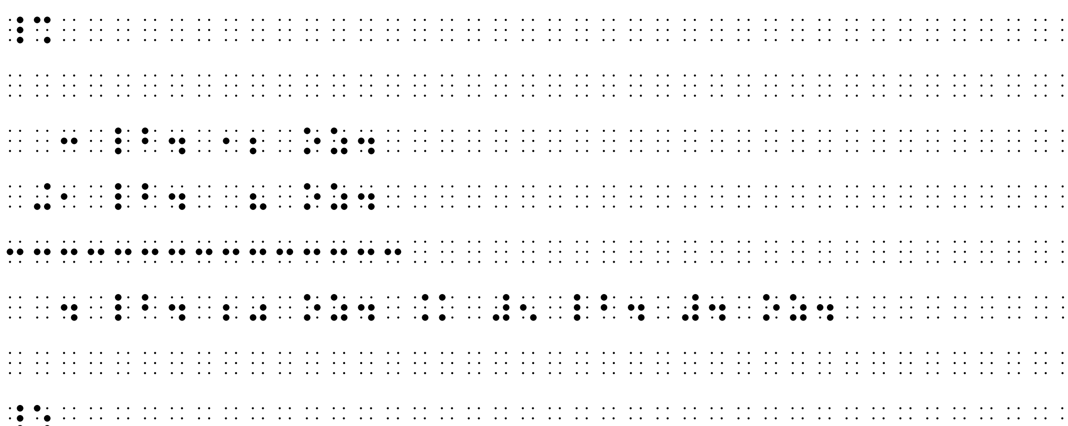
$$\begin{array}{r} 1\frac{7}{8} \\ +6\frac{4}{7} \\ \hline \end{array}$$



(in print, the plus sign is farther to the left than any term in the problem)

Example 25-7: Addition with Separation Line

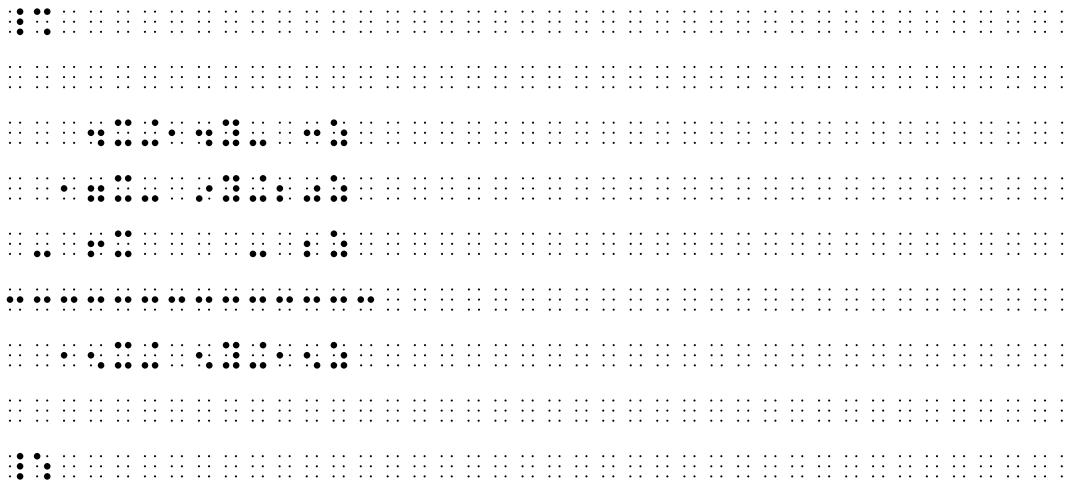
$$\begin{array}{rrl} & 3\text{ lb.} & 12\text{ oz.} \\ + & 1\text{ lb.} & 8\text{ oz.} \\ \hline & 4\text{ lb.} & 20\text{ oz.} = 5\text{ lb. } 4\text{ oz.} \end{array}$$



(in print, the plus sign is farther to the left than any term in the problem)

Example 25-8: Polynomial Subtraction with Separation Line

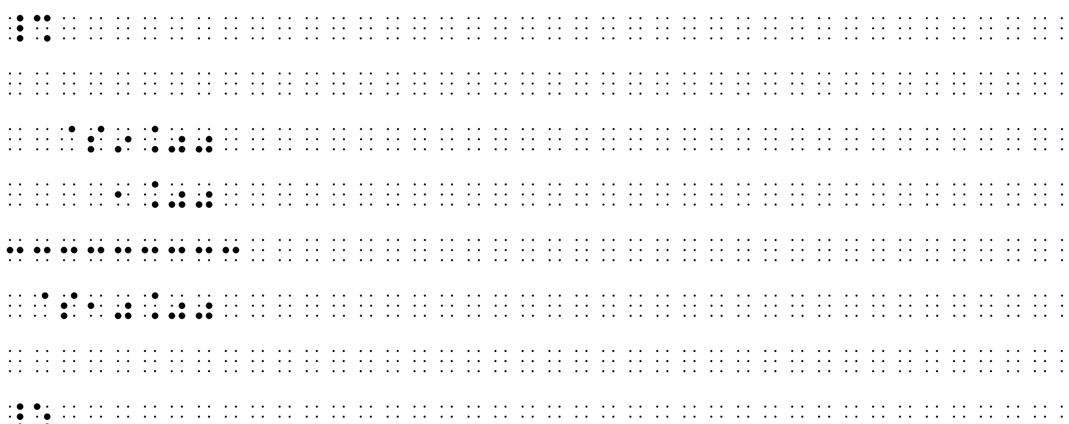
$$\begin{array}{rrrrrr} & 4x & + 14y & - & 3z \\ & 17x & - \ 9y & + & 20z \\ - & 6x & & - & 2z \\ \hline & 15x & + \ 5y & + & 15z \end{array}$$



(in print, the minus sign is farther to the left than any term in the problem)

Example 25-9: Addition with Dollar Sign and Separation Line

$$\begin{array}{r} \$9.00 \\ 1.00 \\ \hline \$10.00 \end{array}$$



(if there is no plus sign in print, a plus sign is not inserted in braille)

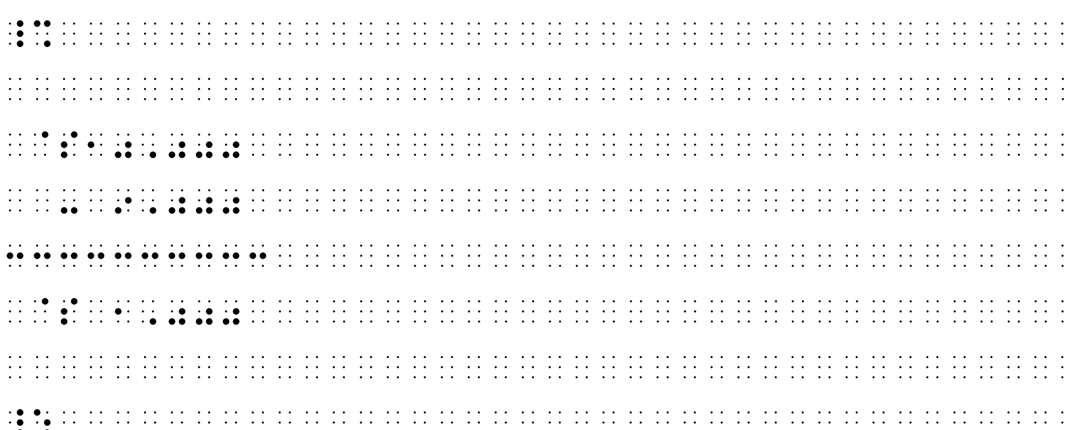
Example 25-10: Addition with Dollar Sign and Separation Line

$$\begin{array}{r} \$7.45 \\ 10.92 \\ +84.00 \\ \hline \$102.37 \end{array}$$

(in print, there is no space between the dollar signs and the following digits, and the plus sign is to the left of the first dollar sign)

Example 25-11: Subtraction with Dollar Sign

$$\begin{array}{lr} \$ & 10{,}000 \\ - & 9{,}000 \\ \hline \$ & 1{,}000 \end{array}$$



(in print, the dollar signs and the minus sign occupy the same position as in braille)

25.3.5 **Regrouping in Addition.** When regrouping numbers appear in an addition arrangement, the appropriate regrouping indicator is inserted between the regrouping numbers and the arrangement to which they apply. The regrouping indicator begins one cell to the left of the separation line and extends to the same point that the separation line ends.

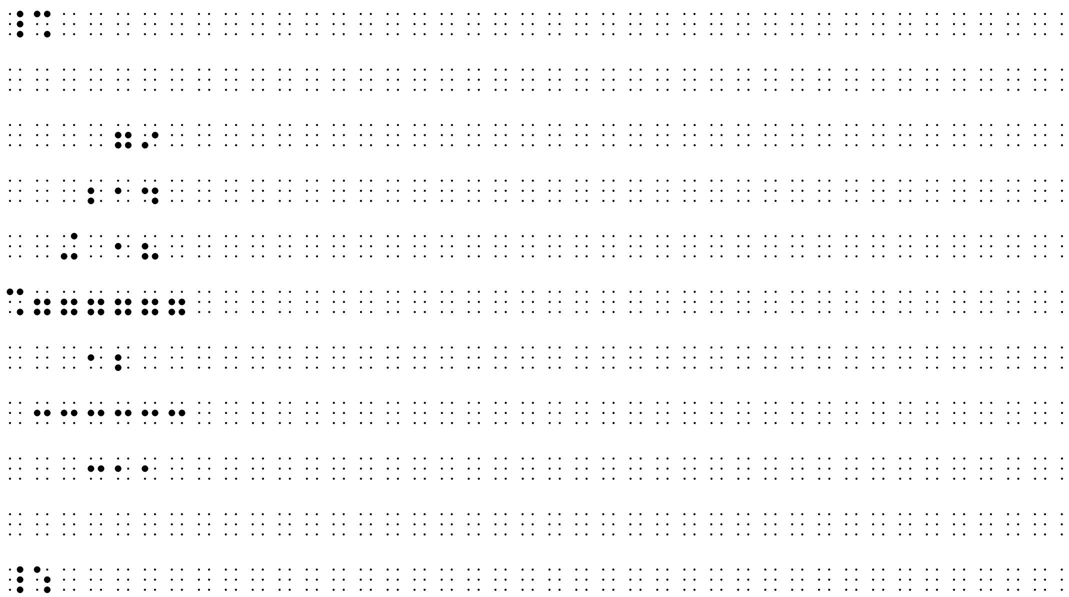
Example 25-12: Regrouping Numbers Above the Arrangement

$$\begin{array}{r} {}^{1}\,{}^{1} \\ 254 \\ +176 \\ \hline 430 \end{array}$$

(in print, the regrouping numbers are in small type directly above the columns to which they apply)

Example 25-13: Regrouping Numbers Above Separation Line

$$\begin{array}{rccc}
 & & 7 & 9 \\
 & 2 & 1 & 4 \\
+ & & 1 & 8 \\
 & {\scriptstyle 1} & {\scriptstyle 2} & \\
\hline
 & 3 & 1 & 1
\end{array}$$



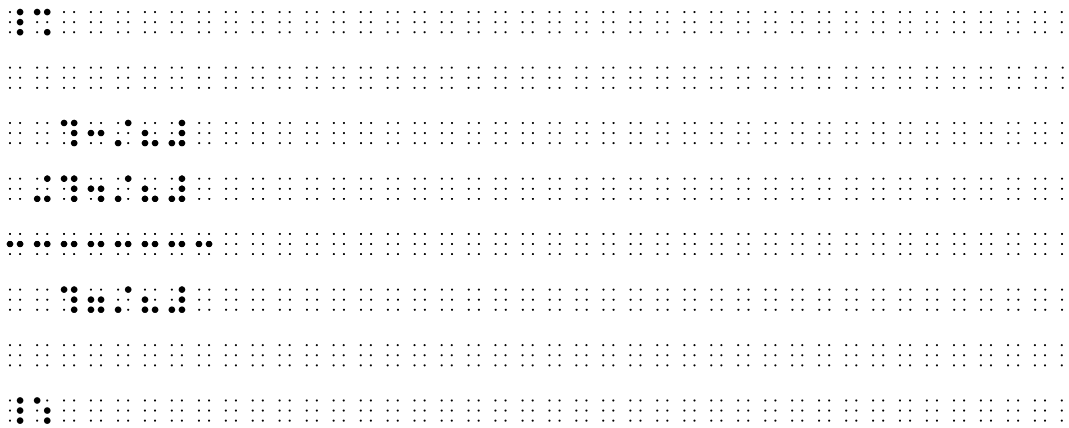
(in print, the regrouping numbers are in small type directly below the columns to which they apply)

25.3.6 **Regrouping Numbers in Subtraction.** When cancellation is shown, it is not necessary to insert a regrouping indicator. See **Rule 12** for use of cancellation indicators in subtraction.

25.3.7 **Fractions.** In an arrangement containing fractions, fraction lines are vertically aligned, and each numerator and denominator must be in contact with its fraction line. All opening fraction indicators, mixed number indicators, closing fraction indicators, and fraction lines must be aligned vertically.

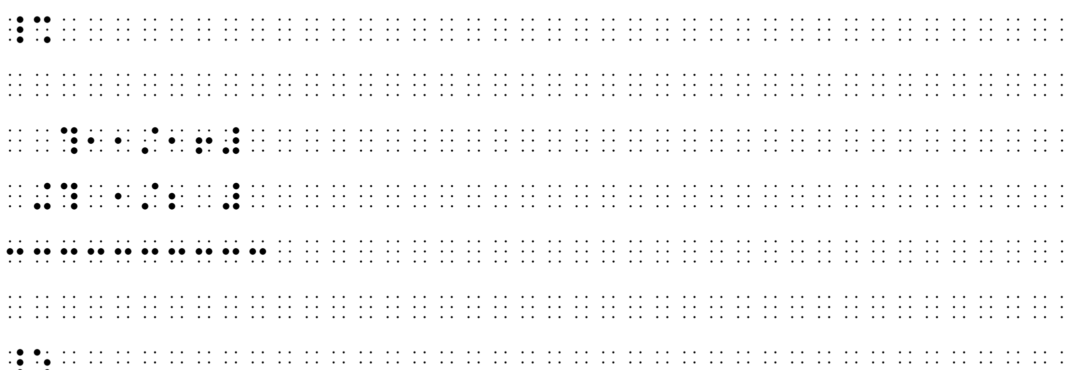
Example 25-14: Addition of Simple Fractions

$$\begin{array}{r} \frac{3}{8} \\ +\ \frac{4}{8} \\ \hline \frac{7}{8} \end{array}$$



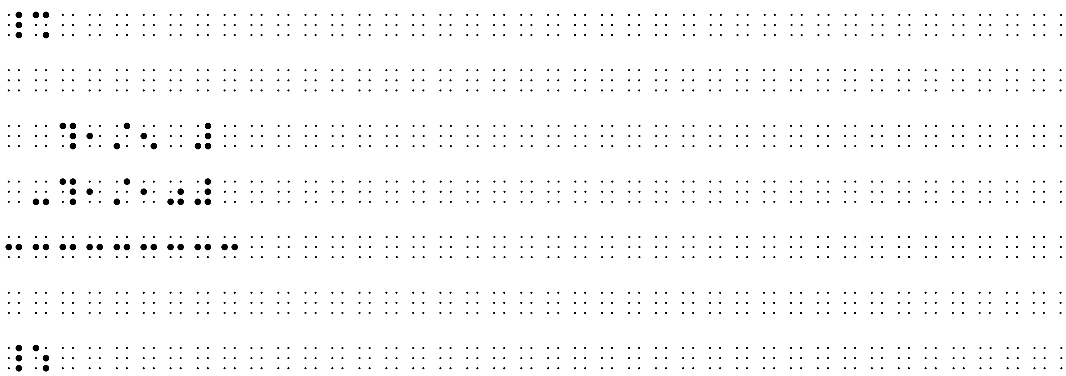
Example 25-15: Addition of Simple Fractions

$$\begin{array}{r} \frac{11}{16} \\ +\ \frac{1}{2} \\ \hline \end{array}$$



Example 25-16: Subtraction of Simple Fractions

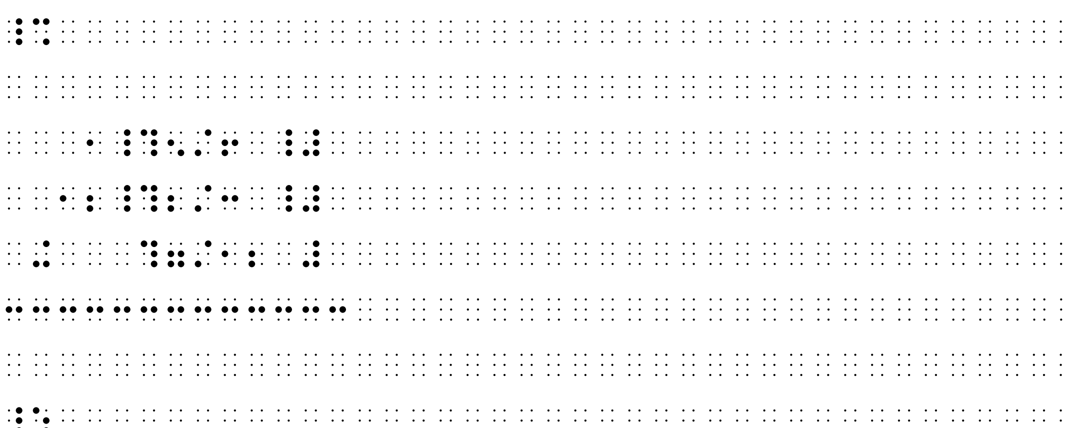
$$\begin{array}{r} \frac{1}{5} \\ -\ \frac{1}{10} \\ \hline \end{array}$$



25.3.8 In an arrangement containing mixed numbers and fractions, the whole numbers, opening mixed number indicators, fraction lines, and closing mixed number indicators must be aligned vertically.

Example 25-17: Addition with Mixed Numbers

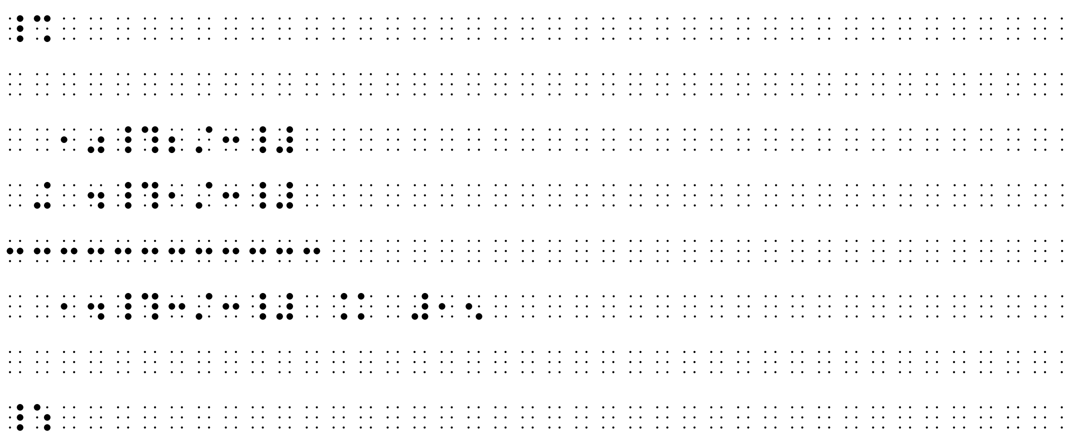
$$\begin{array}{rr} & 1\frac{5}{6} \\ & 12\frac{2}{3} \\ + & \frac{7}{12} \\ \hline \end{array}$$



25.3.9 A number that is not part of the spatial arrangement may require a numeric indicator, according to the rules regarding the use of numeric indicators.

Example 25-18: Number Not Part of the Spatial Arrangement

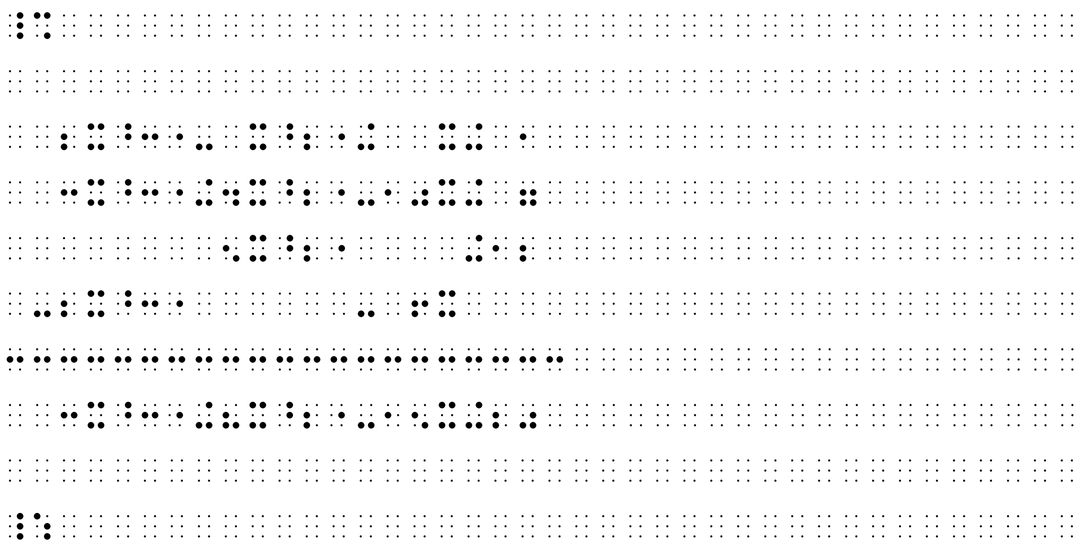
$$\begin{array}{r} 10\frac{2}{3} \\ +\ \ 4\frac{1}{3} \\ \hline 14\frac{3}{3} \end{array} = 15$$



25.3.10 In an arrangement containing polynomials, terms are vertically aligned. Within each term, symbols of operation, coefficients, letters, superscript indicators, superscripts, and baseline indicators are vertically aligned. Within each coefficient and superscript, corresponding symbols are vertically aligned. When the baseline indicator is required, it is placed in the first possible position consistent with this required alignment.

Example 25-19: Addition with Polynomials

$$
\begin{array}{r}
2x^3 - x^2 + x + 1 \\
3x^3 + 4x^2 - 10x + 7 \\
5x^2 + 12 \\
-\ 2x^3 - 6x \\
\hline
3x^3 + 8x^2 - 15x + 20
\end{array}
$$



25.4 Multiplication

25.4.1 The numeric indicator is not used in spatially arranged multiplication problems.

25.4.2 In a spatial arrangement for multiplication, the symbols comprising the multiplier and multiplicand are aligned in the transcription as the corresponding signs are aligned in print.

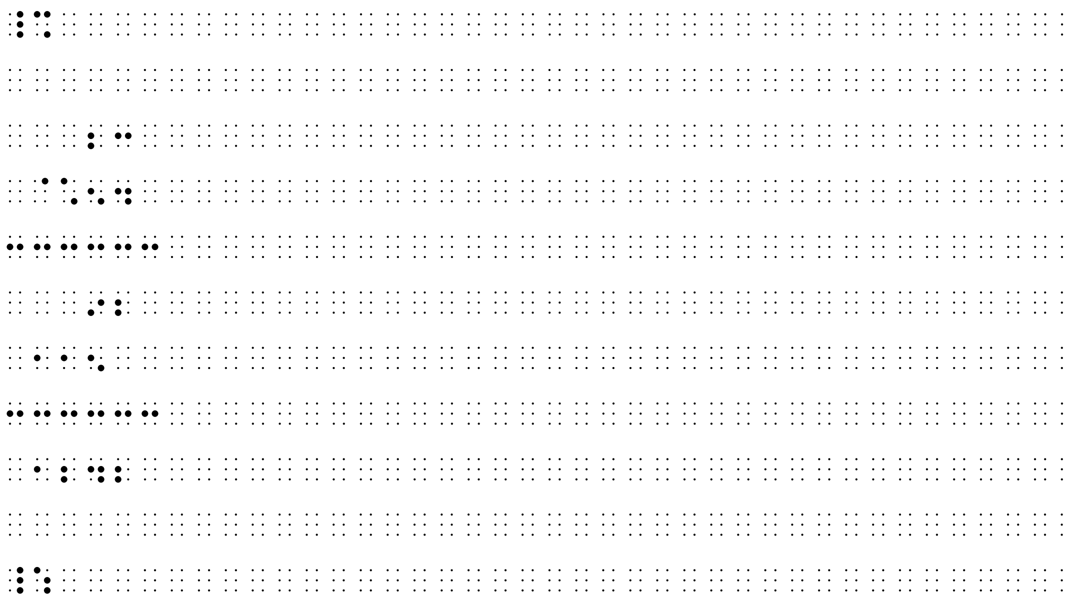
25.4.3 The multiplication symbol, if the corresponding sign is present in print, is placed immediately to the left of the multiplier.

25.4.4 The partial products (following the first separation line) are aligned as designated for spatial addition problems.

25.4.5 The separation lines which appear in a multiplication arrangement are one cell longer at either end than the overall width of the arrangement. Separation lines in an arrangement are all the same length.

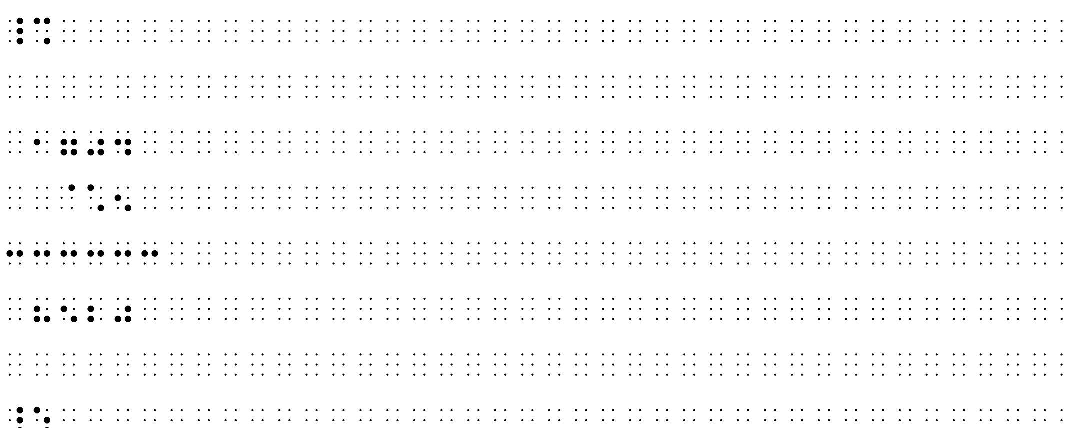
Example 25-20: Multiplication and Partial Products

$$\begin{array}{r} 23 \\ \times 54 \\ \hline 92 \\ 115 \\ \hline 1242 \end{array}$$



Example 25-21: Multiplication

$$\begin{array}{r} 1704 \\ \times \quad 5 \\ \hline 8520 \end{array}$$

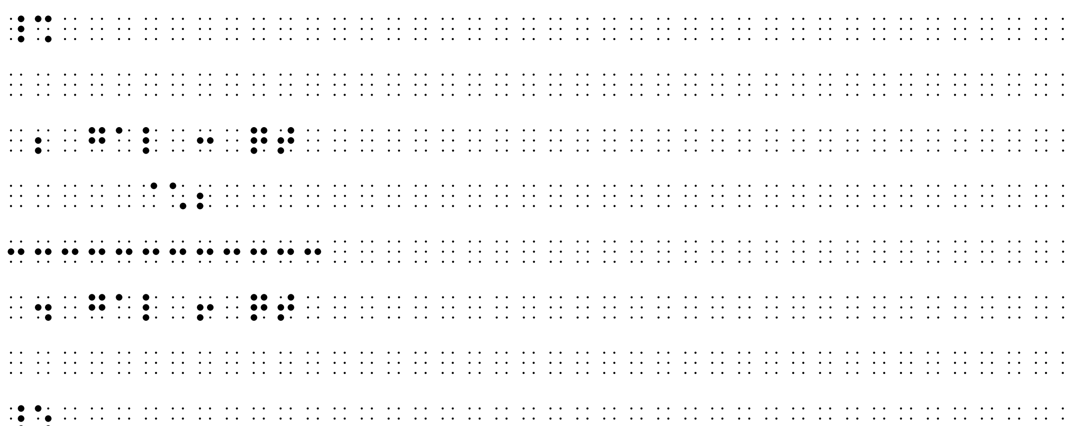


Example 25-22: Multiplication and Print Alignment

$$\begin{array}{r} 132 \\ \times\,300 \\ \hline 39600 \end{array}$$

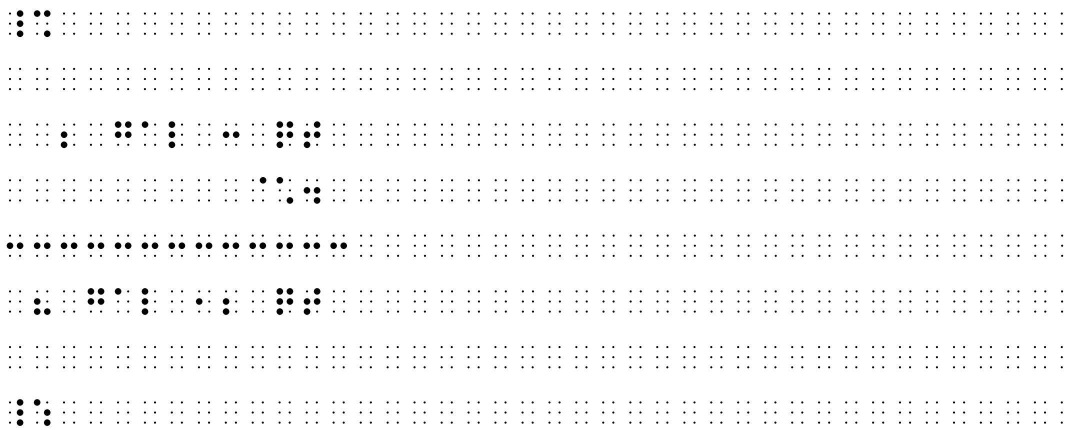
Example 25-23: Multiplication with Alignment

$$\begin{array}{r} 2\text{ gal }3\text{ qt} \\ \times\,2 \\ \hline 4\text{ gal }6\text{ qt} \end{array}$$



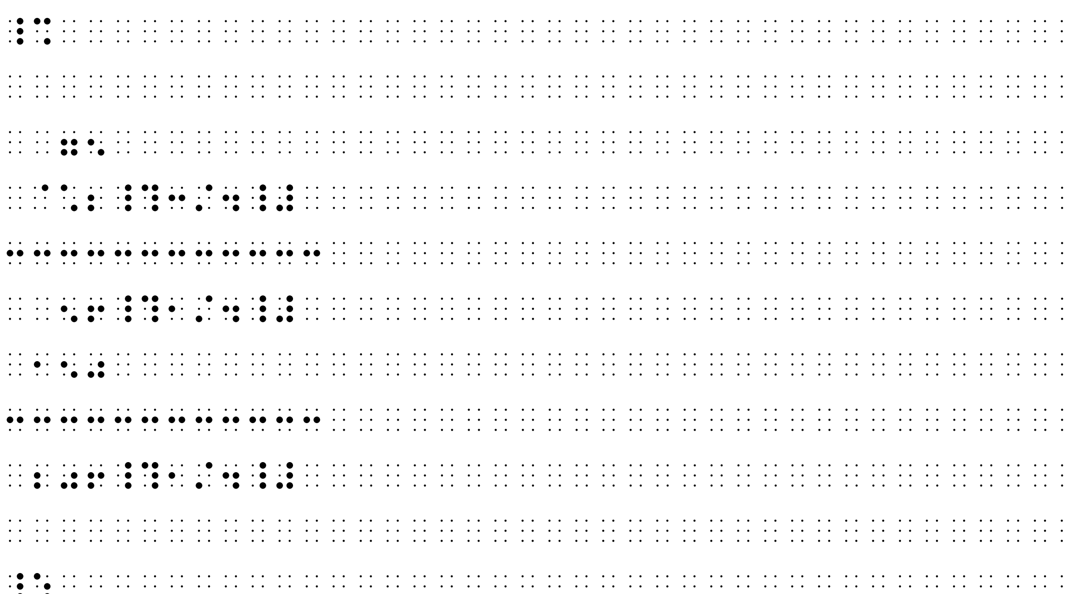
Example 25-24: Multiplication with Units of Measure

$$\begin{array}{r} 2\text{ gal }3\text{ qt} \\ \times 4 \\ \hline 8\text{ gal }12\text{ qt} \end{array}$$



Example 25-25: Multiplication and Mixed Numbers

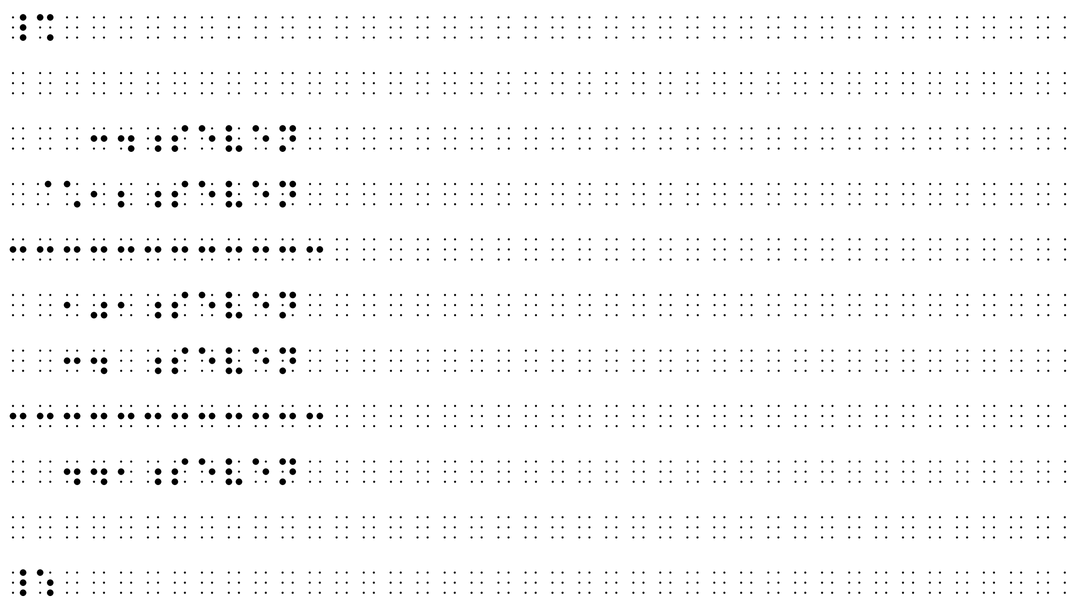
$$\begin{array}{r} 75 \\ \times 2\frac{3}{4} \\ \hline 56\frac{1}{4} \\ 150\phantom{\frac{1}{4}} \\ \hline 206\frac{1}{4} \end{array}$$



25.4.6 In arrangements which show multiplication to non-decimal bases in which subscripts appear, the subscript indicator is placed in the first possible position consistent with the alignment required for the addition of partial products.

Example 25-26: Multiplication and Subscripted Words

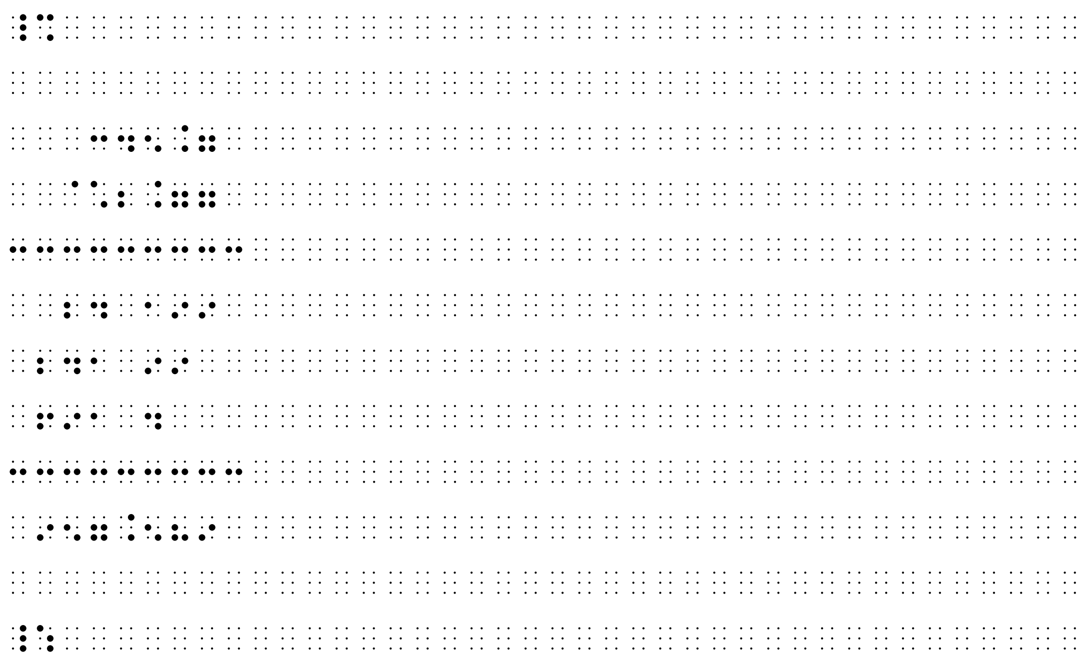
$$
\begin{array}{r}
34_{\text{seven}} \\
\times\ 12_{\text{seven}} \\
\hline
101_{\text{seven}} \\
34_{\text{seven}} \\
\hline
441_{\text{seven}}
\end{array}
$$



25.4.7 When commas or the decimal point occur in the answer of a multiplication arrangement, a blank column of cells is left above these in the partial products.

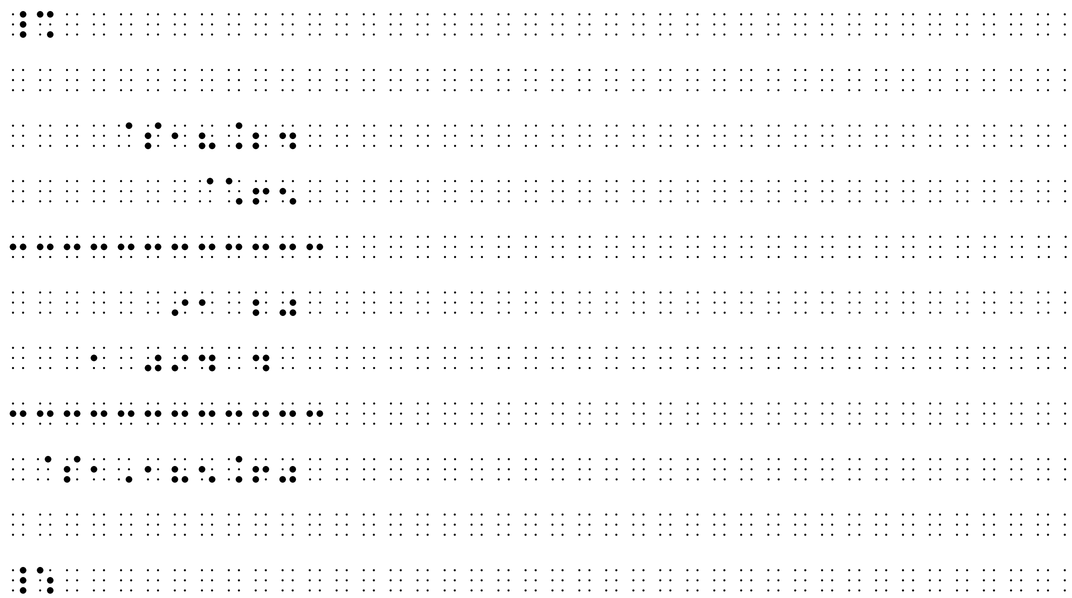
Example 25-27: Multiplication and Spacing for Decimal Points

$$
\begin{array}{r}
345.7 \\
\times 2.77 \\
\hline
24199 \\
24199 \\
6914 \\
\hline
957.589
\end{array}
$$



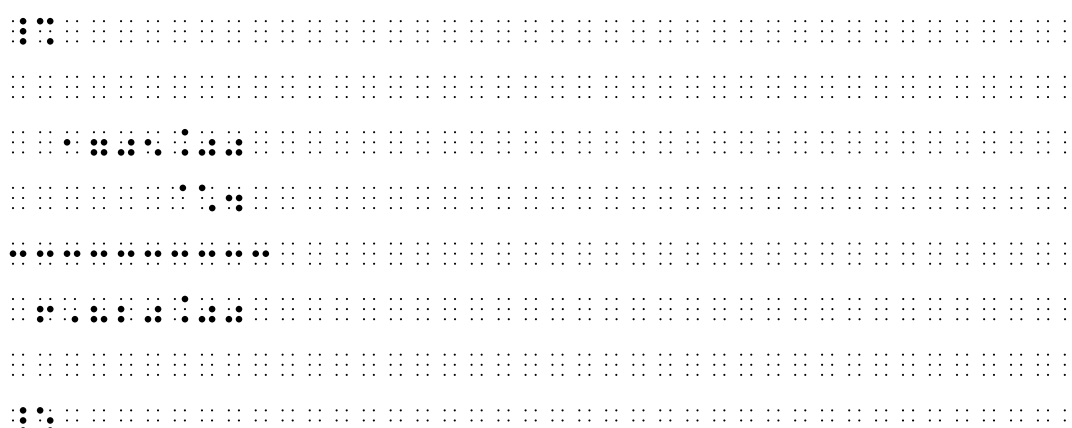
Example 25-28: Blank Space for Decimal Point

```
   $18.24
  ×    65
  -------
    9120
  10944
---------
$1,185.60
```



Example 25-29: Multiplication with Decimal Number

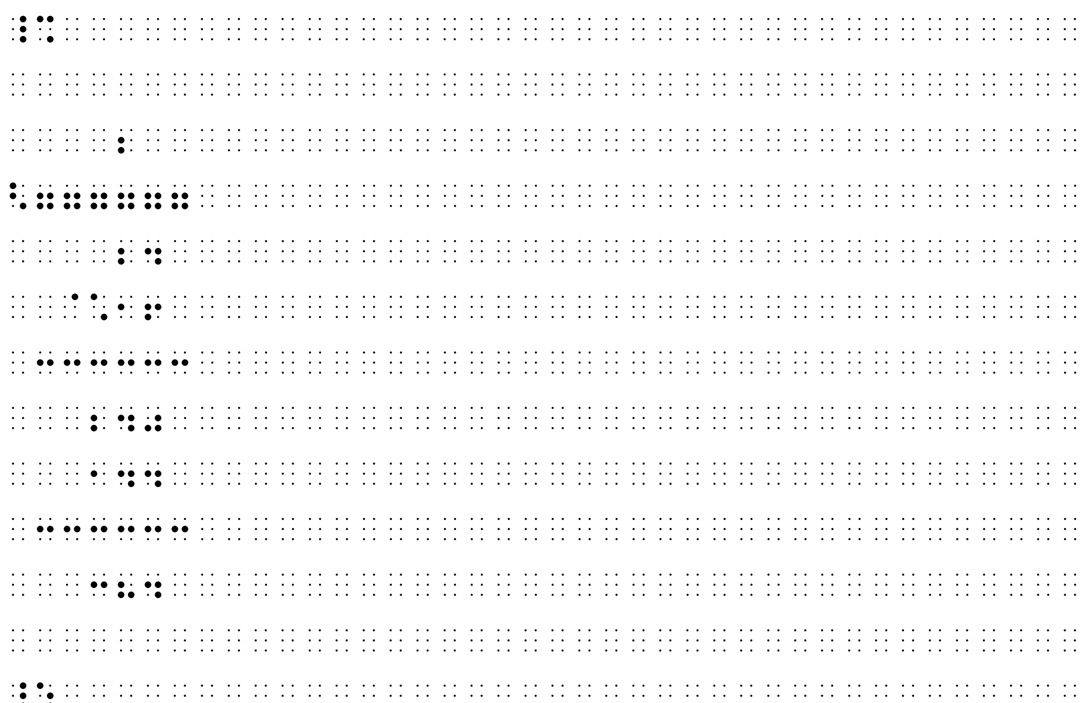
```
 1705.00
 ×     4
--------
6,820.00
```



25.4.8 **Regrouping in Multiplication.** When regrouping numbers appear in a multiplication arrangement, the appropriate regrouping indicator is inserted between the regrouping numbers and the arrangement to which they apply. The regrouping indicator begins one cell to the left of the separation line and extends to the same point that the separation line ends.

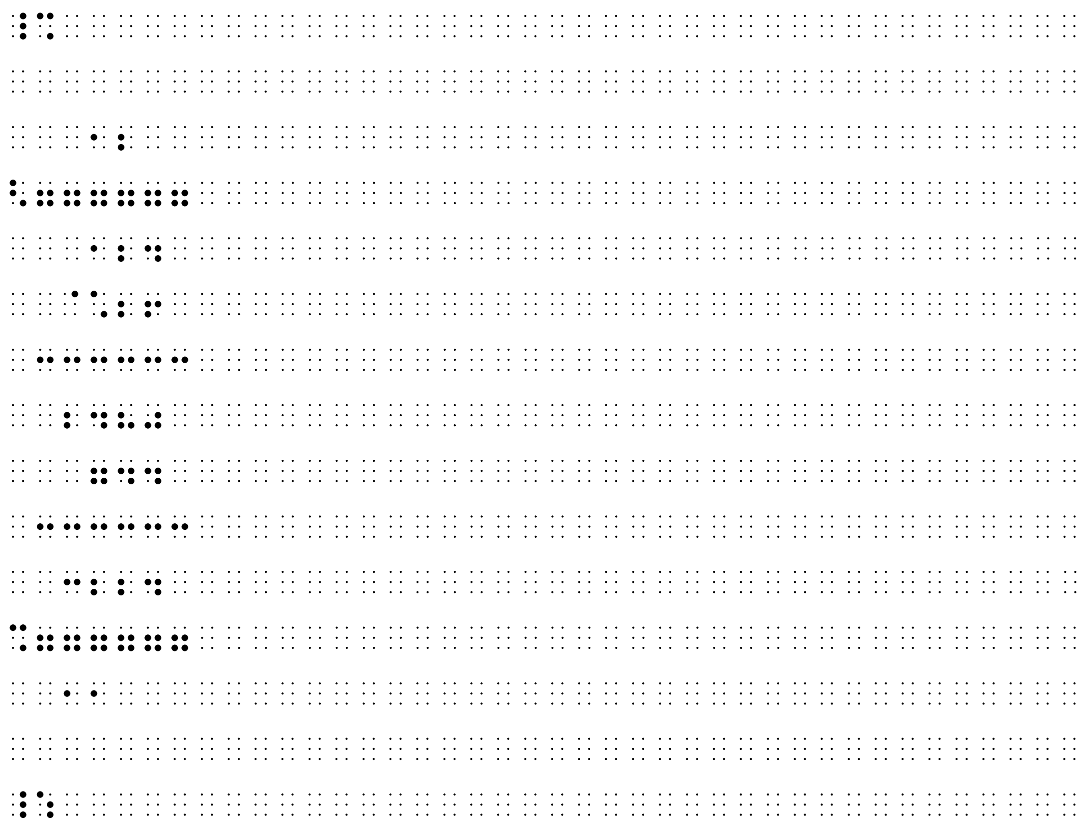
Example 25-30: Regrouping Number Above Arrangement

$$
\begin{array}{rrr}
 & \overset{2}{2} & 4 \\
\times & 1 & 6 \\
\hline
2 & 4 & 0 \\
1 & 4 & 4 \\
\hline
3 & 8 & 4
\end{array}
$$



Example 25-31: Regrouping Numbers Above and Below

$$\begin{array}{rrrr} & \overset{1}{1} & \overset{2}{2} & 4 \\ \times & & 2 & 6 \\ \hline 2 & 4 & 8 & 0 \\ & 7 & 4 & 4 \\ \hline \underset{1}{3} & \underset{1}{2} & 2 & 4 \end{array}$$



25.5 Division

25.5.1 A division problem is transcribed as a spatial arrangement if it includes a quotient, partial products and differences, or if there are spaces within the dividend.

Note: When the division arrangement contains only a divisor and a dividend but no quotient, no partial products and differences, and no spaces, the separation line, whether shown above or below the dividend, is omitted in braille. In this case, the division arrangement is not regarded as spatial. In particular, the numeric indicator is used in the appropriate place, and a blank line is not left above or below such a division problem.

Example 25-32: Non-Spatial Division Problem

$6\overline{)48}$

Example 25-33: Non-Spatial Division Problem

$7\overline{)104.58}$

Example 25-34: Non-Spatial Division Problem

$5\overline{)\$125}$

Example 25-35: Non-Spatial Division Problem

$x+7\overline{)x^2+10x+21}$

Example 25-36 Spatial Division Problem

$6\overline{)1\text{ ft. }6\text{ in.}}$

(spatial because there are spaces in the dividend)

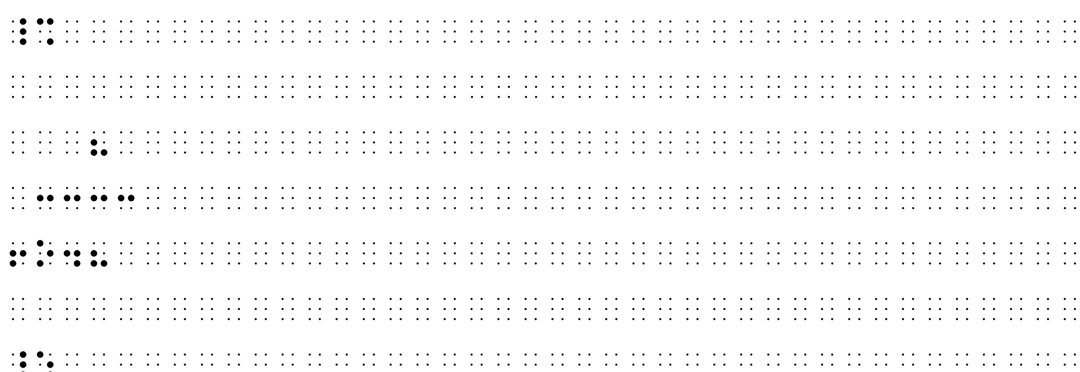
25.5.2 In a spatial arrangement for division the symbols comprising the dividend, the partial products, and the differences are aligned as in print. Symbols in the quotient are aligned with their corresponding symbols in the dividend unless they are intentionally misaligned as an exercise for the student.

25.5.3 The division symbol is placed in the same location as it appears in print, in the cell directly before or directly following the dividend. There is no space between the division symbol and the divisor or between the division symbol and the dividend. A quotient may be printed above or below the separation line or next to a division symbol. Follow print for placement of a quotient. If a quotient is printed next to a division symbol, no space is left between them. (See example **25-38**.) If the divisor is underlined, this line is ignored in the transcription. (See example **25-42**.)

25.5.4 Each separation line which appears in a division arrangement begins in the column containing a division symbol and ends in the column containing the other division symbol, if the latter appears in print. Otherwise, each separation line ends in a cell one column beyond the overall arrangement. For exceptions related to division with vertical lines see sections **25.5.8**, **25.6**.

Example 25-37: Spatial Division Problem

$$\begin{array}{r} 8 \\ 6\overline{)48} \end{array}$$



Example 25-38: Spatial Division Problem

$$6\overline{)48}(8$$

```
⠸⠩

⠀⠒⠒⠒⠒
⠖⠕⠲⠦⠪⠦

⠸⠱
```

Example 25-39: Spatial Division Problem with Quotient Below

$$6\underline{)636}$$
$$106$$

```
⠸⠩

⠖⠕⠖⠒⠖
⠀⠒⠒⠒⠒⠒
⠀⠀⠂⠴⠖

⠸⠱
```

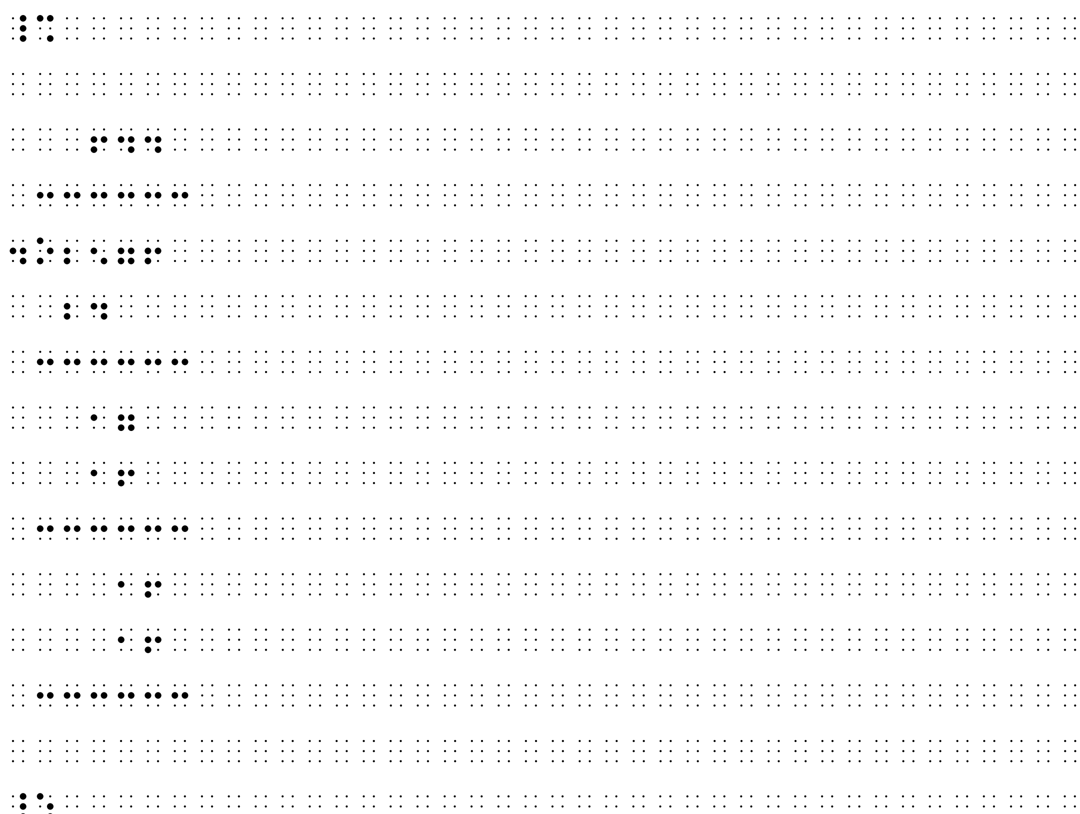
Example 25-40: Spatial with Straight Division Indicator

$$\begin{array}{r|l} & \overline{106} \\ 6 & 636 \end{array}$$

```
⠸⠩

⠀⠀⠂⠴⠖
⠀⠒⠒⠒⠒⠒
⠖⠸⠖⠒⠖

⠸⠱
```

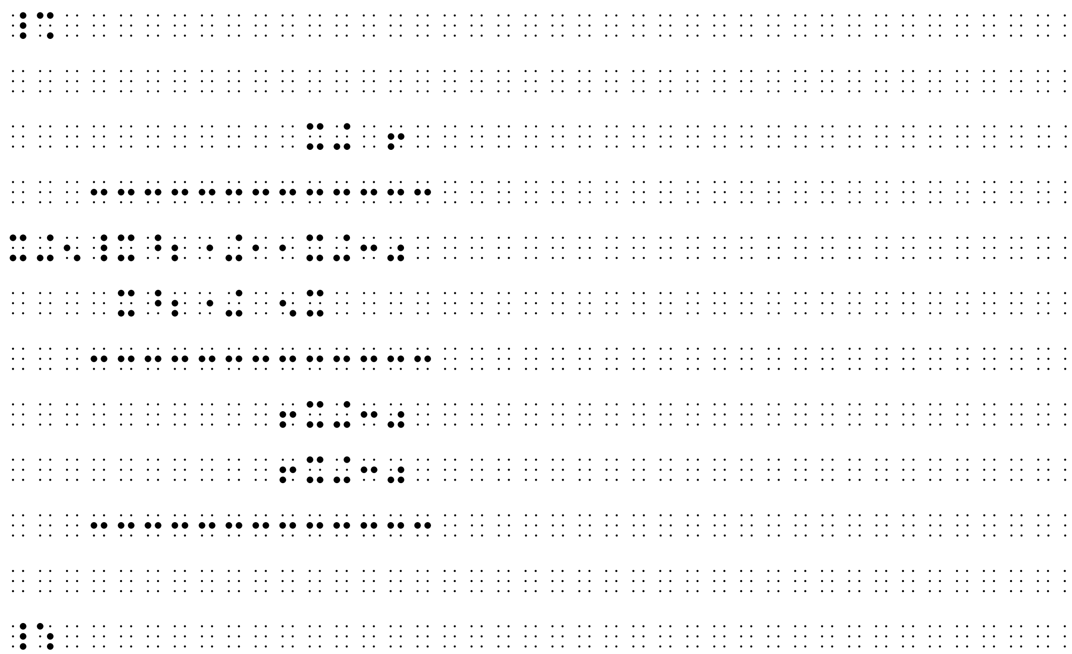
Example 25-41: Spatial Long Division Problem

```
   644
4)2576
  24
   17
   16
    16
    16
```



Example 25-42: Polynomial Long Division

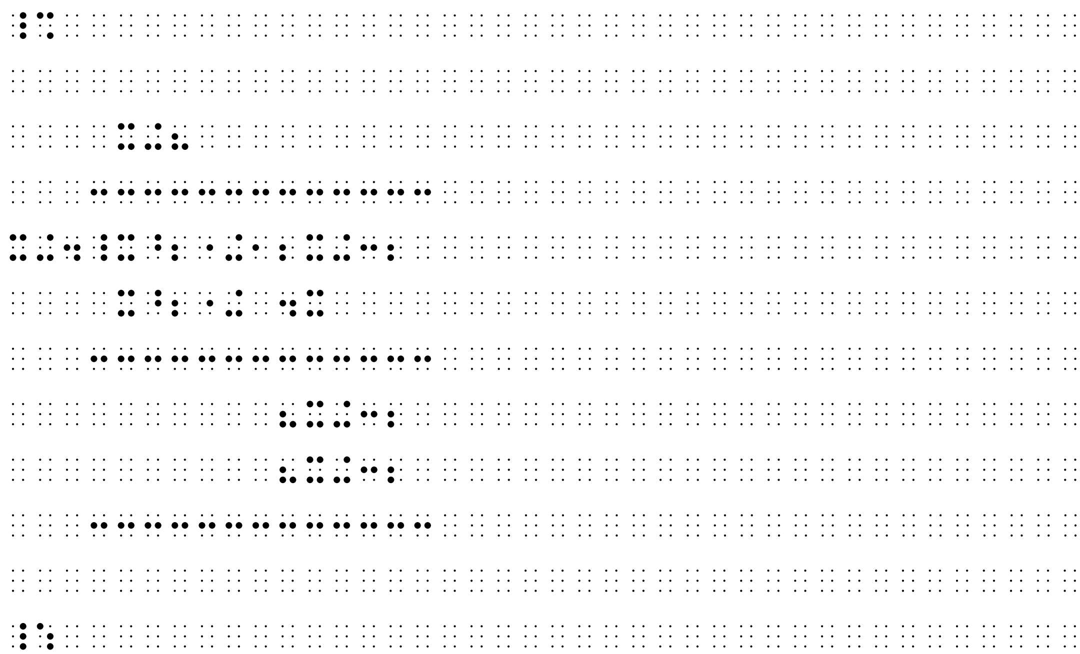
$$
\begin{array}{r}
x + \quad 6 \\
\underline{x+5}\,\big|\overline{x^2 + 11x + 30} \\
\underline{x^2 + \ \ 5x} \qquad\quad \\
6x + 30 \\
\underline{6x + 30}
\end{array}
$$



(in print, it is clear that the quotient is aligned with the dividend, and there is a horizontal line under the divisor)

Example 25-43: Polynomial Long Division

$$\begin{array}{r} x+8 \\ x+4\overline{\big)x^2+12x+32} \\ \underline{x^2+\ \ 4x} \\ 8x+32 \\ \underline{8x+32} \end{array}$$

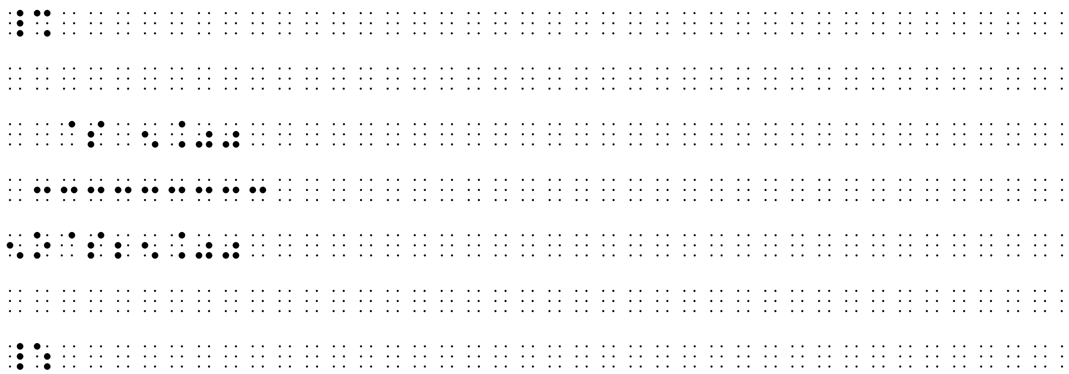


(in print, the quotient is clearly not aligned with the terms in the dividend)

25.5.5 When commas or the decimal point occur in the dividend of a division arrangement, a blank column of cells is left where these occur in the entire arrangement except within separation lines. When a caret occurs in a dividend, a blank column of two cells is left where this occurs in the entire arrangement except within the separation lines and the quotient. In the quotient, the decimal point corresponding to the caret is right-justified in the two cells allotted to the caret.

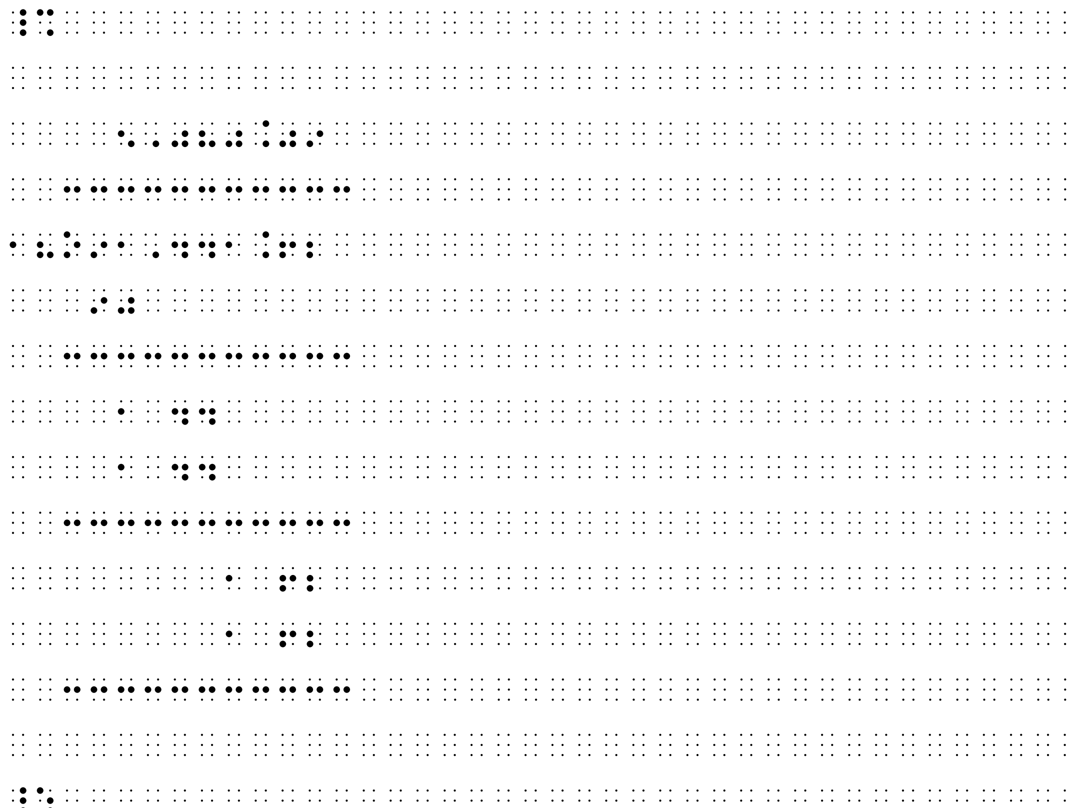
Example 25-44: Spatial Division Aligned as in Print

$$\begin{array}{r} \$\ \ 5.00 \\ 5\overline{)\$25.00} \end{array}$$



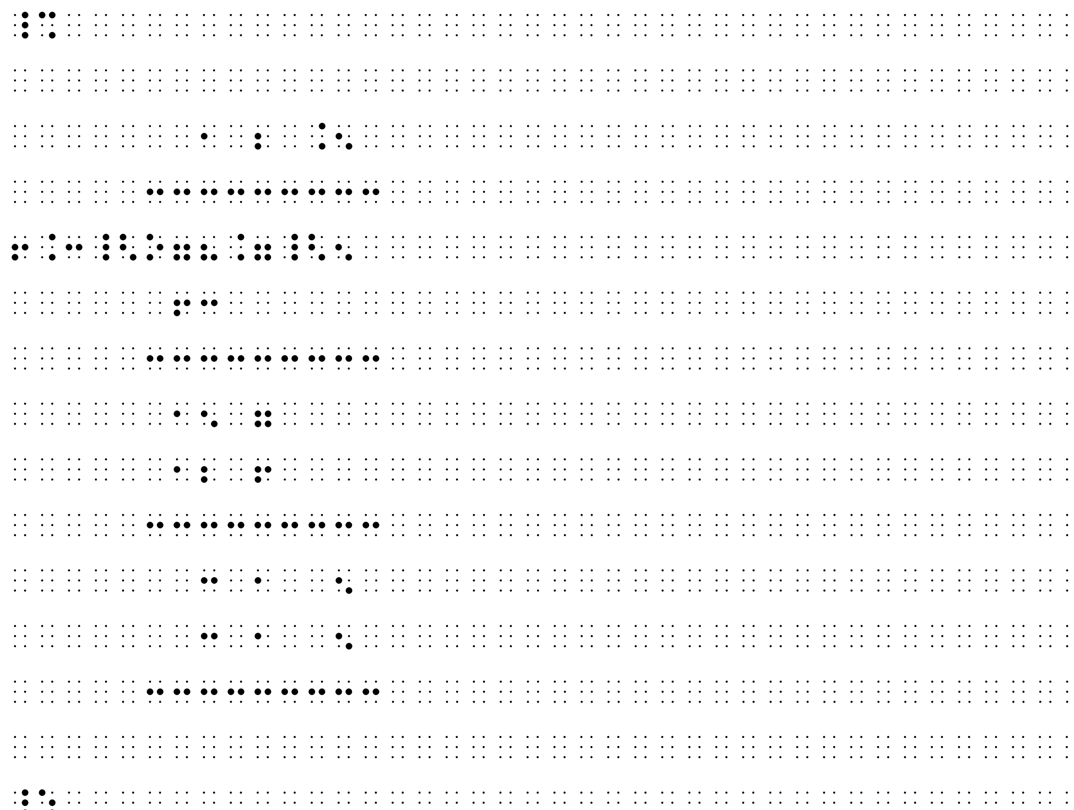
Example 25-45: Spatial Division with Space for a Decimal Point

```
      5,080.09
18)91,441.62
   90
     1 44
     1 44
          1 62
          1 62
```



Example 25-46: Spatial Division with Carets

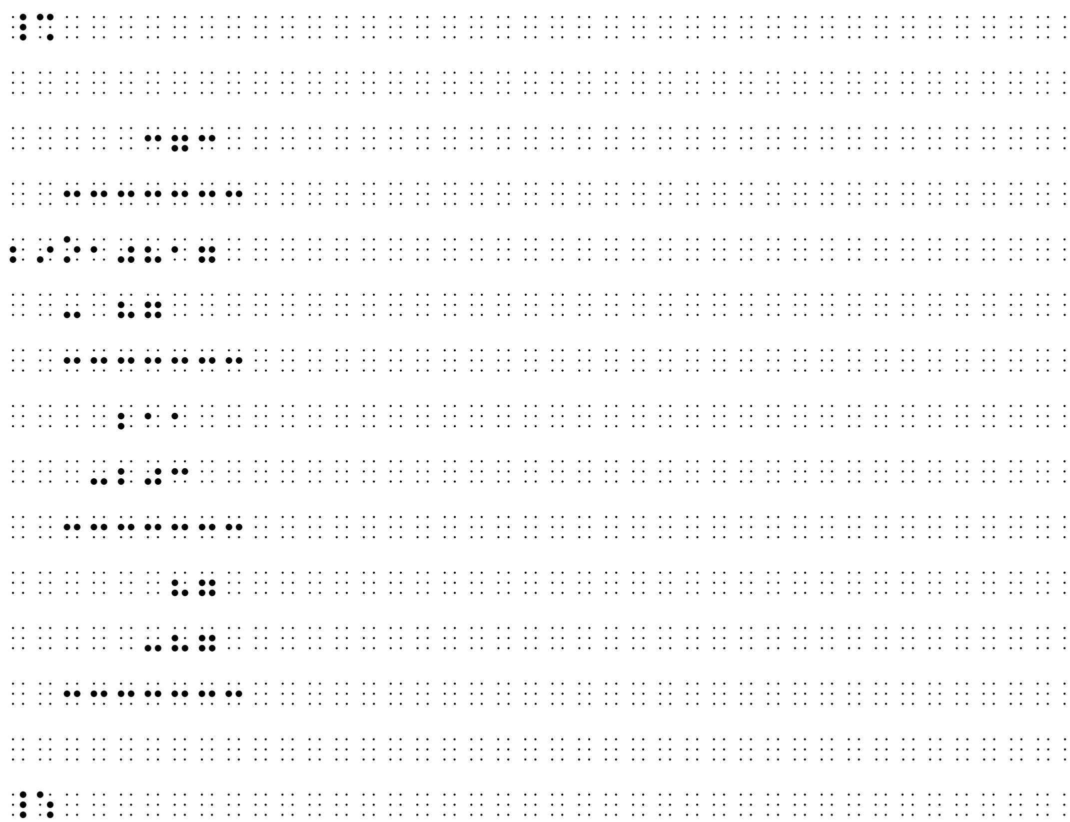
```
         1 2.5
6.3^)78.7^5
     63
     15 7
     12 6
       3 1 5
       3 1 5
```



25.5.6 A minus sign appearing in the long division portion of the problem is placed as follows: On the first line below the dividend, the minus symbol is placed under the division symbol. In further partial subtractions in the same problem, the minus symbol aligns according to the rules for spatial subtraction. See **25.3**.

Example 25-47: Spatial Division Problem with Minus Sign

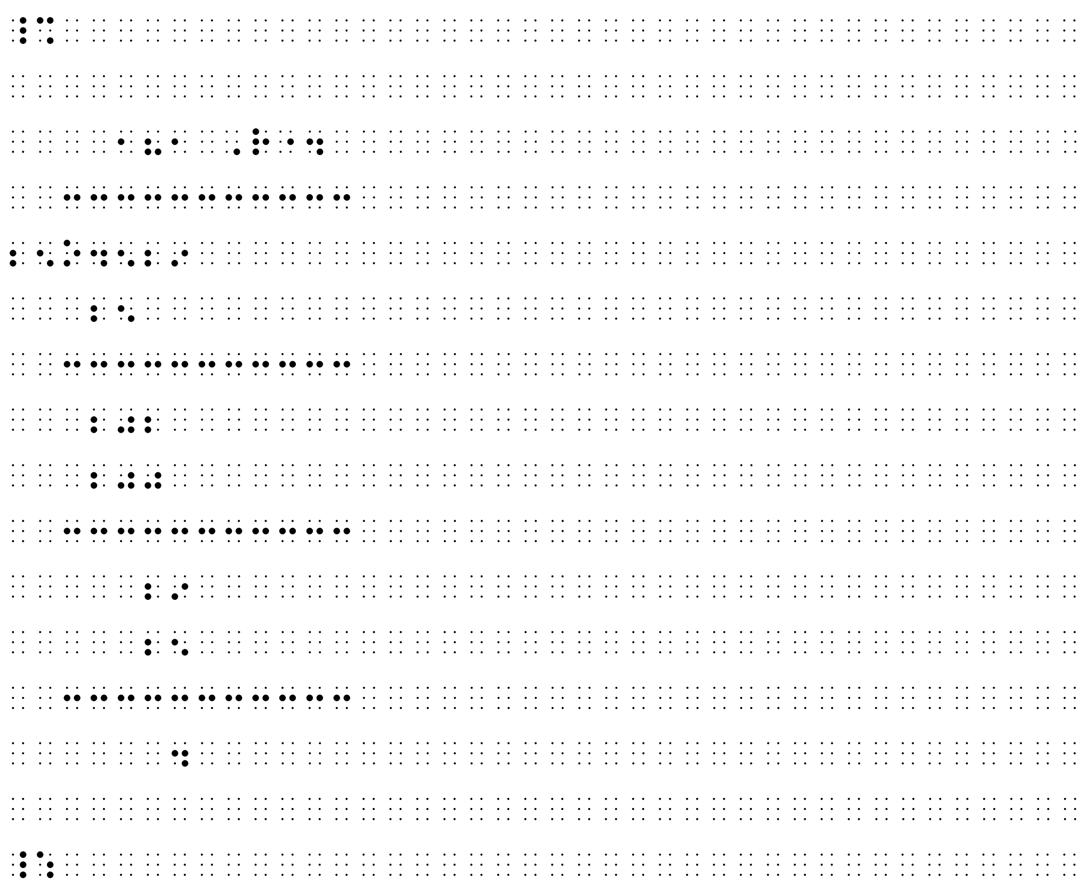
```
      373
29)10817
   -87
    211
   -203
      87
    - 87
```



25.5.7 When there is a remainder identified by the letter "r" or "R", the letter is preceded by a space in braille. See **24.1b** for use of the multipurpose indicator with a number following a letter.

Example 25-48: Spatial Division with Remainder

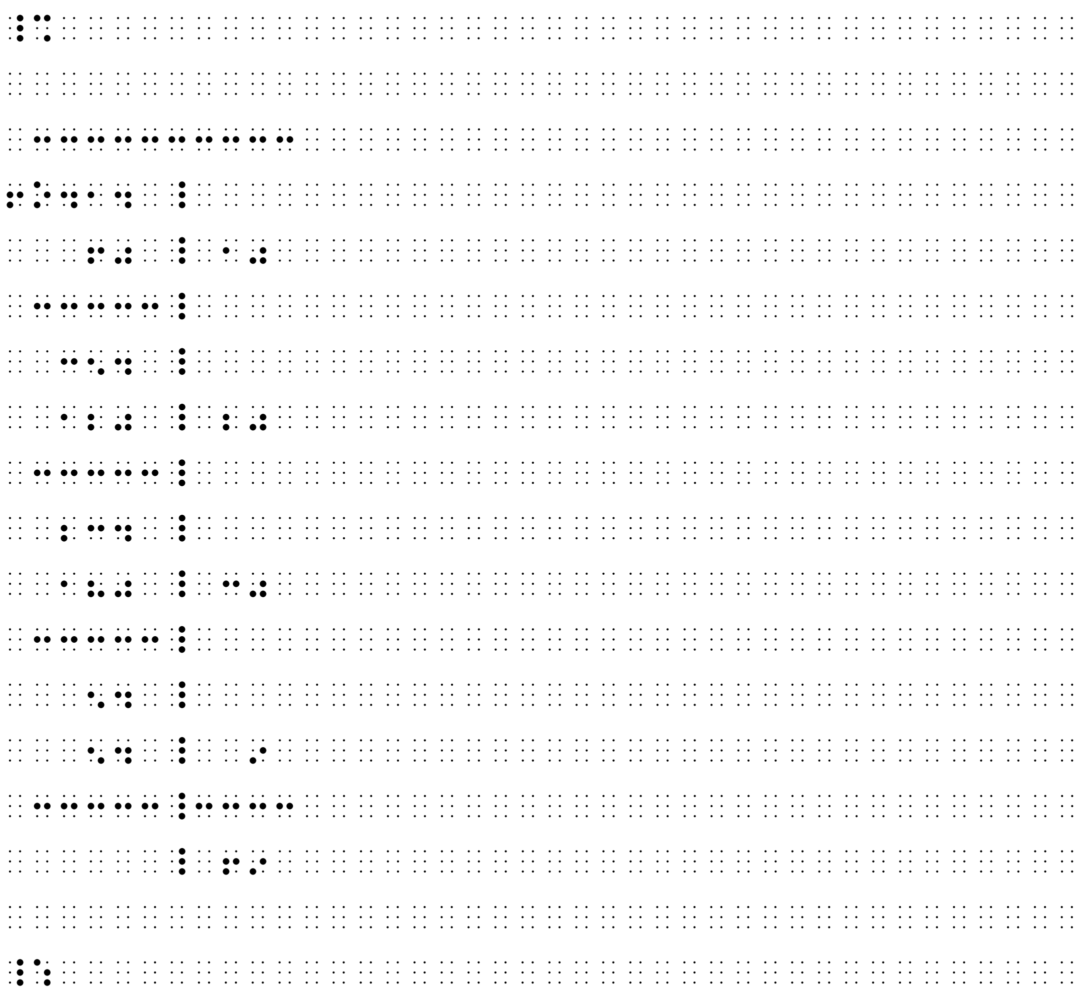
```
       181 R4
    ________
25) 4529
    25
    ___
    202
    200
    ___
     29
     25
     __
      4
```



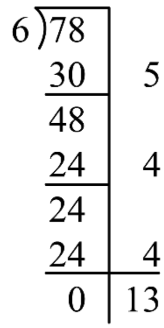
25.5.8 If a vertical line is part of a division arrangement, it is represented by a column of dots 456 (⠸). A space is left between the vertical line and any digit which precedes or follows it. No space is inserted between a separation line and the vertical line.

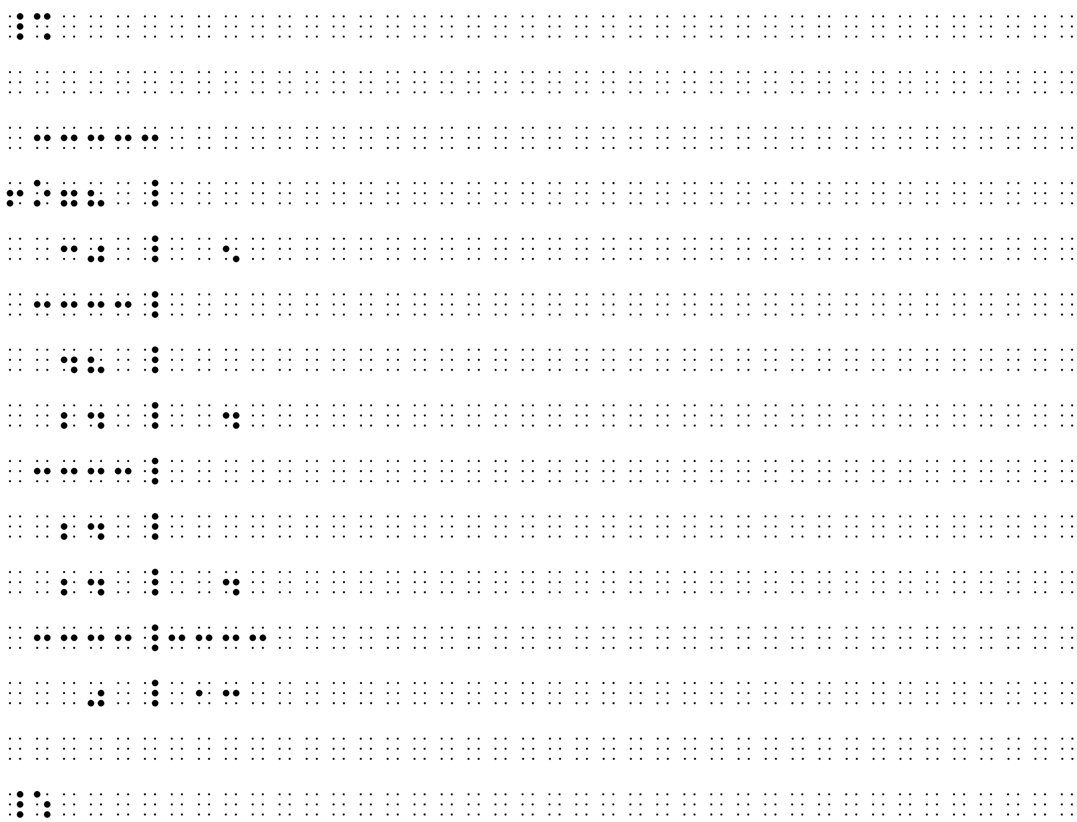
Example 25-49 Spatial Division with Vertical Lines

$$
\begin{array}{r|r}
6\,\overline{\smash{)}\,414} & \\
60 & 10 \\
\hline
354 & \\
120 & 20 \\
\hline
234 & \\
180 & 30 \\
\hline
54 & \\
54 & 9 \\
\hline
 & 69
\end{array}
$$



Example 25-50: Spatial Division with Vertical Lines





25.5.9 **Regrouping in Division.** When regrouping numbers appear in a division arrangement, the regrouping indicator is transcribed above or below the dividend, depending on the position of the separation line. When the separation line is printed above the dividend, the regrouping indicator "for numbers below the arrangement" is transcribed below the dividend. When the separation line is printed below the dividend, the regrouping indicator "for numbers above the arrangement" is transcribed above the dividend. The regrouping indicator is one cell longer on the left than the separation line. A blank space is left in the dividend and in the quotient where necessary to accommodate a regrouping number.

Example 25-51: Regrouping Numbers Within the Dividend

$$\begin{array}{r} 2\;4 \\ 3\overline{)7^{1}2} \end{array}$$

Example 25-52: Regrouping Numbers Below the Dividend

$$\begin{array}{r} 7\;\;6\;\;9 \\ 7\overline{)53_{4}8_{6}3} \end{array}$$

Example 25-53: Regrouping Numbers Above the Dividend

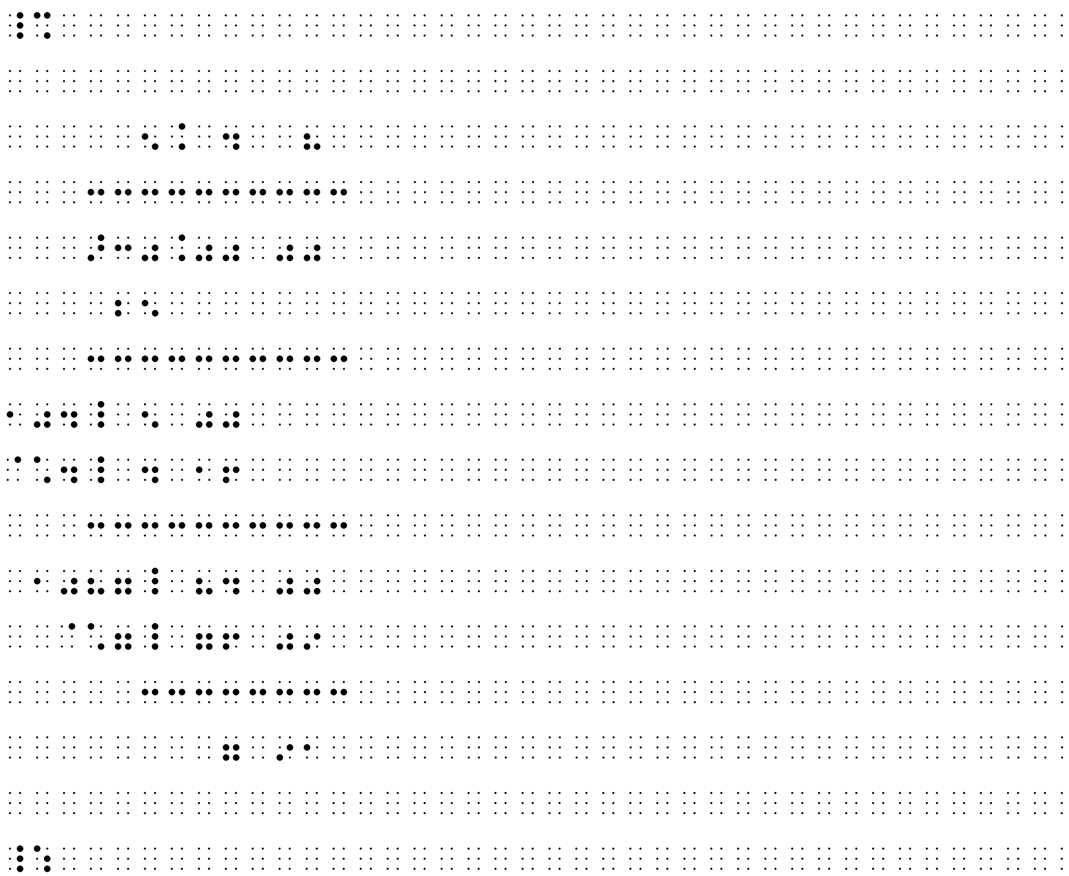
$$\begin{array}{l} 4\,)\,\underline{9\;{}^{1}5\;{}^{3}0.\;{}^{2}0} \\ \quad\;\; 2\;\;3\;\;\;7.\;\;\;5 \end{array}$$

25.6 Square Root

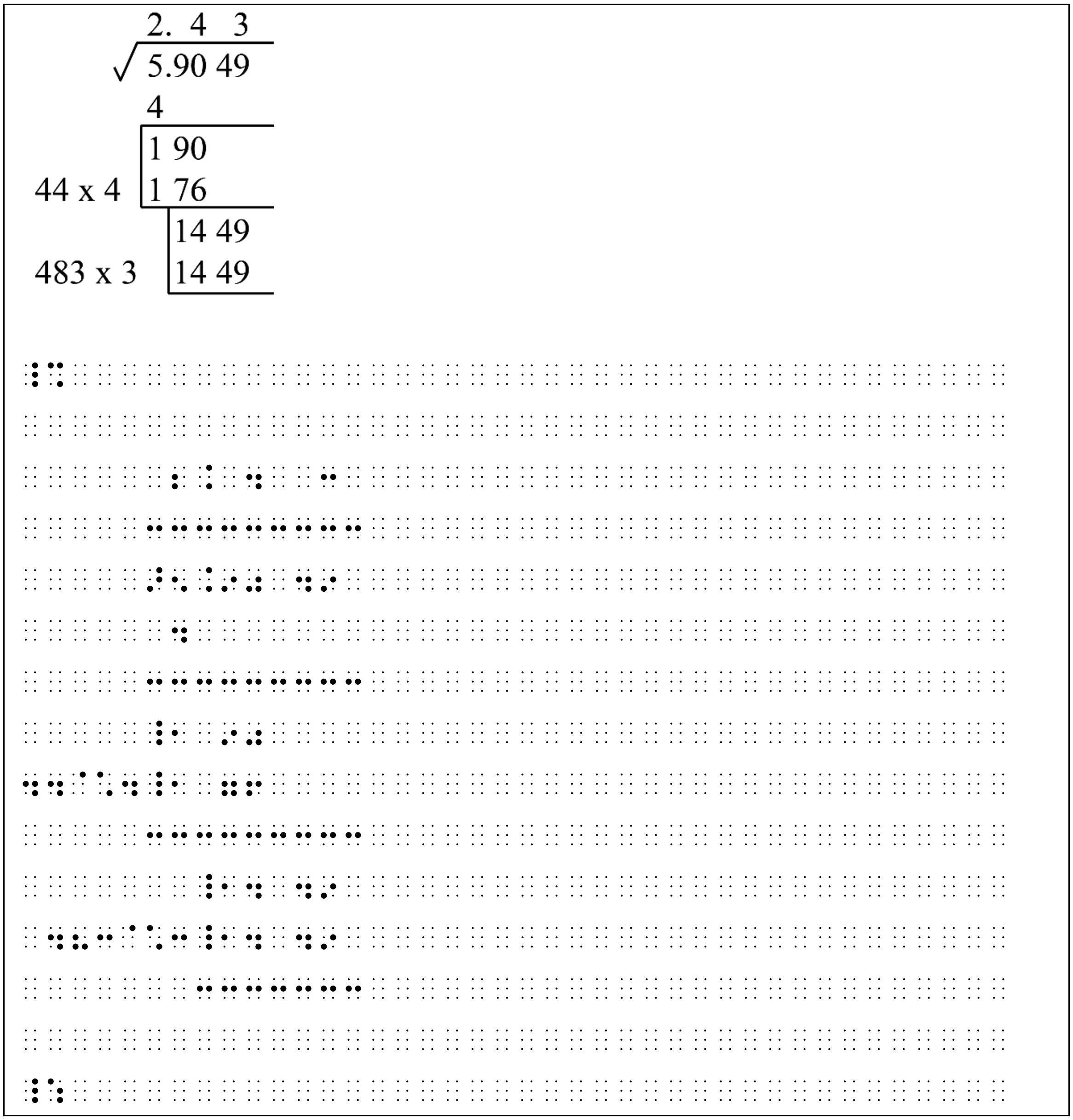
A square root arrangement is similar to a division arrangement except that no divisor is present. The arrangement should be adapted to resemble the arrangement in print as closely as possible. The vinculum is transcribed as a separation line. A spatial square root arrangement does not require a termination indicator.

Example 25-54: Spatial Square Root Arrangement

```
          5. 4  8
       √ 30.00 00
         25
104   | 5 00
 x4   | 4 16
1087    | 84 00
  x7    | 76 09
           7 91
```



Example 25-55: Spatial Square Root Arrangement



25.7 Synthetic Division

25.7.1 In a synthetic division arrangement, the numeric symbols in the synthetic dividend, synthetic product, and synthetic quotient are aligned by place value. Symbols of operation, when present, are also aligned. There must be at least one column of blank cells between adjacent columns of a synthetic division arrangement.

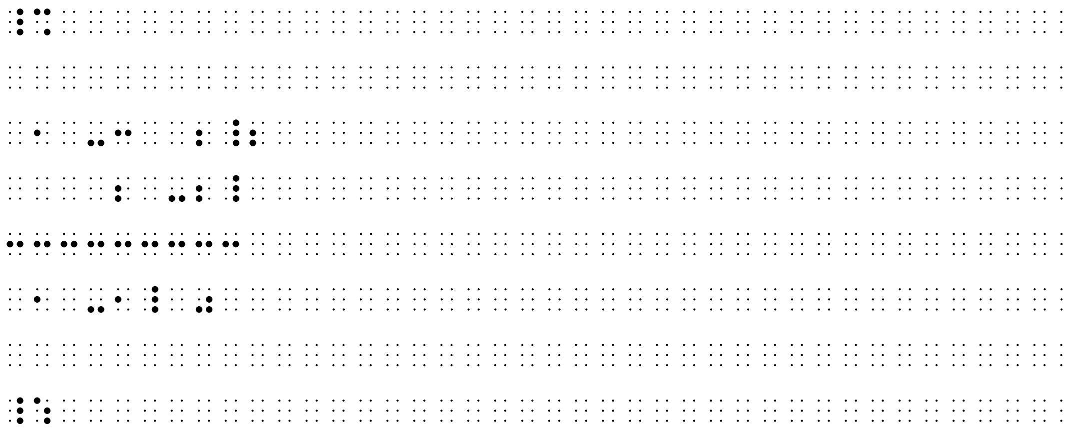
25.7.2 A vertical line is used to the left or to the right of the synthetic division arrangement according as the synthetic divisor appears to the left or to the right. This vertical line is unspaced from the synthetic dividend and from the synthetic divisor. One part of the vertical bar appears on the line containing the synthetic dividend, and another part of the line appears on the line containing the synthetic product. The separation line begins directly under the vertical line at one end, and terminates one cell beyond the over-all synthetic arrangement at the other end. If the synthetic divisor appears in print as boxed-in on two sides, this is ignored in the transcription. When a vertical line is used between the synthetic quotient and the synthetic remainder, it is placed in the column of blank cells as shown in print.

Example 25-56: Synthetic Division with Vertical Line on Left

$$\begin{array}{r|rrr}
2 & 1 & -3 & 2 \\
 & & 2 & -2 \\
\cline{2-4}
\multicolumn{1}{r}{} & 1 & -1 & \multicolumn{1}{|r}{0}
\end{array}$$

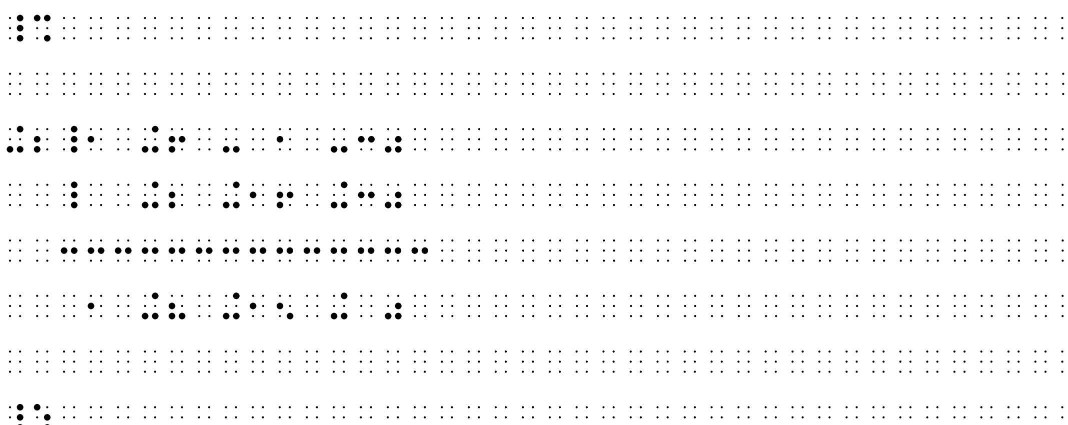
Example 25-57: Synthetic Division with Vertical Line on Right

$$\begin{array}{rrr|l} 1 & -3 & 2 & 2 \\ & 2 & -2 & \\ \hline 1 & -1 & \multicolumn{1}{|r}{0} & \end{array}$$



Example 25-58: Synthetic Division with Divisor on the Left

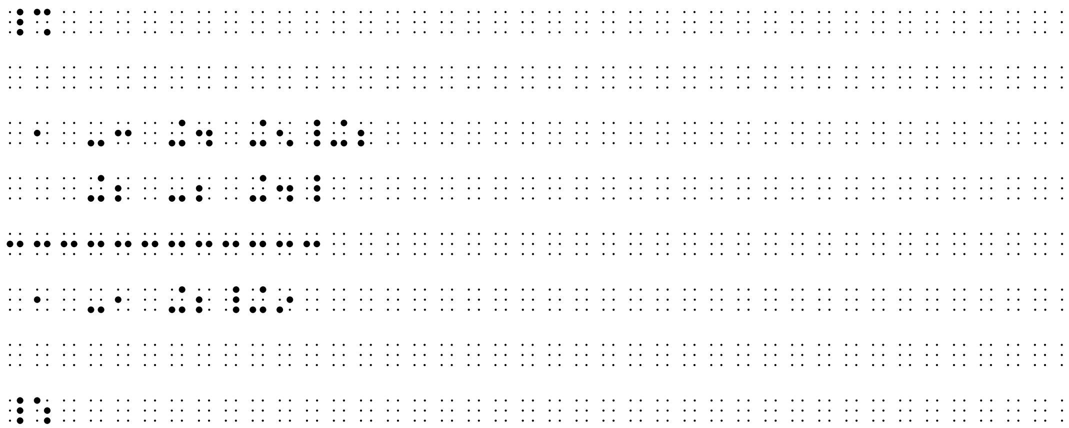
$$\begin{array}{lrrrr} \llcorner{+2} & 1 & +6 & -1 & -30 \\ & & +2 & +16 & +30 \\ \cline{2-5} & 1 & +8 & +15 & +0 \end{array}$$



(in print, the divisor is boxed-in on two sides; there is no vertical line after the divisor)

Example 25-59: Synthetic Division with Divisor on the Right

$$\begin{array}{cccc|c} 1 & -3 & +4 & +5 & \underline{+2} \\ & +2 & -2 & +4 & \\ \hline 1 & -1 & +2 & +9 & \end{array}$$



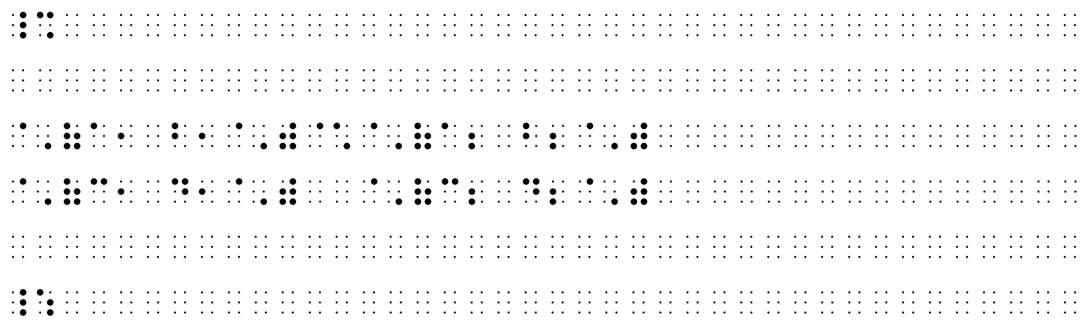
(in print, the divisor is boxed-in on two sides; there is no vertical line after the divisor)

25.8 Determinants and Matrices

25.8.1 In determinants or matrices each entry is left-justified (moved as far left as possible) in the column to which it belongs, and top-justified (moved as far up as possible) in the row to which it applies. Regardless of the print copy, centering or other forms of alignment are not permitted. One column of blank cells is left between columns. Guide dots are not used. A blank entry in a matrix or determinant is represented by a short dash. Punctuation, mathematical signs, and other applicable symbols are transcribed on the top line of the array. See rule **19.6** examples for placement of punctuation with enlarged grouping symbols. A numeric indicator is required on numbers within a matrix or determinant. The English-letter indicator is not used on a single letter within a matrix or determinant; see rule **6.4.4**.

Example 25-60: Matrices with Multiplication Sign

$$\begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix} \times \begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix}$$



25.8.2 Every effort must be made to confine the entire arrangement to a single braille page. To achieve this goal, the following techniques may be used:

a. The arrangement may begin in cell 1.

b. An entry may be run over to other braille lines and each continuation indented two cells from the column margin. When space saving is a factor, runovers may be made without regard to any hierarchy preferences (as listed in **26.9.1**). Successive rows in a column are transcribed without skipping a line between them. (See example **25-61**.)

c. An entry may be run over to other braille lines and each continuation left-justified in its column. When space saving is a factor, runovers may be made without regard to any hierarchy preferences. Successive rows in a column must be transcribed with a skipped line between them. When the technique described in **a** above is effective in providing the required space, it is used in preference to the technique described here. (See example **25-62**.)

d. Additional space may be saved by drawing the grouping symbols instead of using their braille equivalents.

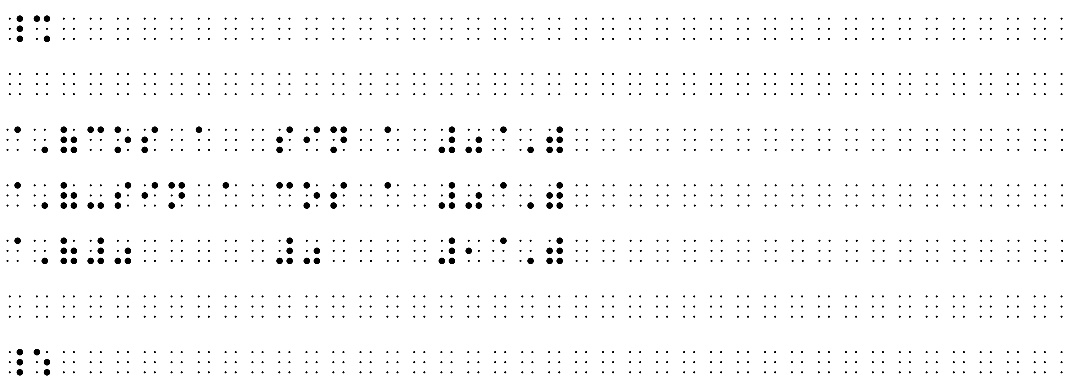
e. When an entry is a fraction, the fraction may be represented spatially if necessary to save space.

However, the row containing such a fraction must then have a line skipped above and below it. See example **25-64**. All fractions within a single determinant or matrix are transcribed the same way, either spatially or linearly.

f. The technique of keying may be employed for one or several entries if no other space-saving technique is effective. (See **26.9**.)

Example 25-61: Matrix Arrangement with Functions

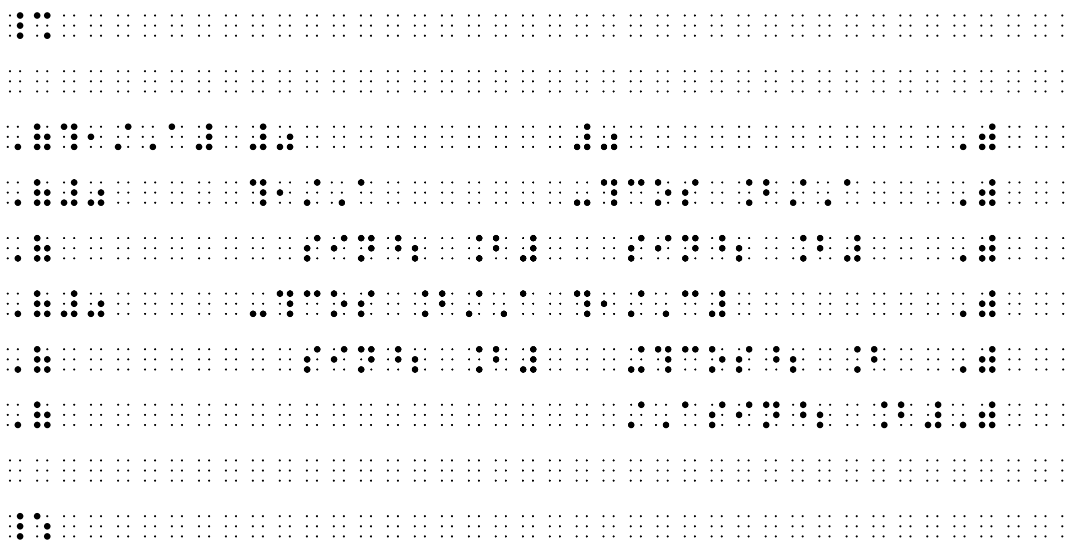
$$\begin{bmatrix} \cos a & \sin a & 0 \\ -\sin a & \cos a & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



(in print, each entry is centered in the column to which it belongs)

Example 25-62: Matrix Arrangement with Runovers

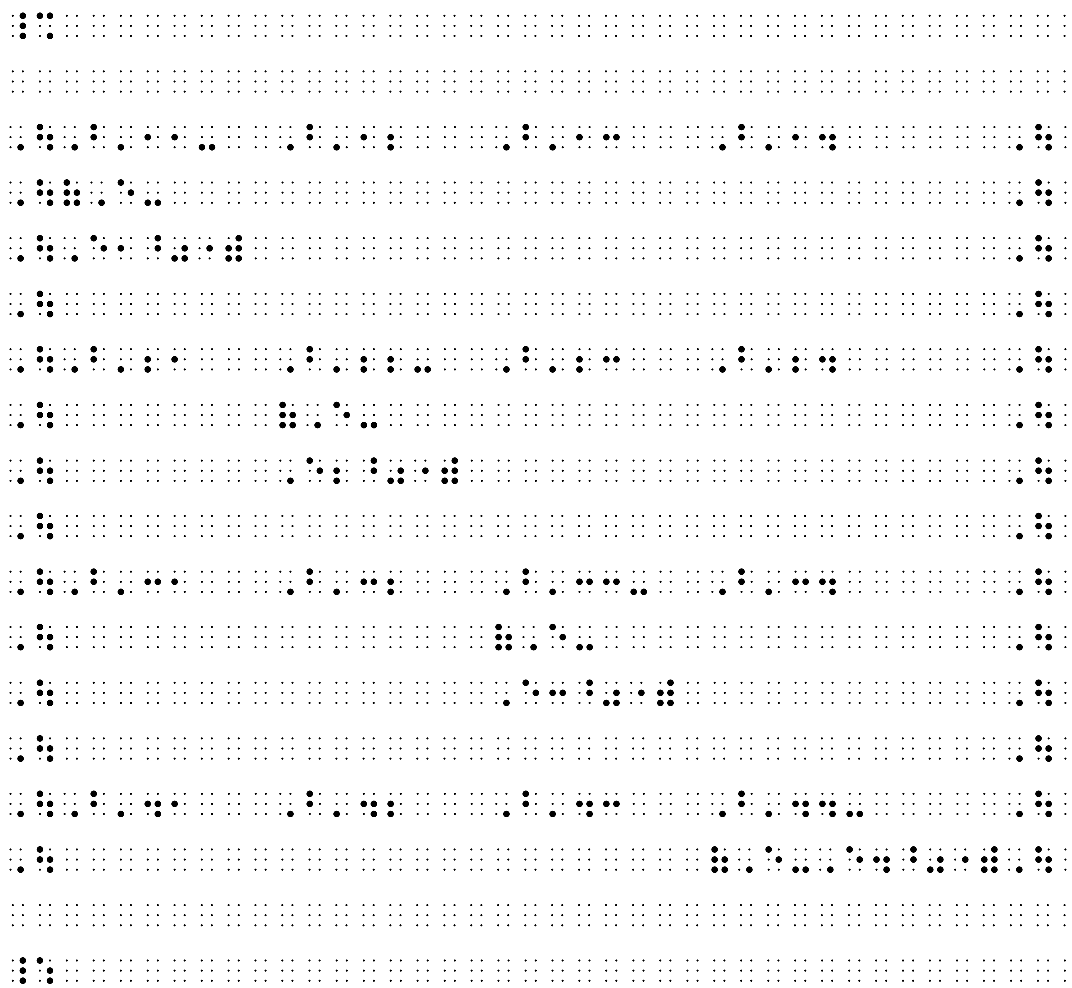
$$\begin{pmatrix} \frac{1}{A} & 0 & 0 \\ 0 & \frac{1}{A\sin^2\beta} & -\frac{\cos\beta}{A\sin^2\beta} \\ 0 & -\frac{\cos\beta}{A\sin^2\beta} & \frac{1}{C}+\frac{\cos^2\beta}{A\sin^2\beta} \end{pmatrix}$$



(in print, each entry is centered in the column to which it belongs)

Example 25-63: Matrix with Runovers

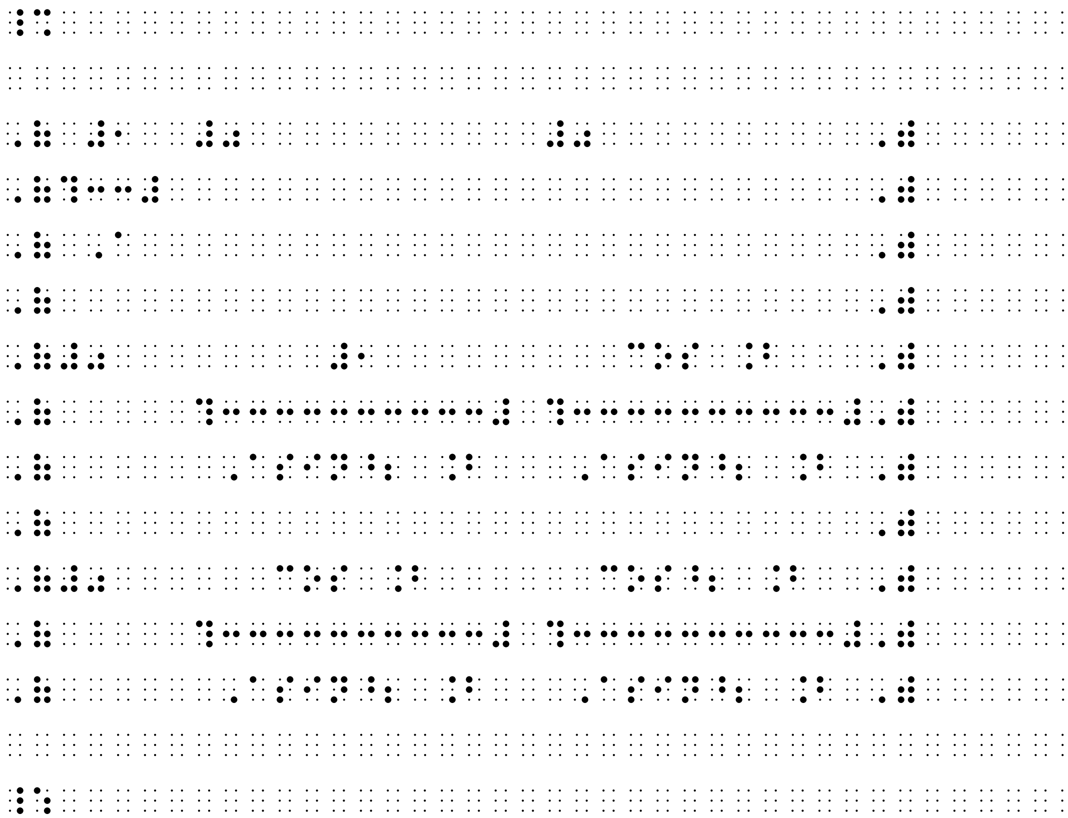
$$\begin{vmatrix} B'_{11}-\left(E-E_1^0\right) & B'_{12} & B'_{13} & B'_{14} \\ B'_{21} & B'_{22}-\left(E-E_2^0\right) & B'_{23} & B'_{24} \\ B'_{31} & B'_{32} & B'_{33}-\left(E-E_3^0\right) & B'_{34} \\ B'_{41} & B'_{42} & B'_{43} & B'_{44}-\left(E-E_4^0\right) \end{vmatrix}$$



(in print, each entry is centered in the column to which it belongs)

Example 25-64: Matrix with Spatial Fractions

$$\begin{pmatrix} \frac{1}{A} & 0 & 0 \\ 0 & \frac{1}{A\sin^2\beta} & \frac{\cos\beta}{A\sin^2\beta} \\ 0 & \frac{\cos\beta}{A\sin^2\beta} & \frac{\cos^2\beta}{A\sin^2\beta} \end{pmatrix}$$



(in print, each entry is centered in the column to which it belongs)

25.8.3 When a dot, an ellipsis, or a sequence of dots signifies the omission of one or more rows or columns, the following rules apply.

a. **Single Dot.** An entry consisting of a single dot is represented in braille by a solid dot (⠨⠡). The symbol is left-justified in its column and top-justified in its row.

b. **Horizontal, Vertical, or Diagonal Ellipsis.** The appropriate symbol from the list of ellipsis symbols is left-justified in its column and top-justified in its row.

c. **Sequence of Dots Across the Width of a Row.** When a sequence of dots is printed across an entire row and is not confined to specific columns, a sequence of dot 3s is transcribed beginning in the first cell of column one and extending to the end of the longest entry in the last column. No transcriber's note is required.

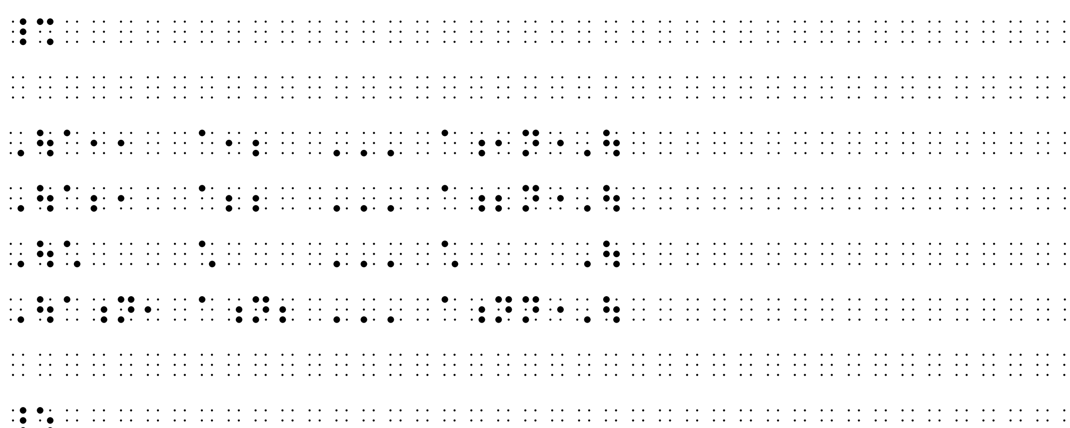
d. **Blank Entry.** A short dash is inserted to represent a blank entry. A transcriber's note is required.

Sample transcriber's note:

In a matrix or determinant, a short dash indicates a blank entry in print.

Example 25-65: Matrix with Ellipses and Single Dots

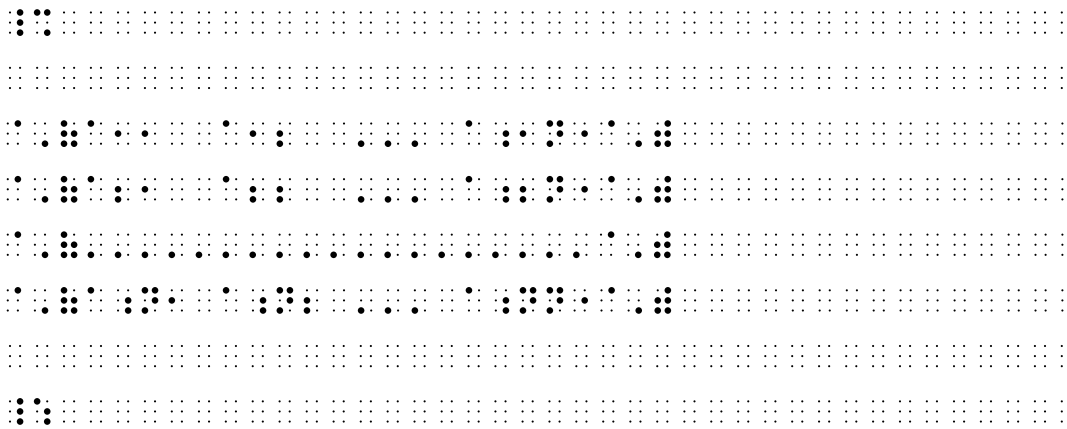
$$\begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ . & . & \dots & . \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{vmatrix}$$



(in print, one dot is shown in each of the first, second, and fourth columns in the third row)

Example 25-66: Matrix with Ellipsis Across All Columns

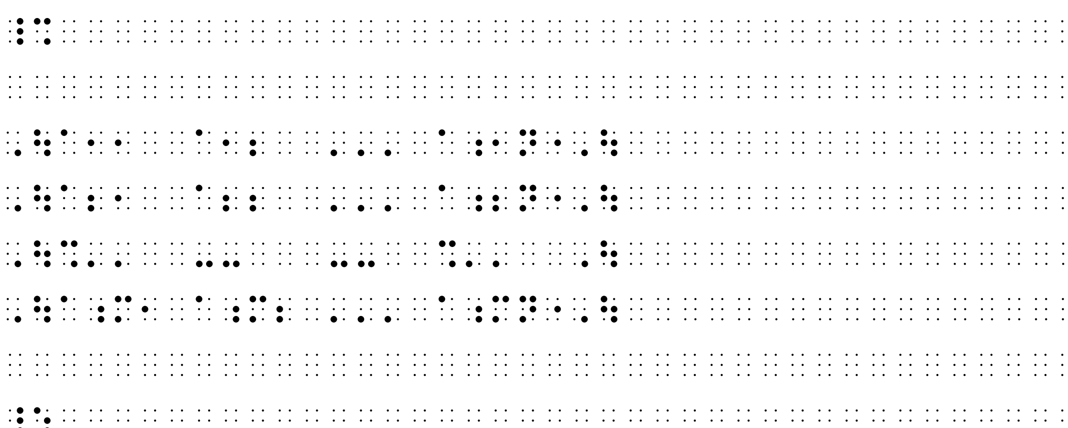
$$\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ . \quad . & . \quad . & . \quad . & . \quad . \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix}$$



(in print, the sequence of dots in the third row is not confined to specific columns)

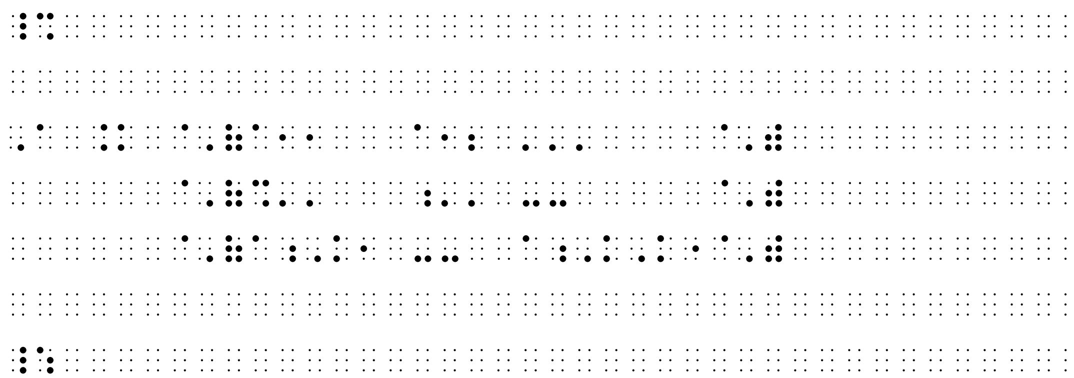
Example 25-67: Determinant with Blank Entries

$$\begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & & & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{vmatrix}$$



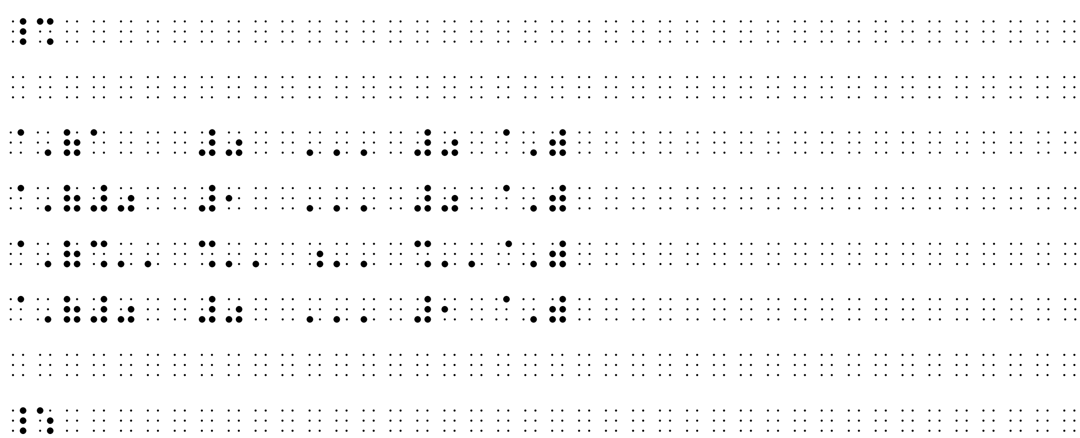
Example 25-68: Matrix with Blank Entries

$$A=\begin{bmatrix} a_{11} & a_{12} & \cdots \\ \vdots & \ddots & \\ a_{K1} & & a_{KK} \end{bmatrix}$$



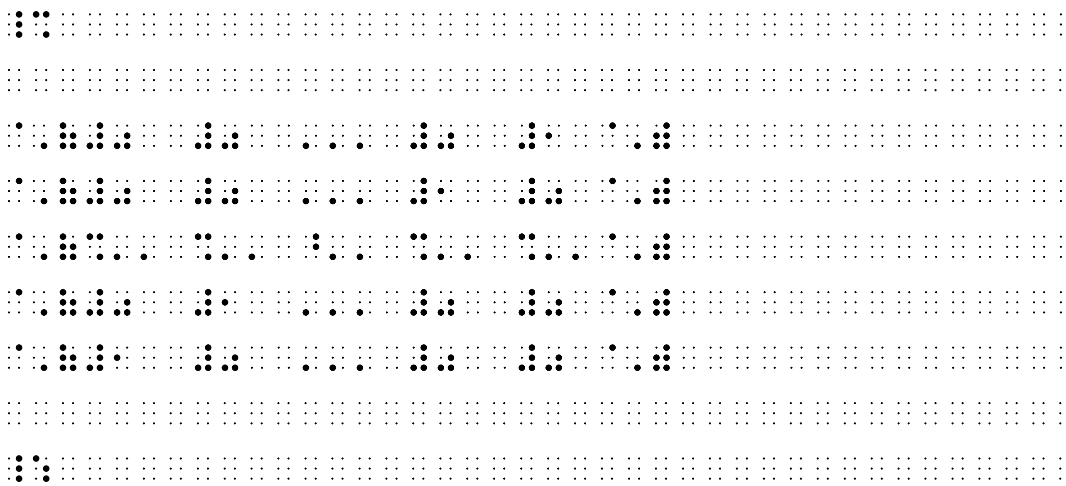
Example 25-69: Matrix with Diagonal and Vertical Ellipses

$$\begin{pmatrix} a & 0 & \ldots & 0 \\ 0 & 1 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & 1 \end{pmatrix}$$



Example 25-70: Matrix with Diagonal and Vertical Ellipses

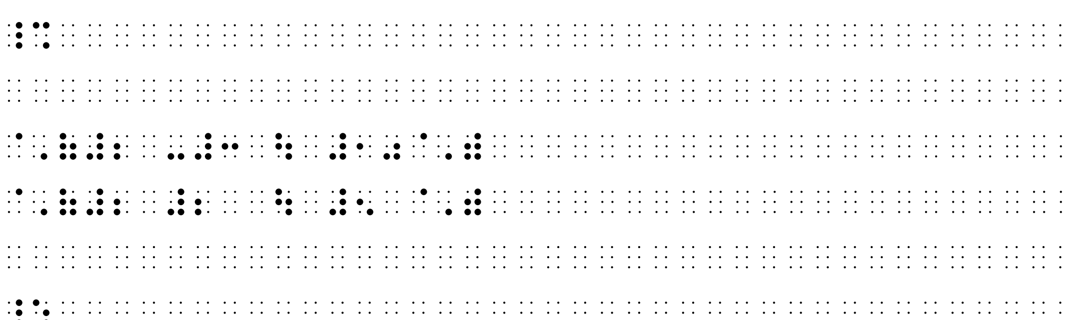
$$\begin{pmatrix} 0 & 0 & \cdots & 0 & 1 \\ 0 & 0 & \cdots & 1 & 0 \\ \vdots & \vdots & ⋰ & \vdots & \vdots \\ 0 & 1 & \cdots & 0 & 0 \\ 1 & 0 & \cdots & 0 & 0 \end{pmatrix}$$



25.8.4 **Augmented Matrix.** An augmented matrix is a matrix obtained by appending columns of two given matrices. In print, a vertical line separates the two matrices. Print may use a solid, dashed, or gray line. In braille, the vertical bar (⠳) represents the vertical line in print. The symbol requires a blank cell before and after. Alternatively, the line may be inserted as a tactile drawing if space is an issue.

Example 25-71: Augmented Displayed Matrix

$$\left[\begin{array}{cc|c} 2 & -3 & 10 \\ 2 & 2 & 5 \end{array}\right]$$



25.9 Systems of Equations

A system of equations is a spatial arrangement. The system, consisting of two or more equations, is preceded and followed by a blank line.

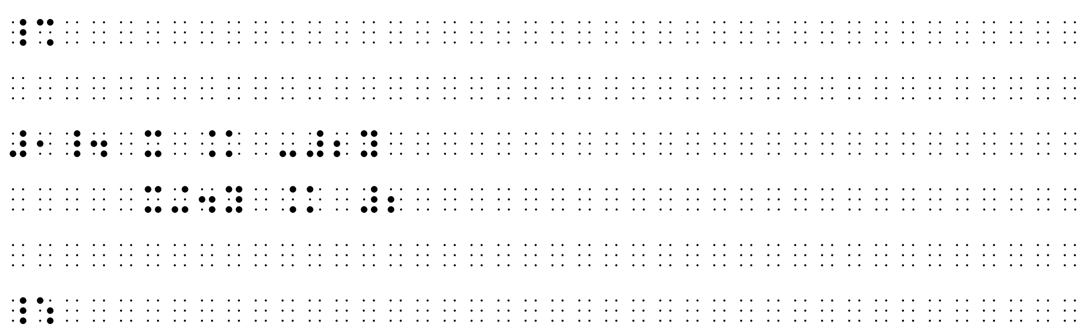
When elements of the system are vertically aligned in print, the alignment is retained in braille. Numeric indicators are required. Do not insert numeric indicators when spaces are inserted *within* an equation to achieve vertical alignment.

Follow print regarding the presence or absence of grouping symbols. Add the transcriber-inserted grouping symbol if there is no grouping symbol in print when a comment is present that applies to all the expressions.

Punctuation, mathematical signs, and other applicable symbols are transcribed on the top line of the system.

Example 25-72: System of Equations with Identifier

1. $x = -2y$
   $x + 4y = 2$



Example 25-73: System of Equations Aligned as in Print

$$\begin{aligned} x + y &= 4 \\ 2x - y &= 5 \end{aligned}$$

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⠼⠆⠭⠤⠽⠀⠨⠅⠀⠼⠢

⠸⠱
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Example 25-74: Partially Aligned System of Equations

$$\begin{aligned} 2x + \quad 2y &= 1,100 \\ 45x + 80y &= 22,750 \end{aligned}$$

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⠀⠼⠆⠭⠬⠀⠆⠽⠀⠨⠅⠀⠼⠂⠠⠂⠴⠴
⠼⠲⠢⠭⠬⠦⠴⠽⠀⠨⠅⠀⠼⠆⠆⠠⠶⠢⠴

⠸⠱
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Example 25-75: System of Equations with Computation

$$\begin{array}{rcr} -45x - 45y &=& -15,750 \\ 45x + 80y &=& 22,750 \\ \hline 35y &=& 7,000 \end{array}$$

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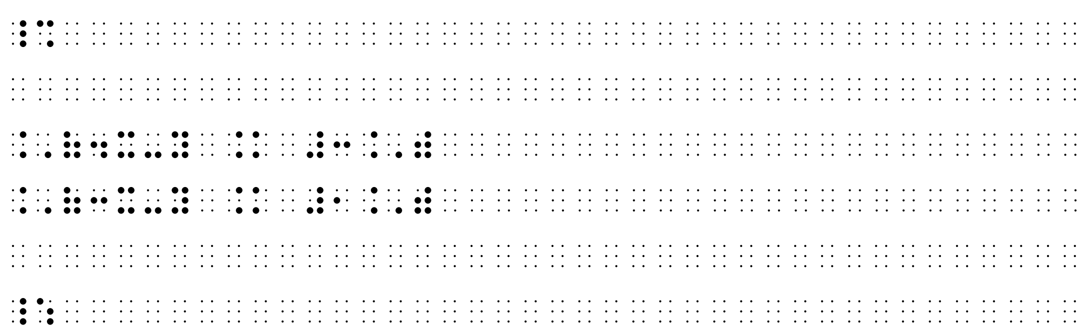
25.10 Unified Expressions

When enlarged grouping symbols are used to unify an expression which is neither a determinant nor a matrix, each item begins in the cell which immediately follows the left enlarged grouping symbol and at least one item ends in the cell which immediately precedes the right enlarged grouping symbol. It is advantageous to draw these enlarged grouping symbols when space saving is a factor. These requirements are waived whenever vertical alignment must be indicated. In that case, at least one item must either begin in the cell which immediately follows the left enlarged grouping symbol or must end in the cell which immediately precedes the right enlarged grouping symbol. The numeric indicator is not required before the first numeric character following the left grouping symbol. Do not insert numeric indicators within an equation whose elements are vertically aligned.

Example 25-76: System of Equations with Left and Right Braces

$$\left\{\begin{matrix} 4x - y = 3 \\ 3x - y = 1 \end{matrix}\right\}$$



Example 25-77: Aligned System of Three Equations

$$\left\{\begin{array}{l} 3x+15y-2z=64 \\ \ \ x+12y+\ \ \ z=51 \\ 7x-\ \ 8y+2z=-16 \end{array}\right\}$$

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⠨⠠⠷⠒⠭⠬⠂⠢⠽⠤⠆⠵⠀⠨⠅⠀⠀⠼⠖⠲⠨⠠⠾
⠨⠠⠷⠀⠭⠬⠂⠆⠽⠬⠀⠵⠀⠨⠅⠀⠀⠼⠢⠂⠨⠠⠾
⠨⠠⠷⠶⠭⠤⠀⠦⠽⠬⠆⠵⠀⠨⠅⠀⠤⠼⠂⠖⠨⠠⠾

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Example 25-78: Aligned System with Left Brace

$$\begin{cases} \ \ 3x-4y=\ 11 \\ -3x+2y=-7 \end{cases}$$

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⠨⠠⠷⠀⠒⠭⠤⠲⠽⠀⠨⠅⠀⠼⠂⠂
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Example 25-79: Unaligned System of Equations with Left Brace

$$\begin{cases} x+y=6 \\ y=2x \end{cases}$$

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⠨⠠⠷⠭⠬⠽⠀⠨⠅⠀⠼⠖
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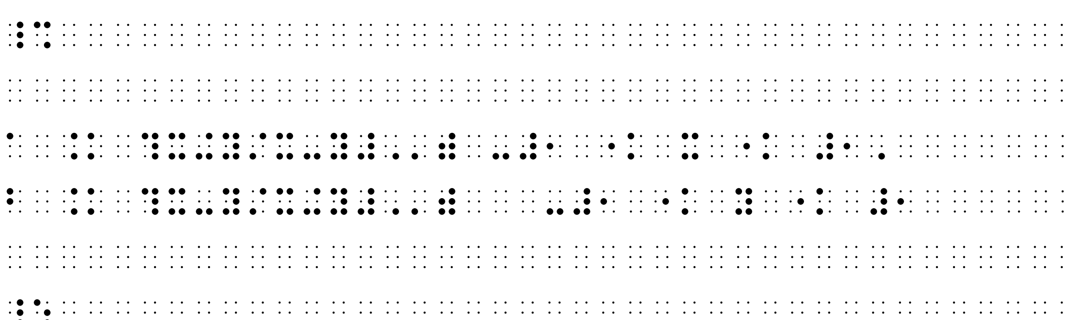
25.10.1 **Transcriber-Inserted Grouping Symbol.** When an explanation or comment refers to more than one print line to which no grouping sign as a whole applies, the implied grouping is indicated by using the transcriber-inserted grouping symbol. The explanation is placed to the right in braille regardless of its placement in print (See example **19-42**). There must be at least one clear column of spaces between transcriber-inserted grouping symbols and the associated explanation. Any text in the comment is transcribed uncontracted. If the explanation requires more braille lines than what is being explained, the transcriber-inserted grouping symbol is extended to cover the explanation, and each runover of the explanation is indented two cells from the column in which the explanation begins. A blank line is required before and after such a grouping.

Example 25-80: Transcriber Inserted Grouping Symbol

$$a = \frac{x+y}{x-y}$$

$$-1 < x < 1,\ -1 < y < 1$$

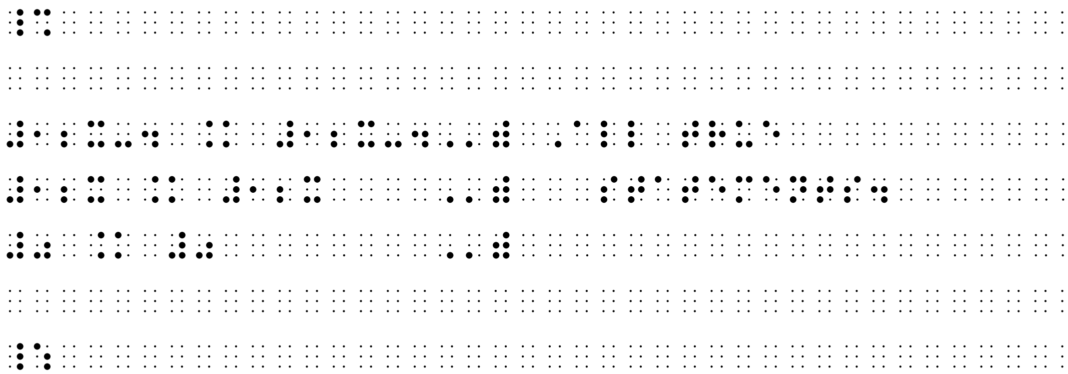
$$b = \frac{x-y}{x+y}$$



(in print, the explanation is centered to the right of the two equations to which it applies)

Example 25-81: Transcriber Inserted Grouping Symbol

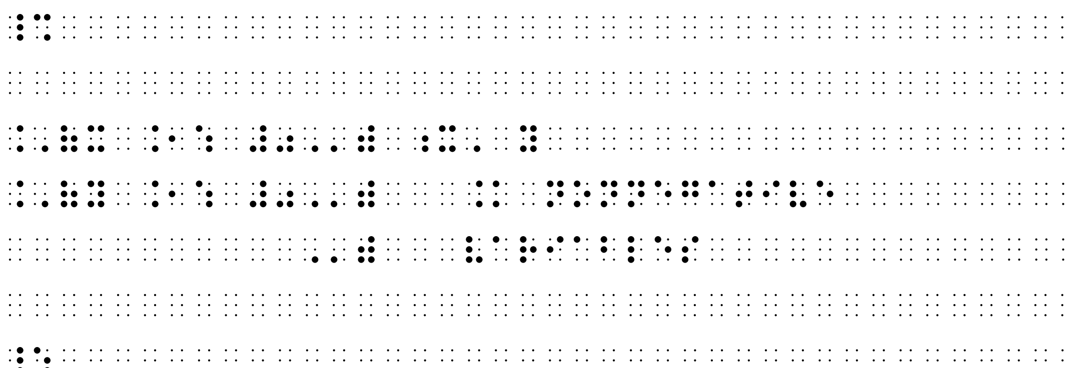
$$\begin{aligned} 12x - 4 &= 12x - 4 \\ 12x &= 12x \qquad\qquad \text{All true statements.} \\ 0 &= 0 \end{aligned}$$



(alignment of equals signs is disregarded)

Example 25-82: Comment is Longer Than the System

$$\begin{cases} x \geq 0 \\ y \geq 0 \end{cases} \quad \text{x, y = nonnegative variables}$$



Rule 26
Format

Format refers to the layout on the braille page including, but not limited to, margins, blank lines, and placement of identifiers. See **Rule 4** for placement of Nemeth switch indicators.

Formatting rules apply throughout a transcription, even in the UEB portions. If a format is not addressed in the Nemeth Code, the guidelines outlined in *Braille Formats, Principles of Print to Braille Transcription* should be followed.

26.1 Margins

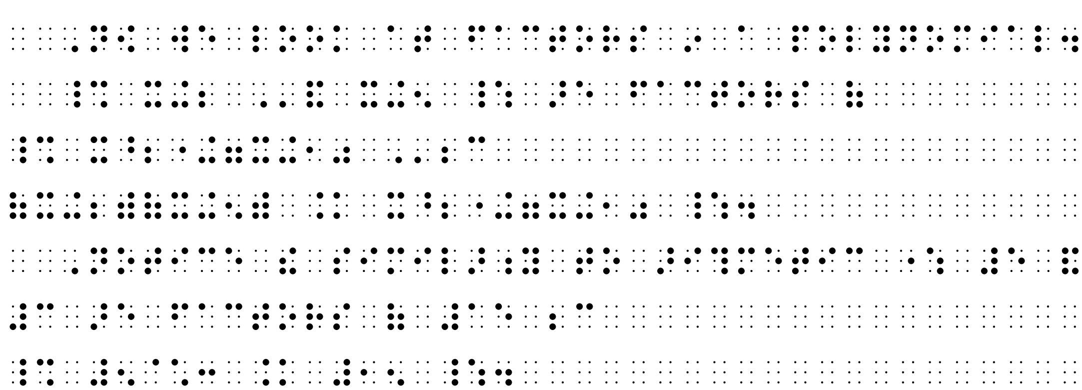
26.1.1 **Narrative Portions of Text.** In a Nemeth transcription, paragraphs begin in cell 3 and are run over in cell 1, although print may show blocked paragraphs. Directions preceding non-itemized material are treated as a narrative paragraph, beginning in cell 3 and runover in cell 1.

Example 26-1: Narrative Paragraph (3-1)

Now we look at factors in a polynomial.

$x + 2$ and $x + 5$ are factors of $x^2 + 7x + 10$ because $(x + 2)(x + 5) = x^2 + 7x + 10$.

Notice the similarity to arithmetic where 5 and 3 are factors of 15 because $5 \times 3 = 15$.



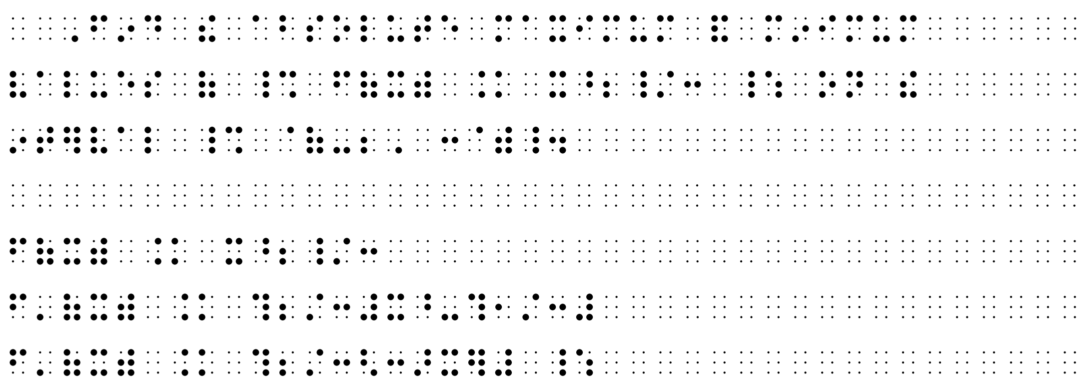
Example 26-2: Narrative Directions (3-1)

Find the absolute maximum and minimum values of $f(x) = x^{2/3}$ on the interval $[-2, 3]$.

$$f(x) = x^{2/3}$$

$$f'(x) = \frac{2}{3}x^{-\frac{1}{3}}$$

$$f'(x) = \frac{2}{3\sqrt[3]{x}}$$



26.1.2 **Itemized Material (Non-Spatial).** When material is identified sequentially by number or letter, as in exercise material, it is referred to as *itemized material*. The number or letter is referred to as the *identifier*. An identifier cannot stand alone on line 25.

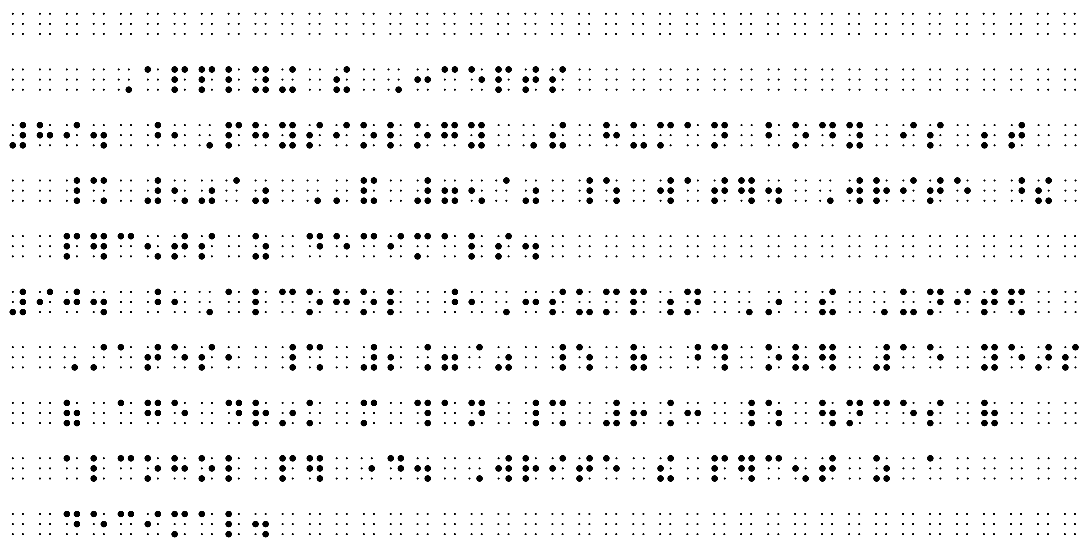
a. **Itemized Material with No Subdivisions.** When non-spatial itemized material contains main divisions only, each identifier begins in cell 1. The associated material is run over in cell 3. A subparagraph within an item begins in cell 5 and is run over in cell 3.

Example 26-3: Itemized Material (1-3)

Applying the Concepts

89. **Physiology** The human body is between 50% and 75% water. Write these percents as decimals.

90. **Alcohol Consumption** In the United States, 2.7% of those over 15 years of age drink more than 6.3 ounces of alcohol per day. Write the percent as a decimal.

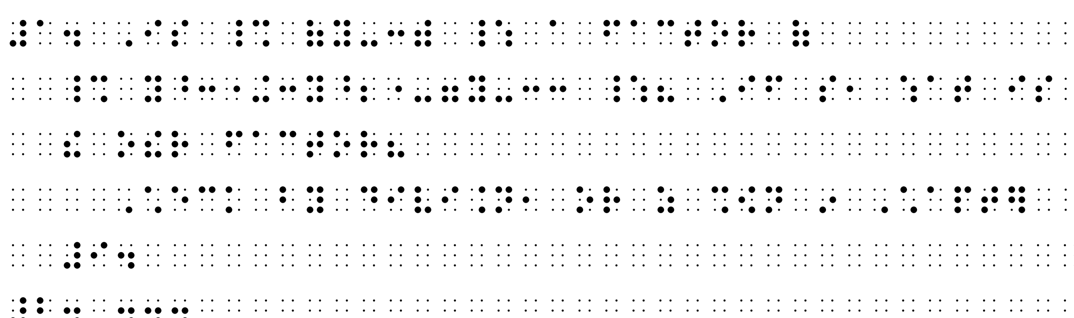


Example 26-4: Itemized Material with Subparagraph (5-3)

1. Is $(y - 3)$ a factor of $y^3 + 3y^2 - 7y - 33$? If so, what is the other factor?

   Check by division, or as shown in Chapter 9.

2. …



b. **Itemized Material with Subdivisions.** When non-spatial itemized material contains main divisions and subdivisions, each main division identifier begins in cell 1.

The associated material is run over in cell 5. Each subdivision identifier begins in cell 3 with runovers in cell 5. Subdivisions to whatever depth follow the same 3-5 pattern.

A subparagraph within an item or within a subitem begins in cell 7 and is run over in cell 5.

When subdivisions of non-spatial material are printed on the same line across the page, each subitem must begin on a new line in the braille transcription.

If the main item has no accompanying text, the identifier is transcribed in cell 1 and the first subdivision begins on the next line in cell 3. Each subitem begins on a new line.

Example 26-5: Itemized Material with Subdivisions (1-5, 3-5)

53. Work each problem.

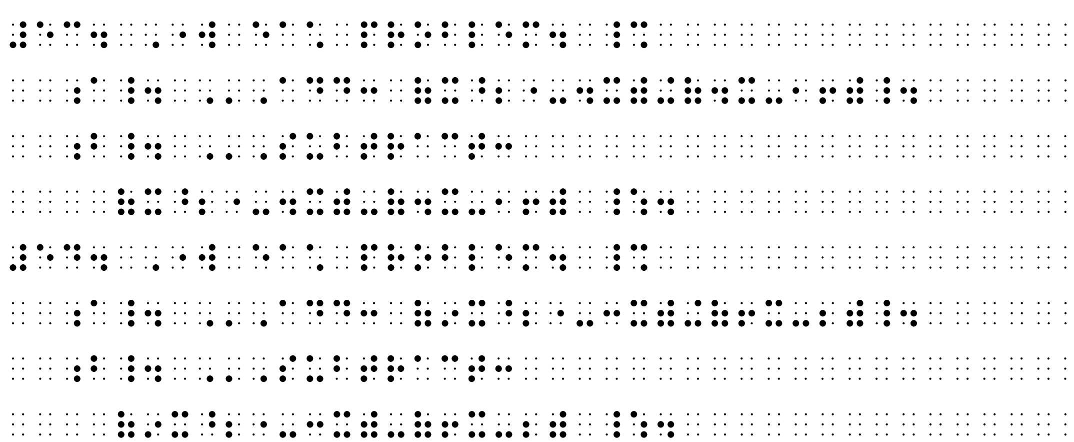
    a. Add: $(x^2 - 4x) + (4x - 16)$.

    b. Subtract: $(x^2 - 4x) - (4x - 16)$.

54. Work each problem.

    a. Add: $(9x^2 - 3x) + (6x - 2)$.

    b. Subtract: $(9x^2 - 3x) - (6x - 2)$.



Example 26-6: Deeper Subdivisions (1-5, 3-5)

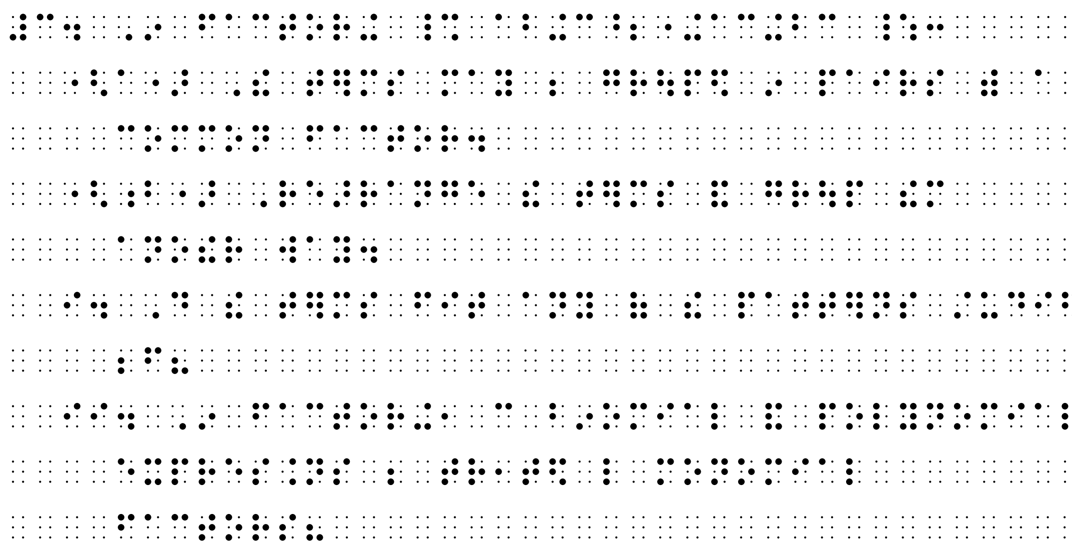
3. In factoring $ab + c^2 + ac + bc$:

   (a) The terms may be grouped in pairs with a common factor.

   (b) Rearrange the terms and group them another way.

      i. Do the terms fit any of the patterns studied before?

      ii. In factoring, can binomial and polynomial expressions be treated like monomial factors?



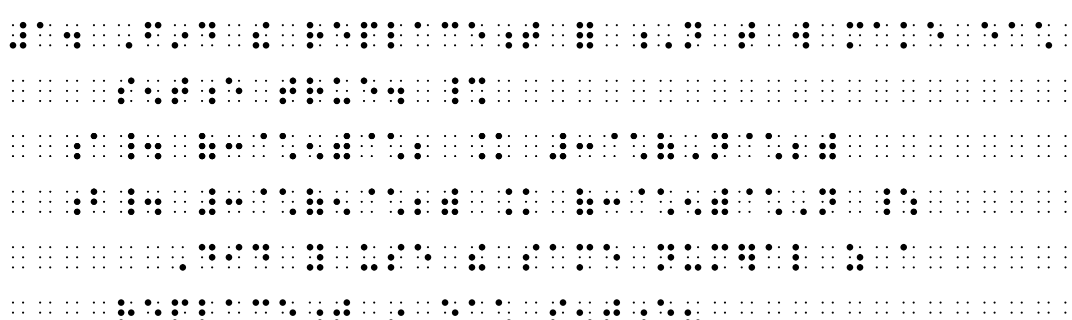
Example 26-7: Subdivisions with a Subparagraph (7-5)

1. Find the replacement for N that will make each sentence true.

   a. $(3 \times 5) \times 2 = 3 \times (N \times 2)$

   b. $3 \times (5 \times 2) = (3 \times 5) \times N$

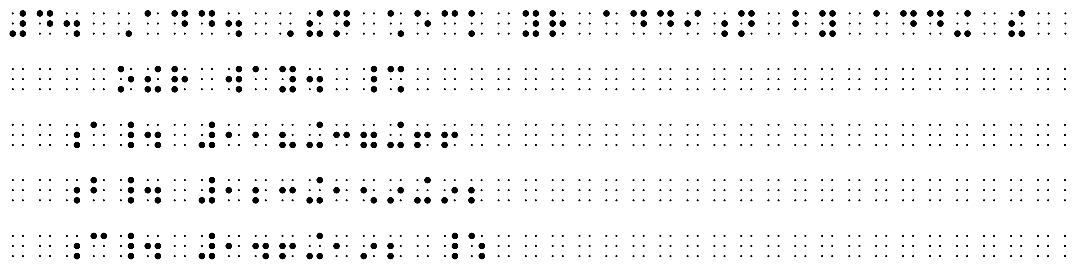
   Did you use the same numeral as a replacement in each sentence?



Example 26-8: Subitems Printed Side-By-Side

4. Add. Then check your addition by adding the other way.

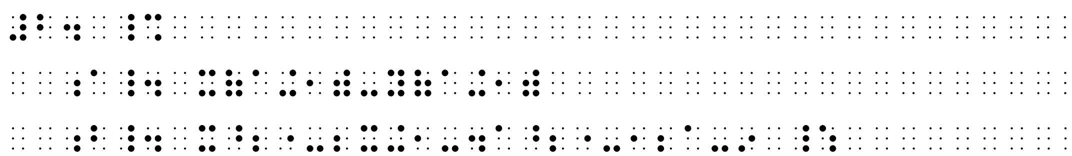
   a. 118 + 37 + 66 b. 123 + 159 + 92 c. 146 + 192



Example 26-9: Subitem Printed on Same Line as Item Number

2. a. $x(a+1) - y(a + 1)$

   b. $x^2 - 2x + 1 - 4a^2 - 12a - 9$



c. **Mixed Margins.** Runover margins for itemized material are determined individually for each item. An item containing only a main division begins in cell 1 with runovers in cell 3. For an item containing both a main division and subdivisions, the main item begins in cell 1 with runovers in cell 5. Each subdivision begins in cell 3 with runovers in cell 5. **NOTE:** This does not apply to tables of contents or outlines.

Example 26-10: Exercise Material with Mixed Margins

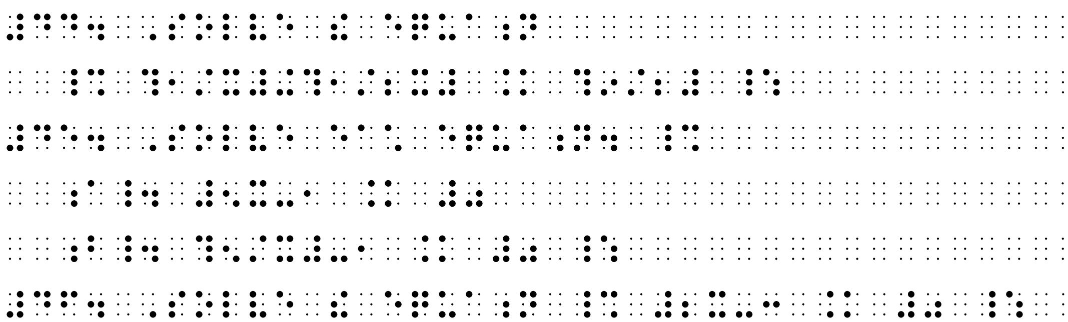
44. Solve the equation $\frac{1}{x}+\frac{1}{2x}=\frac{9}{2}$

45. Solve each equation.

    a. $5x - 1 = 0$

    b. $\frac{5}{x} - 1 = 0$

46. Solve the equation $2x - 3 = 0$



(margins are determined for each listed item individually)

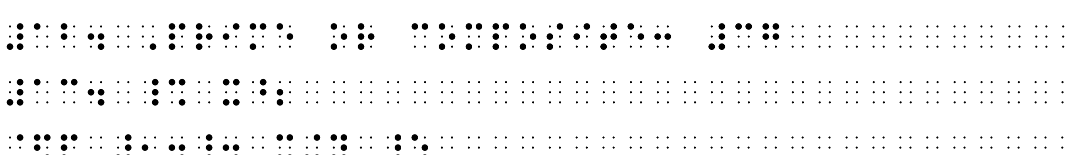
d. **Marked Item with Identifier.** Each identifier or icon begins in the same cell. The list is not aligned by identifier.

Example 26-11: Item with Marked Identifier

12. Prime or composite: 37

13. x^2

 14. $x+y$



(alignment of identifiers is not required)

26.1.3 **Itemized Spatial Material.** See **26.6.2**.

26.1.4 **Instructions.**

a. To be formatted as instructions, the material which follows the instructions must be itemized.

b. Instructions are preceded by a blank line unless they follow a cell-5 or cell-7 heading. Instructions begin in cell 5 and are run over in cell 3. The itemized material follows without a blank line unless the items are spatial arrangements or examples which require a leading blank line. Graphics or unnumbered examples are allowed between instructions and the itemized exercise.

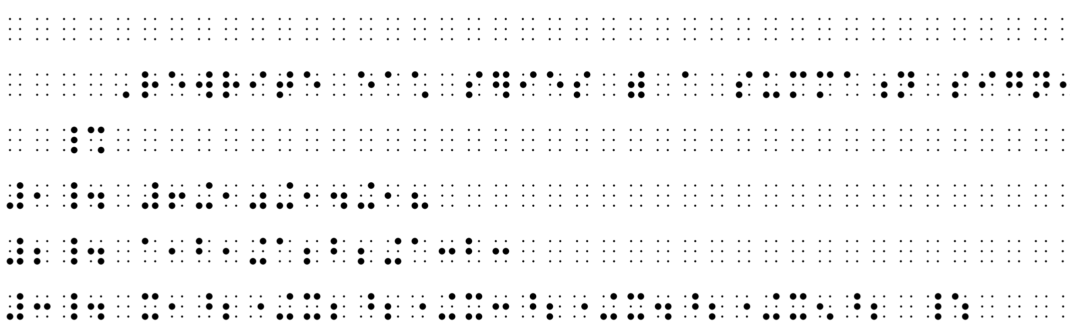
Example 26-12: Instructions (5-3)

Rewrite each series with a summation sign.

1. $6 + 10 + 14 + 18$

2. $a_1b_1 + a_2b_2 + a_3b_3$

3. $x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2$



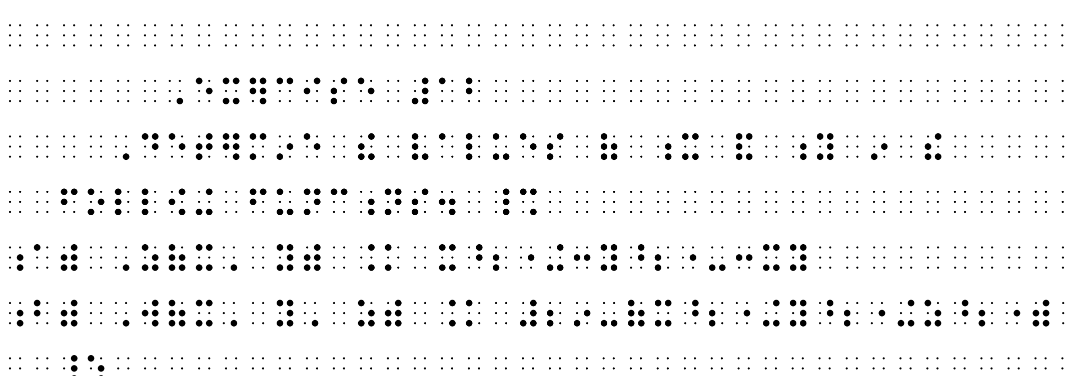
Example 26-13: Instructions Following a Cell-7 Heading

Exercise 12

Determine the values of x and y in the following functions.

a) $Z(x, y) = x^2 + 3y^2 - 3xy$

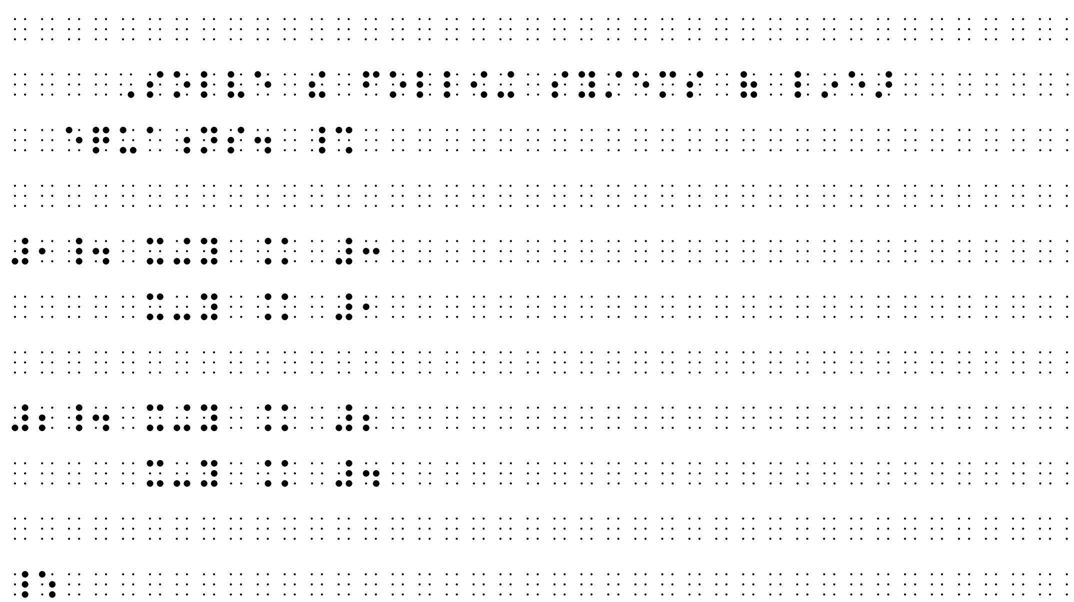
b) $W(x, y, z) = 29 - (x^2 + y^2 + z^2)$



Example 26-14: Instructions Preceding Spatial Problems

Solve the following systems of linear equations.

1. $\begin{array}{l} x + y = 3 \\ x - y = 1 \end{array}$ 2. $\begin{array}{l} x + y = 2 \\ x - y = 4 \end{array}$



c. It is preferable to keep instructions on the same braille page with the exercise. To accomplish this, instructions may need to be moved to the next braille page. However, when there is not sufficient space on that page for the instructions and part of the exercise, instructions may be placed on the preceding page.

26.1.5 **Itemized Material Arranged in Tabular Form.** When itemized material is arranged in tabular form so that rows are identified by number and columns are identified by letter, the following technique is used provided that the entire tabulation can be contained across the braille page.

a. A blank line is left above and below the column headings.

b. The letters which identify the columns are left-justified in the columns to which they apply.

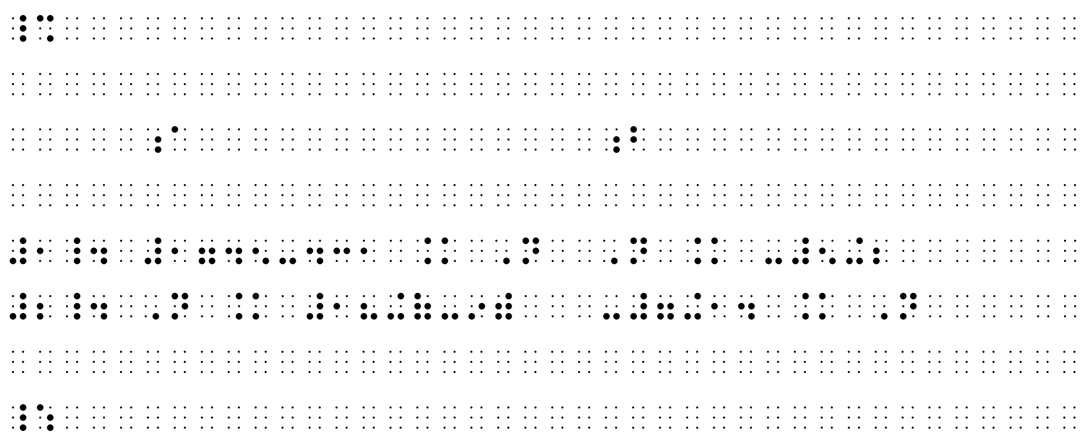
c. Row numbers begin in cell 1.

d. A minimum of two spaces are left between the right-hand margin of one column and the left-hand margin of the next column.

e. If the entire tabulation is too wide to be contained across the braille page using this technique, follow the item/subitem format of **26.1.2.b** (non-spatial) or **26.6.3.c** (spatial), using the row numbers as main item identifiers and using the column letters as subdivision identifiers.

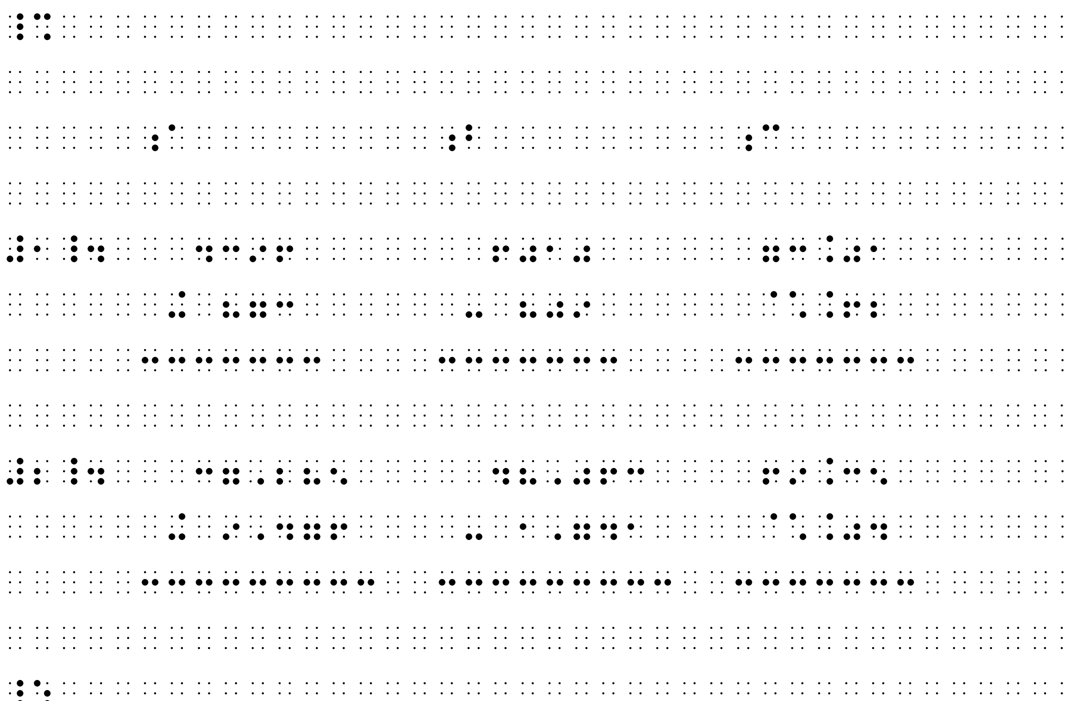
Example 26-15: Tabular Form, Non-Spatial

	a	b
1.	$1745 - 431 = N$	$N = -5 + 2$
2.	$N = 18 + (-9)$	$-7 + 14 = N$



Example 26-16: Tabular Form, Spatial

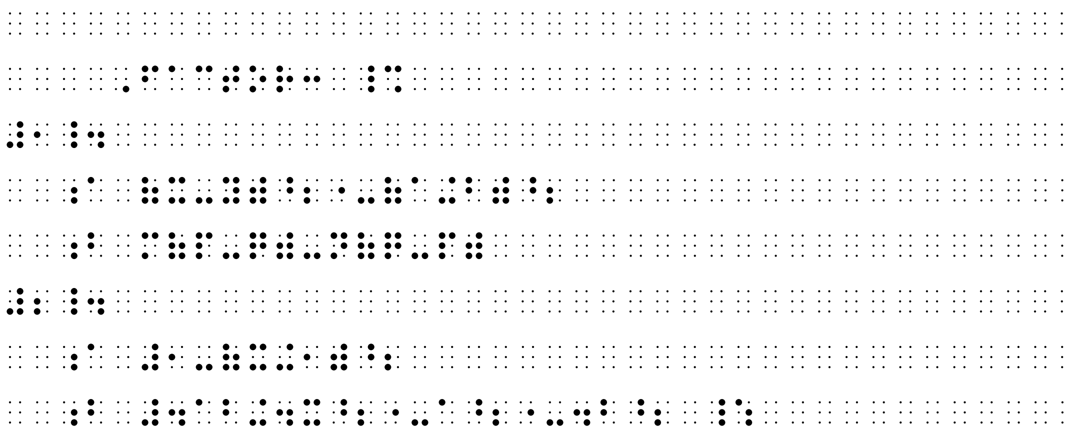
	a	b	c
1.	$\begin{array}{r} 4396 \\ +\ \ 873 \\ \hline \end{array}$	$\begin{array}{r} 6010 \\ -\ 809 \\ \hline \end{array}$	$\begin{array}{r} 73.01 \\ \times\ \ \ .62 \\ \hline \end{array}$
2.	$\begin{array}{r} 37,285 \\ +9,476 \\ \hline \end{array}$	$\begin{array}{r} 48,063 \\ -\ \ 1,741 \\ \hline \end{array}$	$\begin{array}{r} 69.35 \\ \times\ \ \ .04 \\ \hline \end{array}$



Example 26-17: Tabular Form Changed to Item/Subitem Form

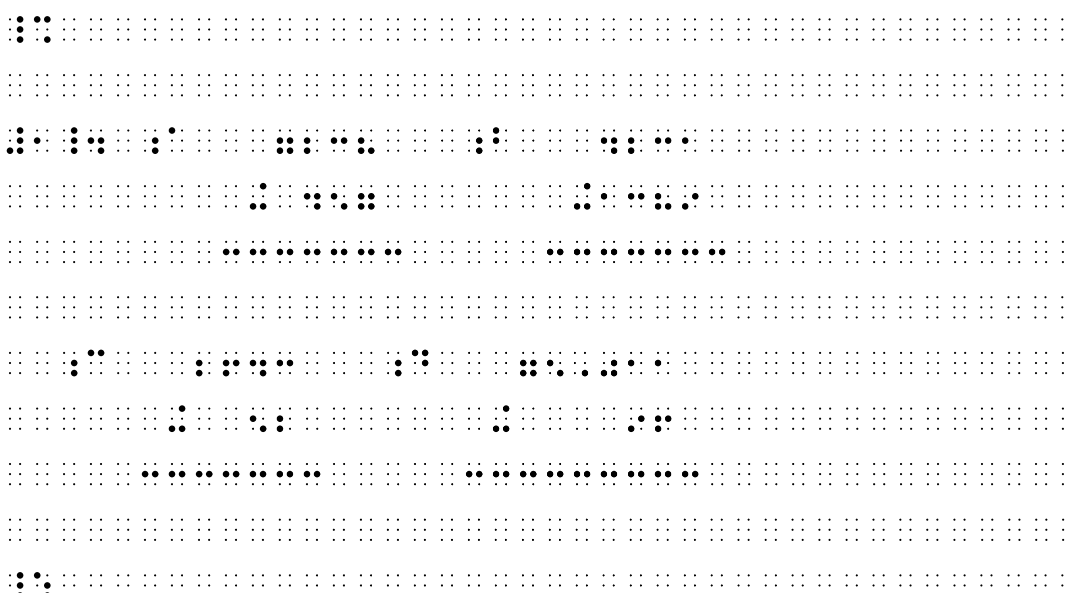
Factor:

	a	b
1.	$(x-y)^2-(a+b)^2$	$m(p-q)-n(q-p)$
2.	$1-(x+1)^2$	$4ab+4x^2-a^2-4b^2$



Example 26-18: Tabular Form Changed to Item/Subitem Form

	a	b	c	d
1.	$\begin{array}{r} 7238 \\ +\ 457 \\ \hline \end{array}$	$\begin{array}{r} 4231 \\ +1389 \\ \hline \end{array}$	$\begin{array}{r} 2643 \\ +\quad 52 \\ \hline \end{array}$	$\begin{array}{r} 75,011 \\ +\qquad 96 \\ \hline \end{array}$



26.2 Mathematical Expressions Requiring Runovers

When a mathematical expression will not fit on one braille line, runover sites should be chosen carefully. Every attempt should be made to keep mathematical units intact. Avoid splitting up mathematical units such as fractions, a numerator, a denominator, mixed numbers, items within grouping symbols, a base and its exponent, an item and its modifiers, functions, radicals, etc. Follow the list below when choosing division sites, starting with step a. If there is still not room, move to an applicable step further down the list. See **26.5** for further considerations regarding division of linked expressions.

a. Before a symbol of comparison on the baseline.

b. Before a symbol of operation on the baseline.

c. Before a mathematical unit such as:

- an opening fraction indicator
- a fraction line if the fraction must be divided
- the baseline indicator which precedes a mathematical unit or the items listed in the paragraph above
- a change-of-level indicator when an item and its exponent or subscript are too long to fit on a single braille line.

d. After a termination indicator.

e. Between items which are enclosed within grouping symbols if the grouping will not fit on one line.

Example 26-19: Divide Before a Symbol of Operation

3. $\mathbf{a}\cdot(\mathbf{b}+\mathbf{c})=\langle a_1, a_2, a_3\rangle\cdot\langle b_1+c_1, b_2+c_2, b_3+c_3\rangle$

(margins are 1-3 itemized material; division is made before the sign of comparison as well as before the operation sign between the grouped items)

Example 26-20: Keep Modified Expressions Intact

… the result is:

$$\mathrm{V}=\lim_{\Delta x\to 0}\sum_{i=1}^{n}\pi\left[f(x_i)\right]^2\Delta x\,.$$

(margins are 3-5 for displayed to narrative; division is made before the sign of comparison as well as after the termination indicator which ends the second modified expression)

Example 26-21: Divide Before a Baseline Indicator

Use the following expression.

$$\mathrm{B}=\frac{7!}{5!(7-5)!}(x^2)^{7-5}\left(-\frac{y}{2}\right)^5$$

(margins are 3-5 for displayed to narrative; division is made before the sign of comparison as well as before the baseline indicator preceding the grouped expression; the base and its exponent are not divided)

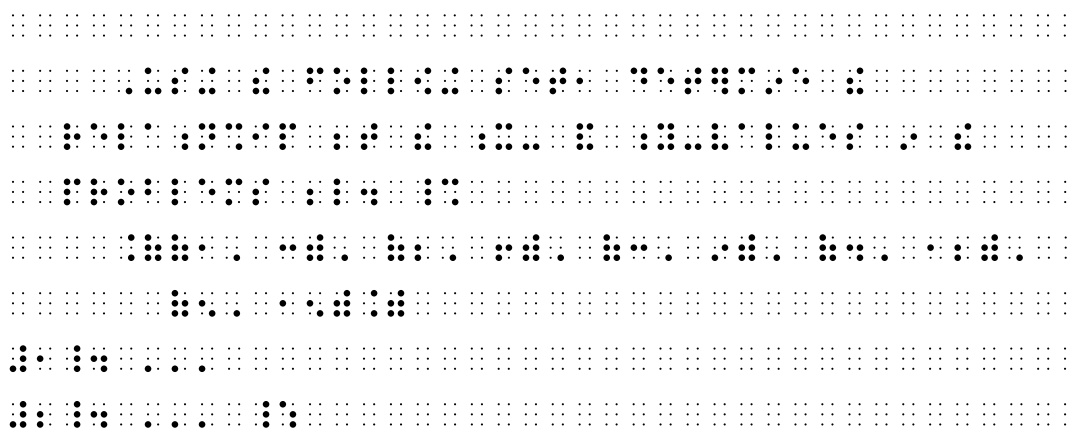
Example 26-22: Divide Between Units in an Enclosed List

Using the following set, determine the relationship between the x- and y-values in the problems below.

$$\{(1, 3), (2, 6), (3, 9), (4, 12), (5, 15)\}$$

1. …

2. ...



(margins are 5-7 for displayed to instructions; division is made between components of an enclosed list)

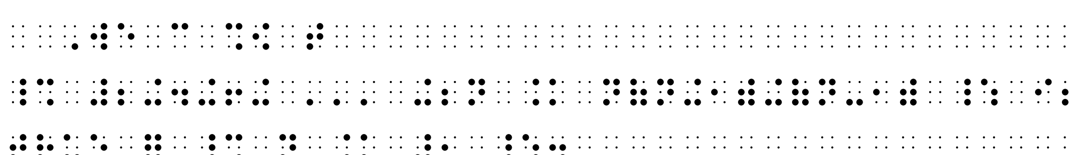
26.3 Embedded Material

26.3.1 **Definition.** When material is not set apart from the surrounding text, it is referred to as *embedded*. This includes material within a displayed passage or expression.

26.3.2 **Mathematical Expression.** A mathematical expression should not be divided between braille lines unless it will not fit within current margins. When a mathematical expression will not fit on one braille line, see **26.2**. See **26.6.4** for format of embedded spatial material.

Example 26-23: Equation Fits on One Line

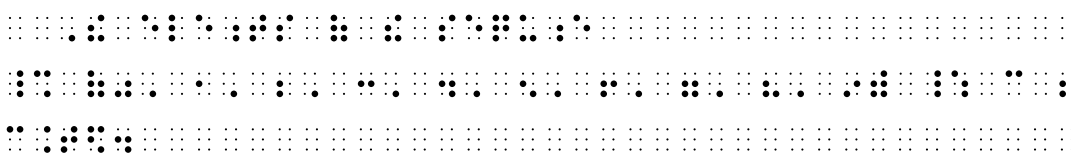
We can show that $2 + 4 + 6 + \ldots + 2n = n(n + 1) + (n - 1)$ is true for $n = 1$.



26.3.3 **Enclosed List.** A sequence of mathematical expressions which occurs in an "enclosed list" should not be divided between braille lines unless it will not fit within current margins. When an enclosed list will not fit on one braille line, utilize as much of the line as possible, then divide the expression after a comma. See **3.5** for the definition of "enclosed list".

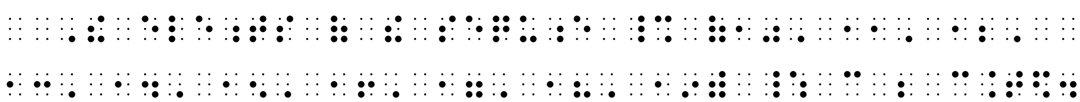
Example 26-24: Enclosed List Fits on One Line

The elements of the sequence (0, 1, 2, 3, 4, 5, 6, 7, 8, 9) can be counted.



Example 26-25: Enclosed List Must be Divided

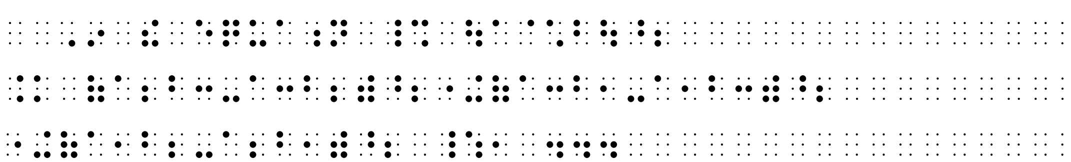
The elements of the sequence (10, 11, 12, 13, 14, 15, 16, 17, 18, 19) can be counted.



26.3.4 **Grouped Expression.** A mathematical expression within grouping symbols should not be divided between braille lines unless it will not fit within current margins. When a grouped expression will not fit on one braille line, see **26.2**.

Example 26-26: Grouped Expression

In the equation $|a \times b|^2 = (a_2b_3 - a_3b_2)^2 + (a_3b_1 - a_1b_3)^2 + (a_1b_2 - a_2b_1)^2, \ldots$



26.3.5 **Abbreviation with a Value.** An abbreviation is not placed on a different braille line from its associated preceding or following numeral or letter, whether in UEB or Nemeth Code.

Example 26-27: Abbreviations in UEB

4 in

3 p.m.

Fig. 6.10

x ft.

Example 26-28: Abbreviations in Nemeth Code

.4 in

2.5 cm

30°C

N 30° W

(abbreviations for degrees Celsius and Fahrenheit are spaced as in print; **10.6.3**)

Example 26-29: Abbreviations in UEB and Nemeth Code

A doctor treats a 190-lb man who, in 2.5 hrs, has ingested four 12-oz beers (48 oz).

26.3.6 **Hyphenated Mathematical Expression.** A hyphenated expression of which one component of the expression is mathematical is not divided between braille lines, whether in UEB or Nemeth Code.

Example 26-30: Hyphenated Expressions in UEB

6-inch

4-sided

x-intercept

Example 26-31: Hyphenated Expression in Nemeth Code

xy-plane

26.3.7 **Mixed Number.** A whole number is not divided from its fractional part in a mixed number.

26.4 Displayed Material

26.4.1 **Definition.** When material is set apart from the body of the text by skipped lines, indentation, or some other means, it is referred to as *displayed*.

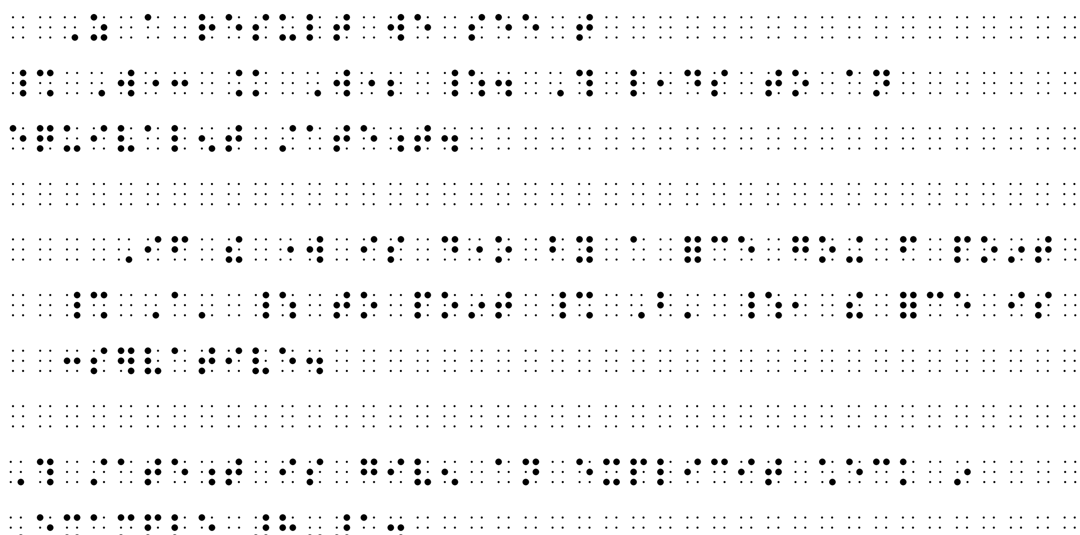
26.4.2 **Displayed Literary Text.** Follow *Braille Formats* rules for transcription of displayed literary text.

Example 26-32: Displayed Literary Text

As a result we see that W3 = W2.This leads to an equivalent statement.

> If the work is done by a force going from point A' to point B', the force is conservative.

This statement is given an explicit check in Example 8-1.



26.4.3 **Displayed Mathematical Expression.** A displayed mathematical expression begins two cells to the right of the runover margin of the material to which it is displayed, whether or not there is an actual runover. No blank lines are inserted unless there is a spatial component. When the displayed math requires more than one braille line, begin its runover two cells to the right of the beginning of the displayed math. See **26.2** for rules regarding division of an expression that requires a runover. See **26.6.5** for displayed spatial material.

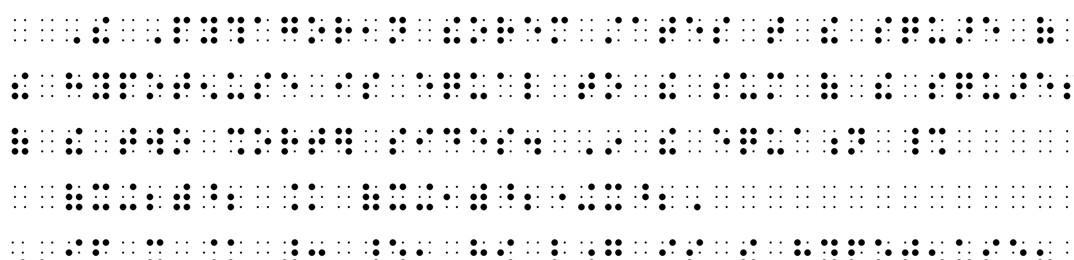
a. **Mathematical Material Displayed to Narrative Text.** When a mathematical expression is displayed to a 3-1 paragraph, the expression begins in cell 3 and is run over in cell 5. (If the displayed material is a nested linked expression, see **26.5.3.b** for definition and format.) When the text following the displayed expression is a continuation of the same paragraph, it continues in the runover cell of that narrative, cell 1. When the text following the displayed expression starts a new paragraph, the new paragraph begins in cell 3.

Example 26-33: Paragraph Interrupted by Displayed Math

The Pythagorean theorem states that the square of the hypotenuse is equal to the sum of the squares of the two shorter sides. In the equation

$$(x + 2)^2 = (x + 1)^2 + x^2,$$

if x = 3, how long is the hypotenuse?

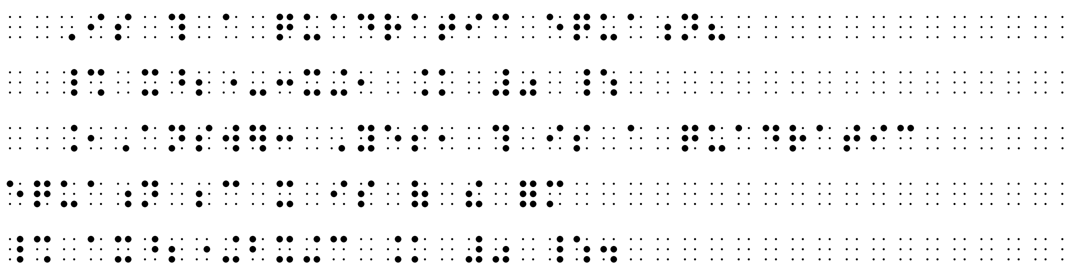


Example 26-34: Math Displayed to 3-1 Narrative Paragraph

Is this a quadratic equation?

$$x^2 - 3x + 1 = 0$$

Answer: Yes, this is a quadratic equation because it is of the form $ax^2 + bx + c = 0$.



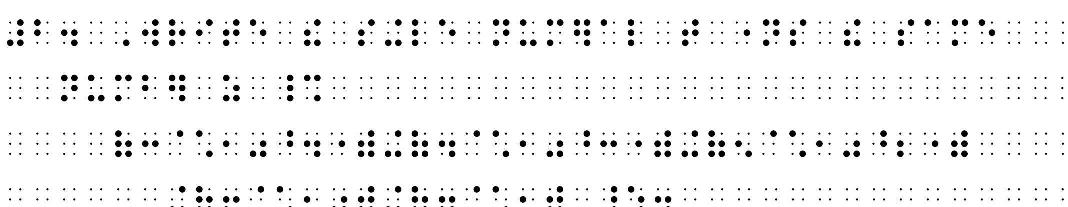
(the displayed equation begins in cell 3 even though the paragraph to which it is displayed has no actual runover)

b. **Mathematical Material Displayed to Itemized Text.** Displayed material begins two cells to the right of the runover cell of the material to which it is displayed. Runovers of displayed material are indented two cells further. Mathematical material displayed to 1-3 itemized material begins in cell 5 and is run over in cell 7. Mathematical material displayed to 1-5 or 3-5 itemized material with subdivisions begins in cell 7 and is run over in cell 9. If the displayed material is a nested linked expression, see **26.5.3.b**.

Example 26-35: Math Displayed to 1-3 Itemized Material (5-7)

2. Write the single numeral that names the same number as

$$(3 \times 10^4) + (4 \times 10^3) + (5 \times 10^2) + (6 \times 10) + (7 \times 1).$$



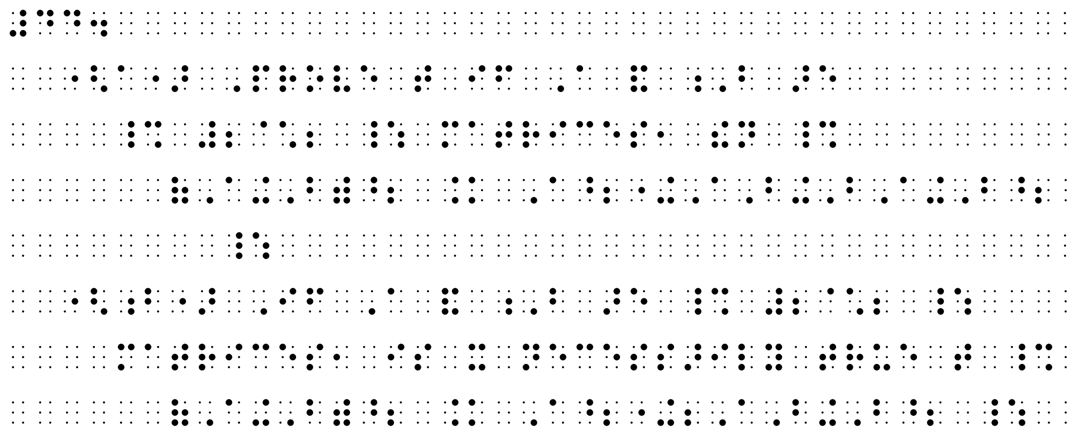
Example 26-36: Math Displayed to a 3-5 Subitem (7-9)

44. (a) Prove that if A and B are 2×2 matrices, then

$$(A + B)^2 = A^2 + AB + BA + B^2$$

(b) If A and B are 2×2 matrices, is it necessarily true that

$$(A + B)^2 = A^2 + 2AB + B^2$$



26.4.4 **Displayed Math Expression with Identifier Printed in the Margin.** If an identifying number or letter is associated with a displayed expression, this number or letter begins in the appropriate cell in accordance with the rules for displayed expressions in **26.3.3**. In print, identifying numbers or letters are sometimes at the right. In braille, numbers or letters are placed uniformly at the left. If identifying numbers or letters occur at the right in print, a transcriber's note concerning the transposition of such numbers or letters is placed at the beginning of each volume in which it occurs. Identifiers may be transcribed in either UEB or Nemeth depending on context. The identifier should be the first symbol on the braille line.

Sample transcriber's note:

> An identifier printed to the right of an expression is transcribed on the left.

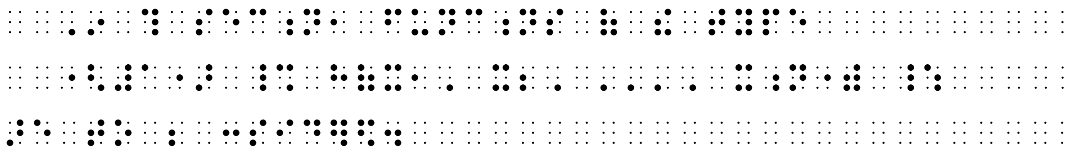
Follow print for location of page references which are associated with an expression.

Example 26-37: Displayed Math Expression with Identifier

In this section, functions of the type

$h(x_1, x_2, \ldots, x_n)$ (1)

are to be considered.

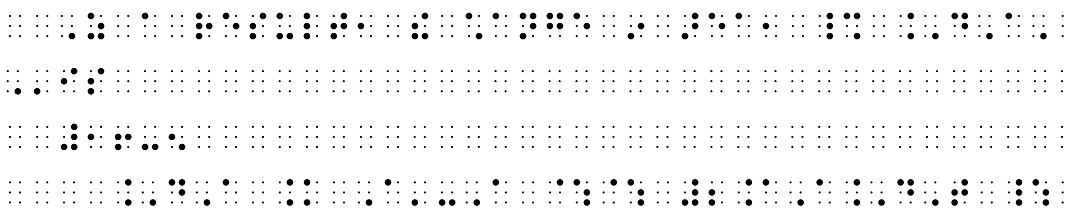


(in print, the identifying number occurs at the right)

Example 26-38: Displayed Math Expression with Identifier

As a result, the change in area, ΔA, is

$\Delta A = A' - A \approx 2\alpha A \Delta T$ 16-5

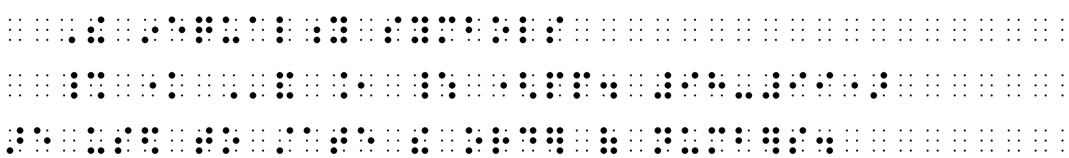


Example 26-39: Expression with Page Number Reference

The inequality symbols

$<$ and $>$ (pp. 98-99)

are used to state the order of numbers.



(in print, the page number references occur at the right-hand margin of the page)

26.4.5 Comments.

a. When comments or remarks appear alternated with math problems, place the comment on the line following the related expression, blocked 4 cells to the right of the runover position for the expression. This format is explained on the transcriber's notes page.

Sample transcriber's note:

> Comments within math problems are placed on the line following the expression, blocked four cells to the right of the runover position of the expression.

See **4.8.9** for directives regarding placement of code switch indicators in this format. See **25.10.1** for transcription of comments referring to more than one print line.

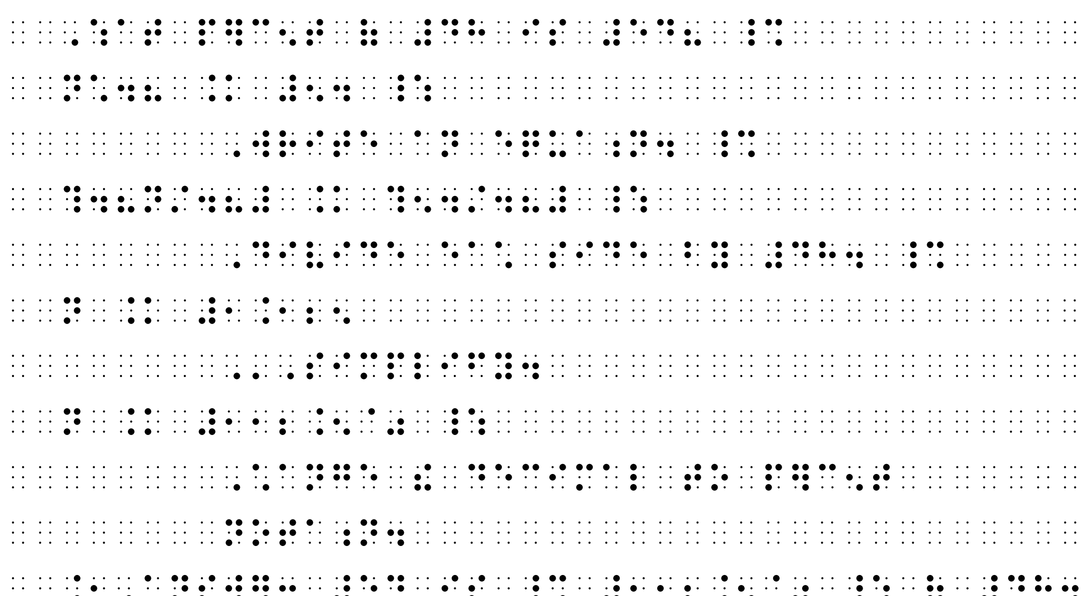
b. When comments accompany a spatial arrangement such as a system of equations, see **25.9**, **25.10.1**.

Example 26-40: Comments and Remarks

What percent of 48 is 54?

$n \cdot 48 = 54$	*Write an equation.*
$\frac{48n}{48} = \frac{54}{48}$	*Divide each side by 48.*
$n = 1.125$	*Simplify.*
$n = 112.5\%$	*Change the decimal to percent notation.*

Answer: 54 is 112.5% of 48.



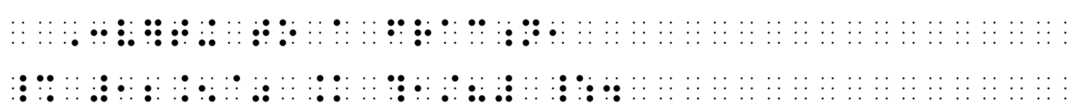
26.5 Linked Expressions

26.5.1 **Definition.** For the purposes of this code, a linked expression is composed of an anchor and one or more links. The component which precedes the first sign of comparison is called the *anchor*. Each *link* begins with a comparison sign. In its simplest form, x = y is a linked expression where "x" is the anchor and "= y" is the link.

26.5.2 **Embedded Linked Expressions.** If the entire embedded linked expression will fit on one braille line, do not divide it. If the entire embedded linked expression will not fit on one braille line, begin the runover line with a link. It is not necessary to divide the expression at every link unless division is made within the anchor or within a link, in which case a division must also be made before each link.

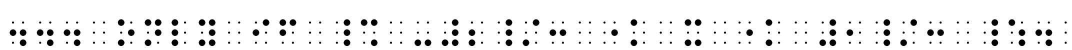
Example 26-41: Embedded Expression with One Link

Converting to a fraction, $12.5\% = \frac{1}{8}$.



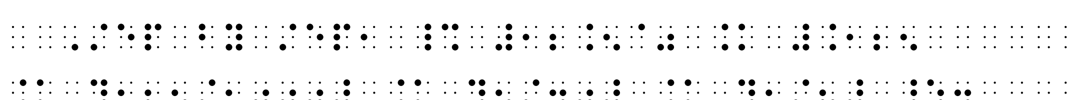
Example 26-42: Embedded Expression with Two Links

… only if $-2/3 < x < 1/3$.



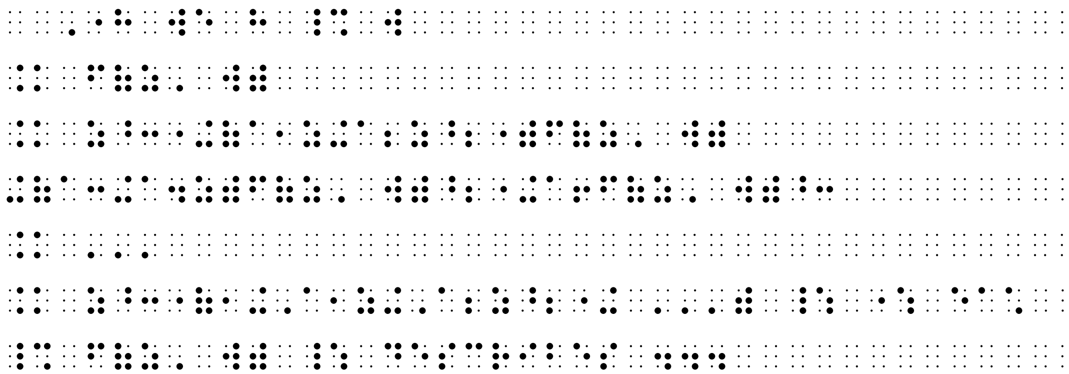
Example 26-43: Embedded Linked Expression with Runover

Step by step, $12.5\% = .125 = \frac{125}{1000} = \frac{5}{40} = \frac{1}{8}$.



Example 26-44: Embedded Expression with Divided Link

Here we have $w = f(z, w) = z^3 + (a_1z+a_2z^2)f(z, w) + (a_3+a_4z)f(z, w)^2 + a_6f(z, w)^3 = \ldots = z^3(1+A_1z+A_2z^2+ \ldots)$ where each $f(z, w)$ describes …



(because the second link must be divided, each link begins a new line)

26.5.3 **Displayed Linked Expressions.** The anchor of a displayed linked expression begins in the appropriate cell for displayed mathematical material as directed in **26.3.3**, that is, two cells to the right of the runover margin of the material to which it is displayed. If there is only one link, or if links are printed without the pattern described in **b**. below, follow the format in **a**. below. If the expression is printed on more than one line with the comparison signs vertically aligned, see **b**.

a. If the entire displayed linked expression will fit on one braille line, do not divide it. If a link will fit entirely on one braille line, do not divide it. If a runover is necessary, begin the runover line two cells to the right of the initial anchor cell. If a division must be made within a link, a division must also be made at the comparison sign that begins that link. It is not necessary to divide the expression at every link unless division is made within the anchor or within a link, in which case a division must be made before each link.

Example 26-45: Linked Expression Displayed to Narrative (3-5)

The product of two monomials is a monomial. For example,

$$\left(3x^2\right)^3 = \left(3x^2\right)\left(3x^2\right)\left(3x^2\right) = \left(9x^4\right)\left(3x^2\right) = 27x^6$$

(there is no division within a link, therefore it is not necessary to divide at every link)

Example 26-46: Linked Expression Displayed to Main Item

17. Evaluate $\lim\limits_{x \to 0} \dfrac{\cos x}{\sin x - 3}$.

$$\lim_{x \to 0} \frac{\cos x}{\sin x - 3} = -\frac{1}{3}$$

Example 26-47: Linked Expression Displayed to a Subitem

1. Follow one of these approaches.

   a. Percentage-to-fraction method:

$$12\frac{1}{2}\% = 12.5\% = .125 = \frac{125}{1000} = \frac{1}{8}$$

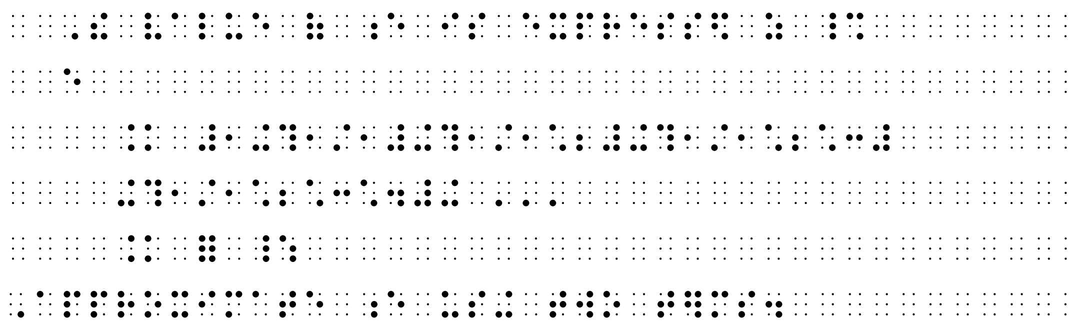
(there is no division within a link, therefore it is not necessary to divide at every link)

Example 26-48: Dividing within a Link

The value of *e* is expressed as

$$e = 1 + \frac{1}{1} + \frac{1}{1 \cdot 2} + \frac{1}{1 \cdot 2 \cdot 3} + \frac{1}{1 \cdot 2 \cdot 3 \cdot 4} + \ldots = ?$$

Approximate *e* using two terms.



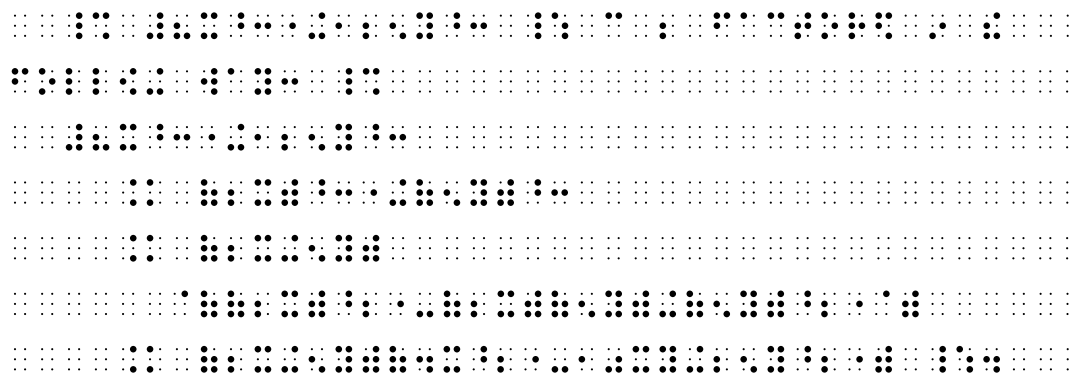
(there is division within a link, therefore there is a division at every link)

b. If a series of linked expressions is printed so that each new line begins with a link and most or all of the comparison signs are vertically aligned, the following format is applied in the braille transcription. Place the anchor in the appropriate cell for displayed mathematical material as directed in **26.3.3**, that is, two cells to the right of the runover margin of the material to which it is displayed. Begin each link on a new line, placing the comparison sign in the runover cell, two cells to the right of the anchor cell. If the anchor or any link requires a runover, indent two cells further—that is, four cells to the right of the initial anchor cell. This displayed format is referred to as "nested links".

Example 26-49: Nested Links Displayed to Narrative

$8x^3 + 125y^3$ can be factored in the following way:

$$\begin{aligned} 8x^3 + 125y^3 &= (2x)^3 + (5y)^3 \\ &= (2x+5y)\left[(2x)^2 - (2x)(5y) + (5y)^2\right] \\ &= (2x+5y)(4x^2 - 10xy + 25y^2). \end{aligned}$$

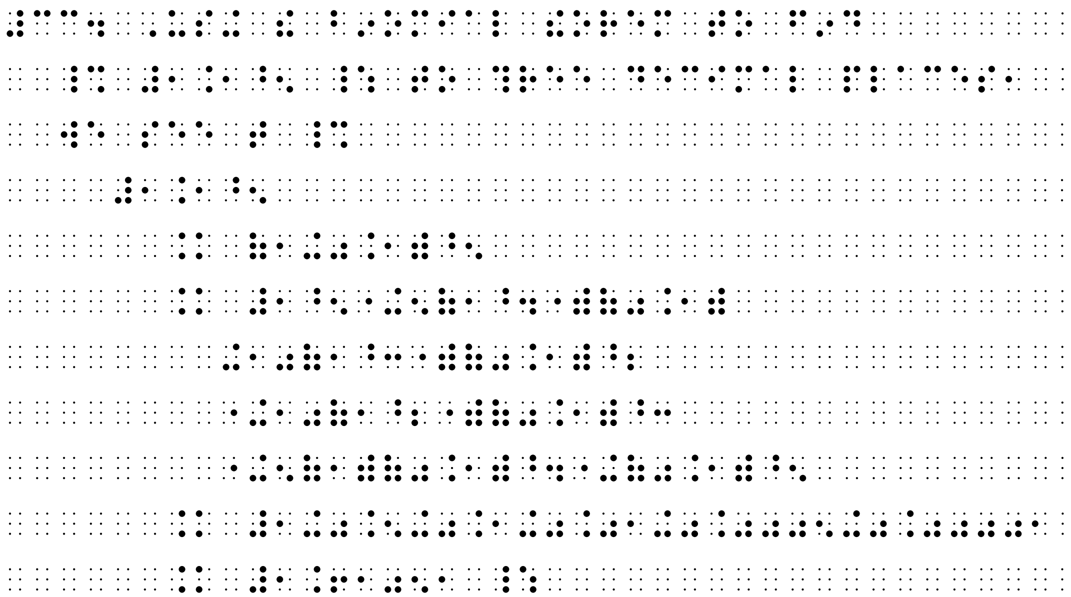


(the margins for the nested displayed material are 3-7, 5-7)

Example 26-50: Nested Links Displayed to Main Item

33. Using the binomial theorem to find 1.1^5 to three decimal places, we see that

$$\begin{aligned} 1.1^5 &= (1 + 0.1)^5 \\ &= 1^5 + 5(1^4)(0.1) + 10(1^3)(0.1)^2 + 10(1^2)(0.1)^3 \\ &\quad + 5(1)(0.1)^4 + (0.1)^5 \\ &= 1 + 0.5 + 0.1 + 0.01 + 0.0005 + 0.00001 \\ &= 1.61051\square \end{aligned}$$



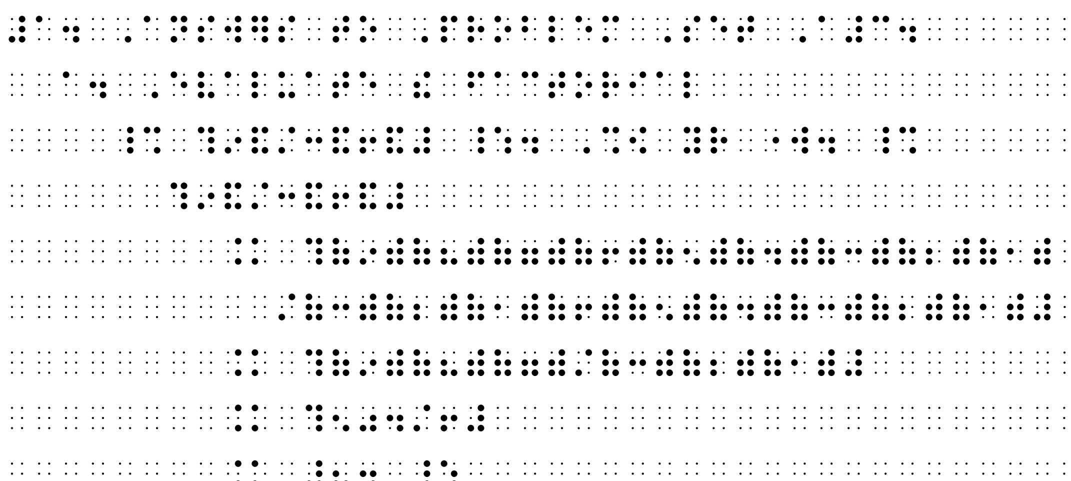
(the margins for the nested displayed material are 5-9, 7-9)

Example 26-51: Nested Links Displayed to Subitem

1. Answers to Problem Set A3.

   a. Evaluate the factorial $\frac{9!}{3!6!}$. Show your work.

$$\begin{aligned}\frac{9!}{3!6!} &= \frac{(9)(8)(7)(6)(5)(4)(3)(2)(1)}{(3)(2)(1)(6)(5)(4)(3)(2)(1)} \\ &= \frac{(9)(8)(7)}{(3)(2)(1)} \\ &= \frac{504}{6} = 84\end{aligned}$$



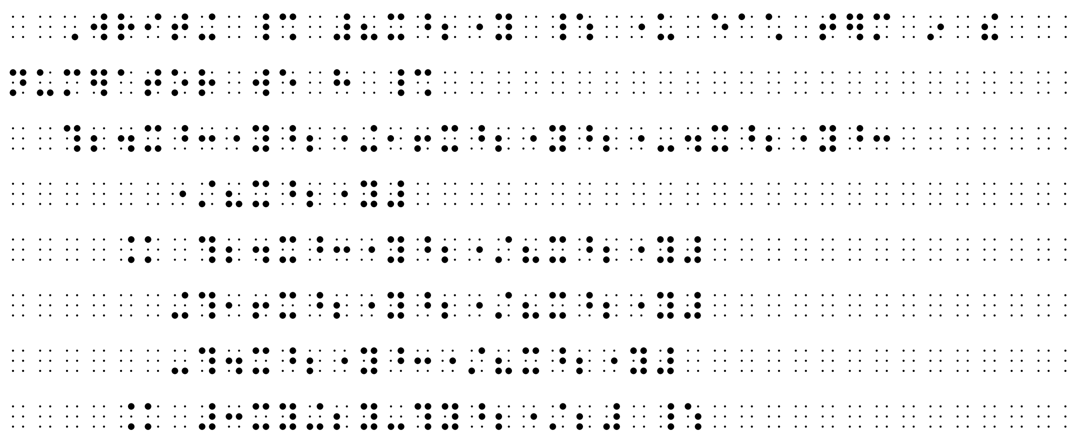
(the margins for the nested displayed material are 7-11, 9-11)

Example 26-52 Nested Links with Runover in Anchor

Writing $8x^2y$ under each term in the numerator we have

$$\frac{24x^3y^2+16x^2y^2-4x^2y^3}{8x^2y}=\frac{24x^3y^2}{8x^2y}+\frac{16x^2y^2}{8x^2y}-\frac{4x^2y^3}{8x^2y}$$

$$=3xy+2y-\frac{y^2}{2}$$



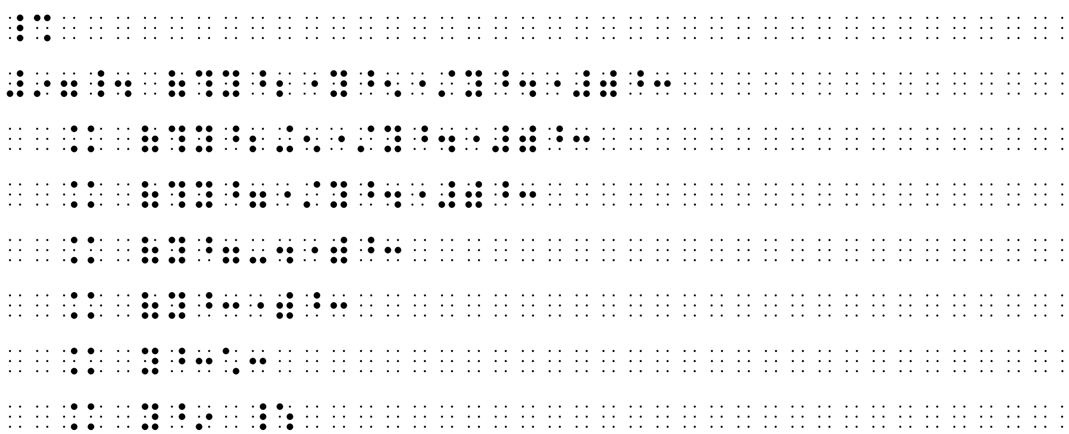
(the margins for the nested displayed material are 3-7, 5-7)

26.5.4 **Itemized Nested Linked Expression with No Narrative.** When nested linked expressions are itemized and immediately follow the identifier, transcribe the anchor on the same line as the identifier. Each link begins a new line, with the comparison symbol two cells to the right of the cell in which the identifier begins. If the anchor or any link requires a runover, indent two cells further—that is, four cells to the right of the cell in which the identifier begins.

Example 26-53: Itemized Nested Linked Expression

97.

$$
\begin{aligned}
\left(\frac{y^2y^5}{y^4}\right)^3 &= \left(\frac{y^{2+5}}{y^4}\right)^3 \\
&= \left(\frac{y^7}{y^4}\right)^3 \\
&= \left(y^{7-4}\right)^3 \\
&= \left(y^3\right)^3 \\
&= y^{3\cdot 3} \\
&= y^9
\end{aligned}
$$



(the margins for the nested material are 1-5, 3-5)

26.6 Spatially Arranged Material

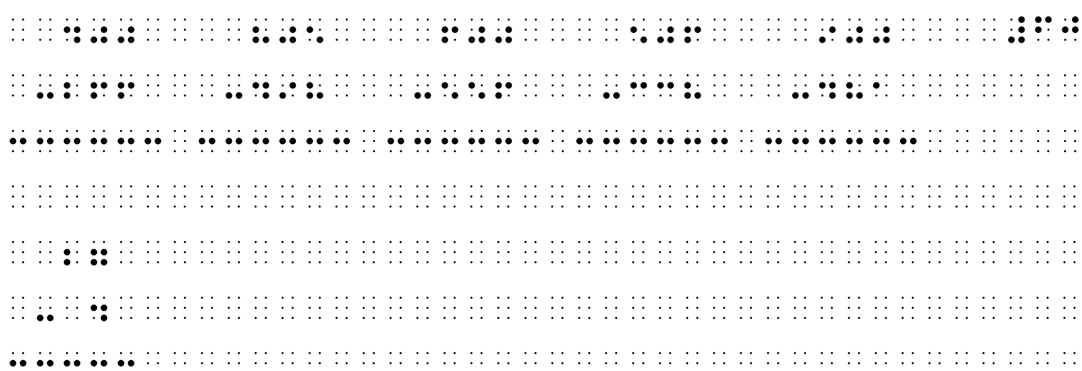
Code switch indicators are not shown in sections **26.6.1**, **26.6.2**, and **26.6.3**. Assume that the examples are within a group of mathematical exercises.

26.6.1 **Layout on the Braille Page.** When the transcription is in the form of a spatial arrangement, a blank line is left both above and below the spatial arrangement even if the spatial arrangement directly precedes or follows the page change indicator or a box line. Transition to a new braille page before beginning or after ending the transcription of a spatial arrangement takes the place of the required blank line. When a running head is used, a line is skipped between the running head and a spatial arrangement. When a spatial arrangement begins on the first or second line of a braille page or ends on the twenty-fourth or the twenty-fifth line of a braille page, there must always be at least three clear columns of cells between the last symbol on any line of the arrangement, including any separation lines, and the first symbol of a page number. If this cannot be achieved, the arrangement begins on line 3 or ends on line 23, respectively. The entire spatial arrangement should be confined to one braille page.

Example 26-54: Page Number Restriction on Line 1

$$\begin{array}{r} 400 \\ -266 \\ \hline \end{array} \qquad \begin{array}{r} 805 \\ -498 \\ \hline \end{array} \qquad \begin{array}{r} 600 \\ -556 \\ \hline \end{array} \qquad \begin{array}{r} 506 \\ -338 \\ \hline \end{array} \qquad \begin{array}{r} 900 \\ -481 \\ \hline \end{array} \qquad \begin{array}{r} 27 \\ -\ \ 4 \\ \hline \end{array}$$

60



(example shows three clear columns of cells between the spatial arrangement and the print page number; the math is a continuation from the previous braille page)

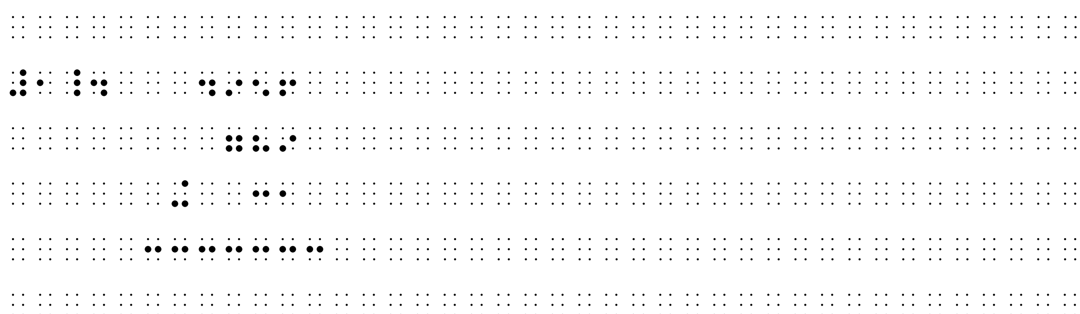
26.6.2 **Identifiers with Spatially Arranged Material.** When a spatial arrangement is identified by a number or a letter, such as in a set of exercises, the identifier is placed on the main line of the arrangement as described in this section. In all cases, there must be one column of blank cells between the identifier and the left-most symbol of the arrangement as a whole, including any separation lines. An identifier cannot stand alone on line 25.

a. **Itemized Addition, Subtraction, and Multiplication Problems.** The identifier is placed on the top line of an addition, subtraction, or multiplication arrangement.

When the regrouping indicator is used in an addition arrangement or when numbers are canceled in a subtraction arrangement, the identifier is placed on the line which contains the first term of the addition arrangement or the minuend of the subtraction arrangement.

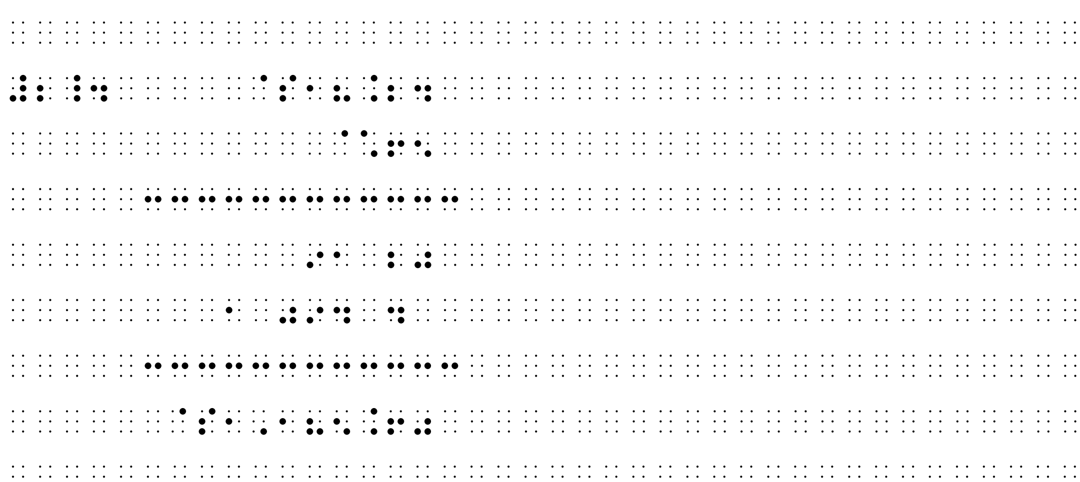
Example 26-55: Identifier with Spatial Addition Problem

1.
$$\begin{array}{r} 4956 \\ 789 \\ +\quad 31 \\ \hline \end{array}$$



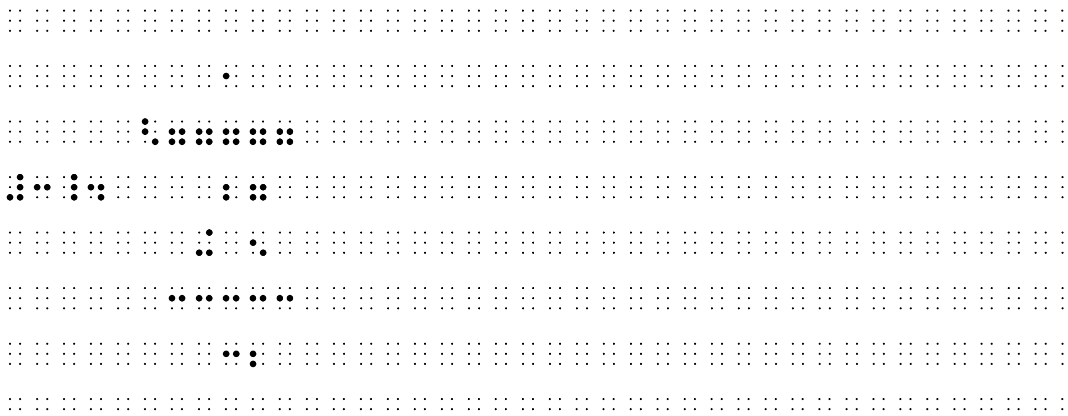
Example 26-56: Identifier with Spatial Multiplication Problem

2.
$$\begin{array}{r} \$18.24 \\ \times 65 \\ \hline 9120 \\ 10944 \\ \hline \$1{,}185.60 \end{array}$$



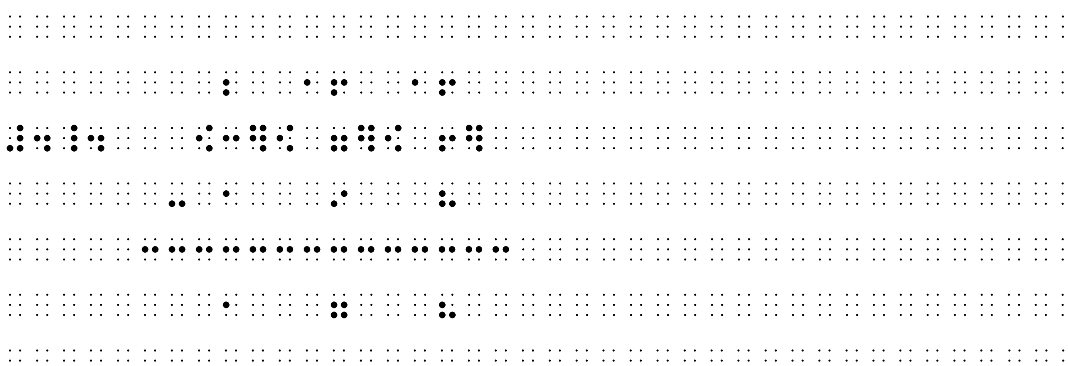
Example 26-57: Identifier with Regrouping Line

3.
$$\begin{array}{r} {\scriptstyle 1} \\ 27 \\ +\ 5 \\ \hline 32 \end{array}$$



Example 26-58: Identifier with Cancellation

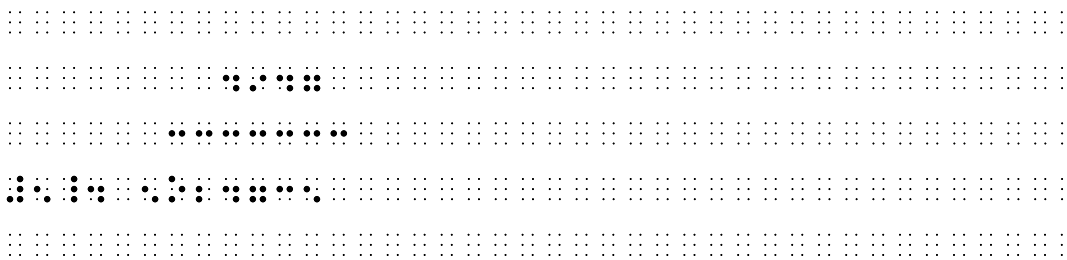
4.
$$\begin{array}{rrrr} & {\scriptstyle 2} & {\scriptstyle 16} & {\scriptstyle 16} \\ & \cancel{3} & \cancel{7} & \cancel{6} \\ - & 1 & 9 & 8 \\ \hline & 1 & 7 & 8 \end{array}$$



b. **Itemized Division Problems and Radical Expressions.** The identifier is placed on the line which contains the dividend in a division arrangement, on the line which contains the radicand in a radical expression and on the line which contains the synthetic dividend in a synthetic division arrangement.

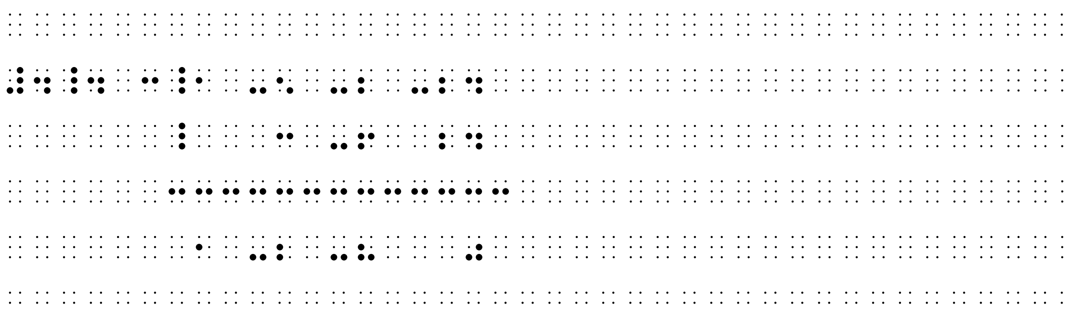
Example 26-59: Identifier with a Division Arrangement

5. $\begin{array}{r} 4947 \\ 5\overline{)24735} \end{array}$



Example 26-60: Synthetic Division Arrangement

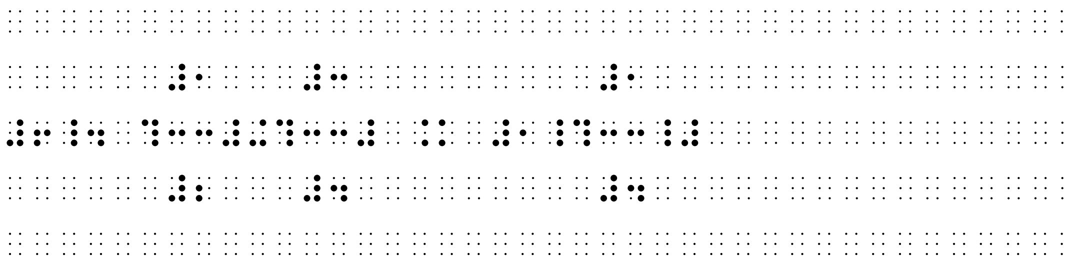
4. $\begin{array}{r|rrrr} 3 & 1 & -5 & -2 & -24 \\ & & 3 & -6 & 24 \\ \hline & 1 & -2 & -8 & 0 \end{array}$



c. **Itemized Spatial Fractions.** In the case of spatial fractions, identifiers are placed on the principal fraction line. In the case of a continued fraction, identifiers are placed on the top fraction line of the arrangement. Centered comparison symbols, symbols of operation, punctuation, and other applicable symbols are placed on the principal fraction line.

Example 26-61: Identifier with Spatial Fractions

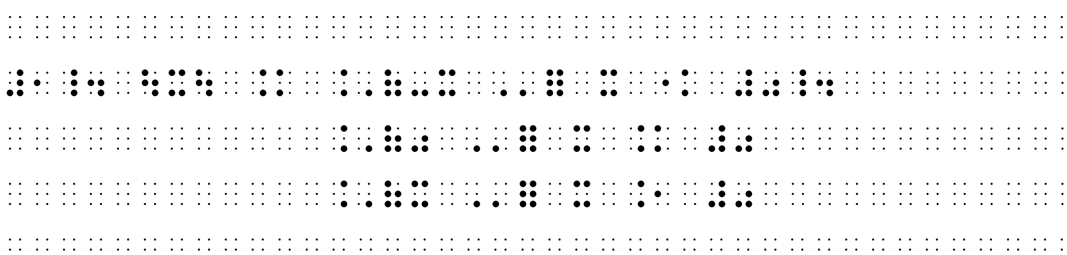
6. $\frac{1}{2}+\frac{3}{4}=1\frac{1}{4}$



d. **Itemized Arrays and Unified Expressions.** In the case of determinants, matrices, and unified expressions (such as piecewise functions or systems of equations), identifiers are transcribed on the top line, regardless of their placement in print, as are comparison symbols, symbols of operation, punctuation, and other applicable symbols.

Example 26-62: Piecewise Function

1. $|x| = \begin{cases} -x & \text{for } x<0 \\ 0 & \text{for } x=0 \\ x & \text{for } x>0. \end{cases}$



(in print, the material outside the enlarged brace is centered and the period follows the last zero)

Example 26-63: Identifier with an Array

7. $D = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = 100$

```
⠼⠶⠸⠲⠀⠠⠙⠀⠨⠅⠀⠠⠳⠁⠂⠂⠀⠁⠂⠆⠠⠳⠀⠨⠅⠀⠼⠂⠴⠴
⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠠⠳⠁⠆⠂⠀⠁⠆⠆⠠⠳
```

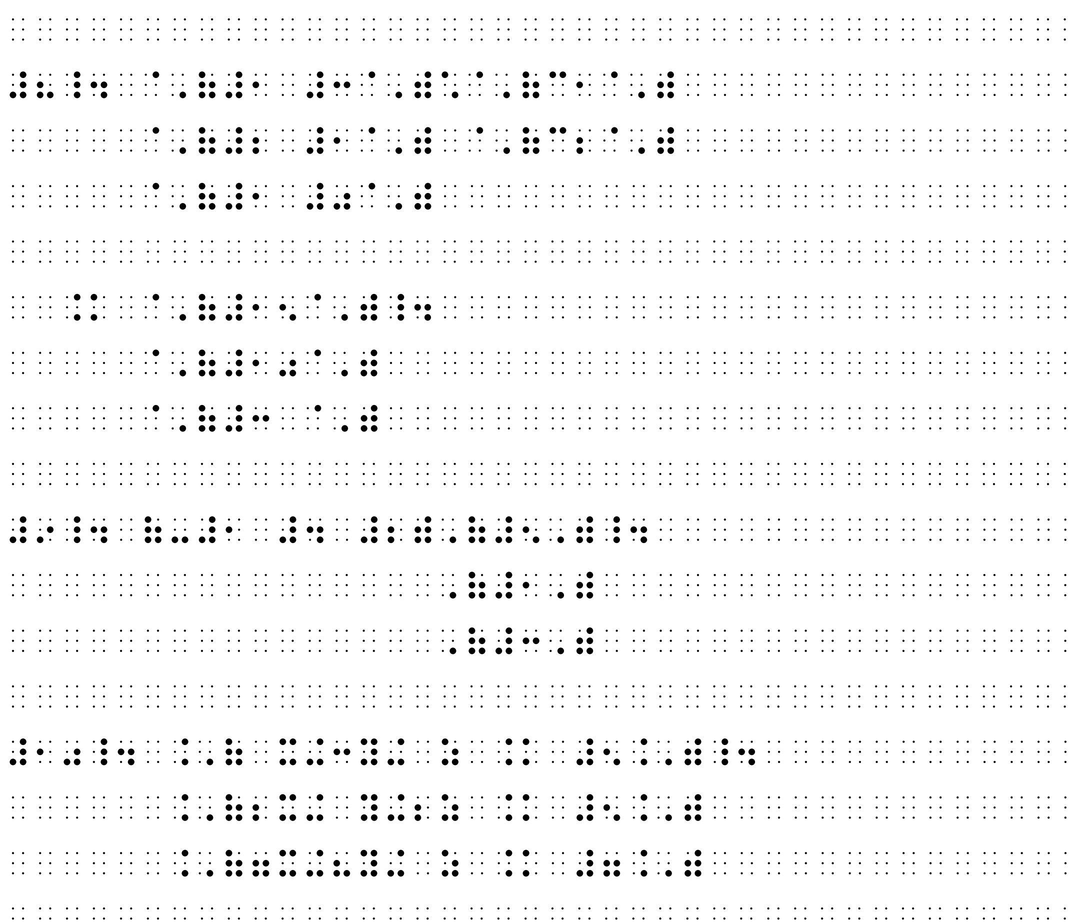
(in print, the material outside the determinant is centered)

Example 26-64: Identifiers and Punctuation with Arrays

8. $\begin{bmatrix} 1 & 3 \\ 2 & 1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 15 \\ 10 \\ 3 \end{bmatrix}.$

9. $\begin{pmatrix} -1 & 4 & 2 \end{pmatrix} \begin{pmatrix} 5 \\ 1 \\ 3 \end{pmatrix}.$

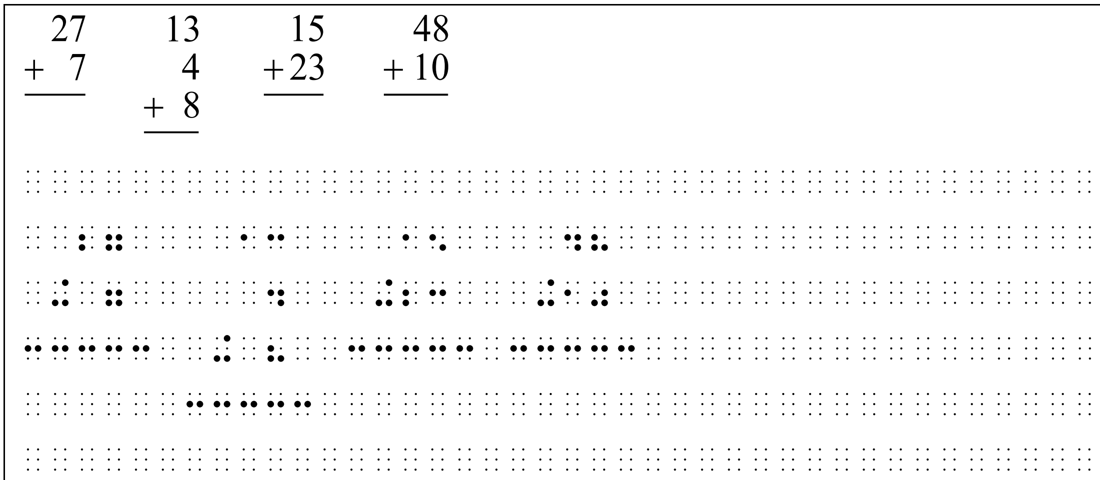
10. $\left\{ \begin{array}{l} x + 3y + z = 5 \\ 2x + y + 2z = 5 \\ 7x + 8y + z = 7 \end{array} \right\}.$



26.6.3 **Side-By-Side Arrangement.** It is preferred that spatial arrangements are placed side-by-side. No symbol in one spatial arrangement may be less than three cells distant from any symbol on any line in, or associated with, a neighboring arrangement other than neighboring ends of separation lines.

a. **Arrangements with No Identifiers.** There must be at least one clear column of blank cells between the end of one separation line and the beginning of the next separation line.

Example 26-65: Side-by-Side Problems



(in print, the examples are side-by-side)

b. **Arrangements with Identifiers.** The first numbered or lettered item begins in cell 1. Subsequent numbered or lettered items may then be transcribed to the right of the preceding spatial arrangement. Identifiers are to be placed on the same braille line across the width of the page. There must be at least two clear columns of blank cells between the end of one separation line and the beginning of the next problem's identifier. As many identifiers and their associated spatial arrangements may occur across the page as can be accommodated. If additional main division numbers remain, begin again in cell 1 after inserting a blank line below the longest spatial arrangement which occurs above.

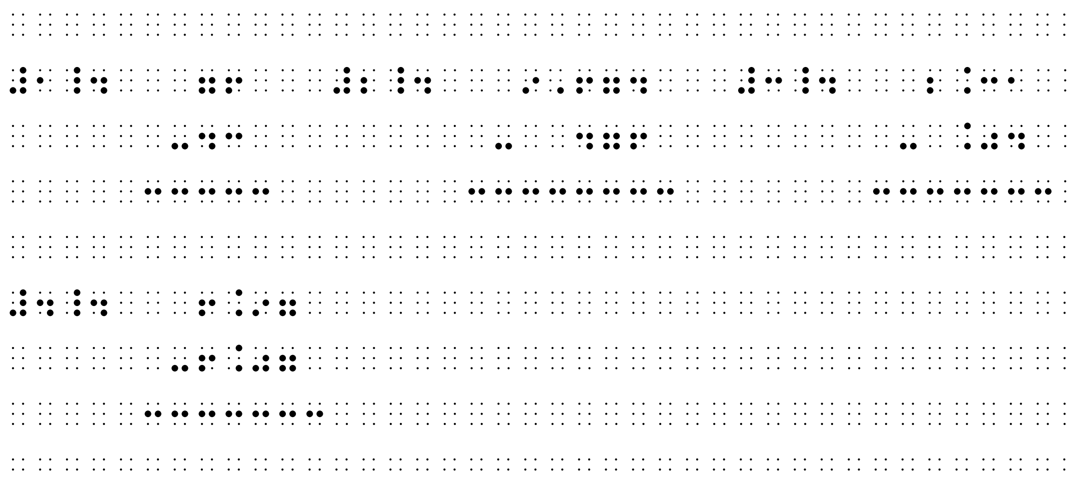
Example 26-66: A Set of Numbered Subtraction Problems

1. $\begin{array}{r} 76 \\ \underline{-43} \end{array}$

2. $\begin{array}{r} 9,674 \\ \underline{-476} \end{array}$

3. $\begin{array}{r} 2.31 \\ \underline{-.04} \end{array}$

4. $\begin{array}{r} 6.97 \\ \underline{-6.07} \end{array}$



(in print, the problems are arranged in two columns, numbered vertically)

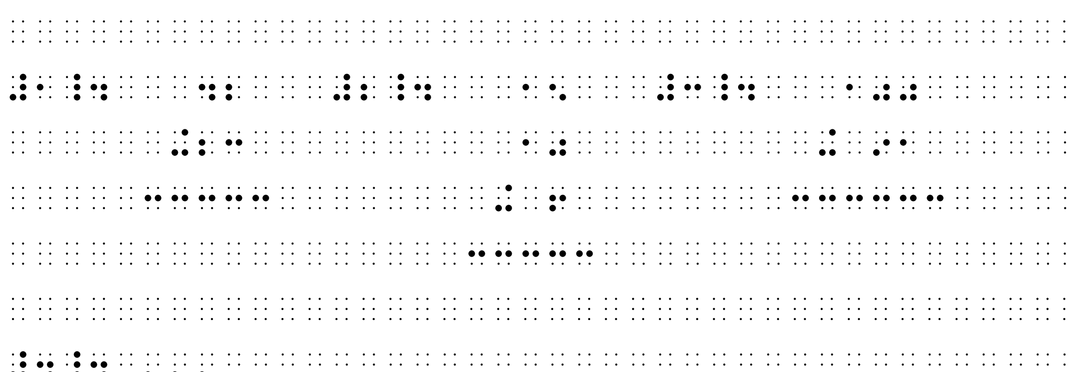
Example 26-67: Side-by-Side Problems with Identifiers

1. $\begin{array}{r} 42 \\ +\underline{23} \end{array}$

2. $\begin{array}{r} 15 \\ 10 \\ \underline{+6} \end{array}$

3. $\begin{array}{r} 100 \\ \underline{+91} \end{array}$

4. …

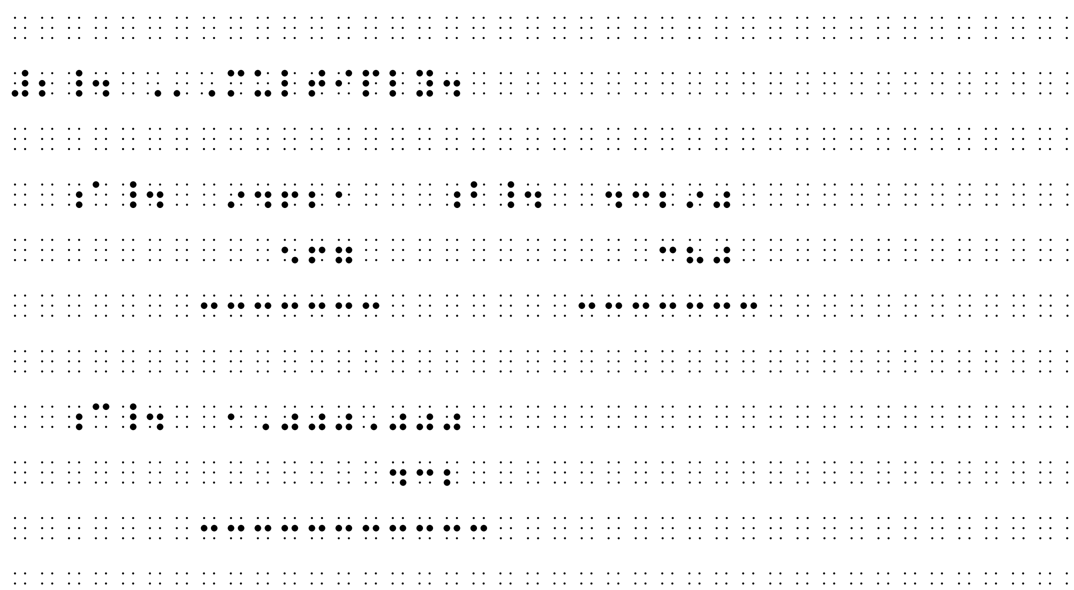


c. **Side-By-Side Arrangement with Subdivisions.** When spatial itemized material contains both main divisions and subdivisions, each main division identifier begins in cell 1. If there is text after the main identifier, the first subdivision begins in cell 3 after the required blank line. If there is no text after the main identifier, the first subdivision may follow on the same braille line. As many additional subdivisions may be transcribed across the line as can be accommodated. If additional subdivisions remain, start in cell 3 after having left a blank line below the longest spatial arrangement which occurs above.

Example 26-68: Lettered Problems Follow Identified Text

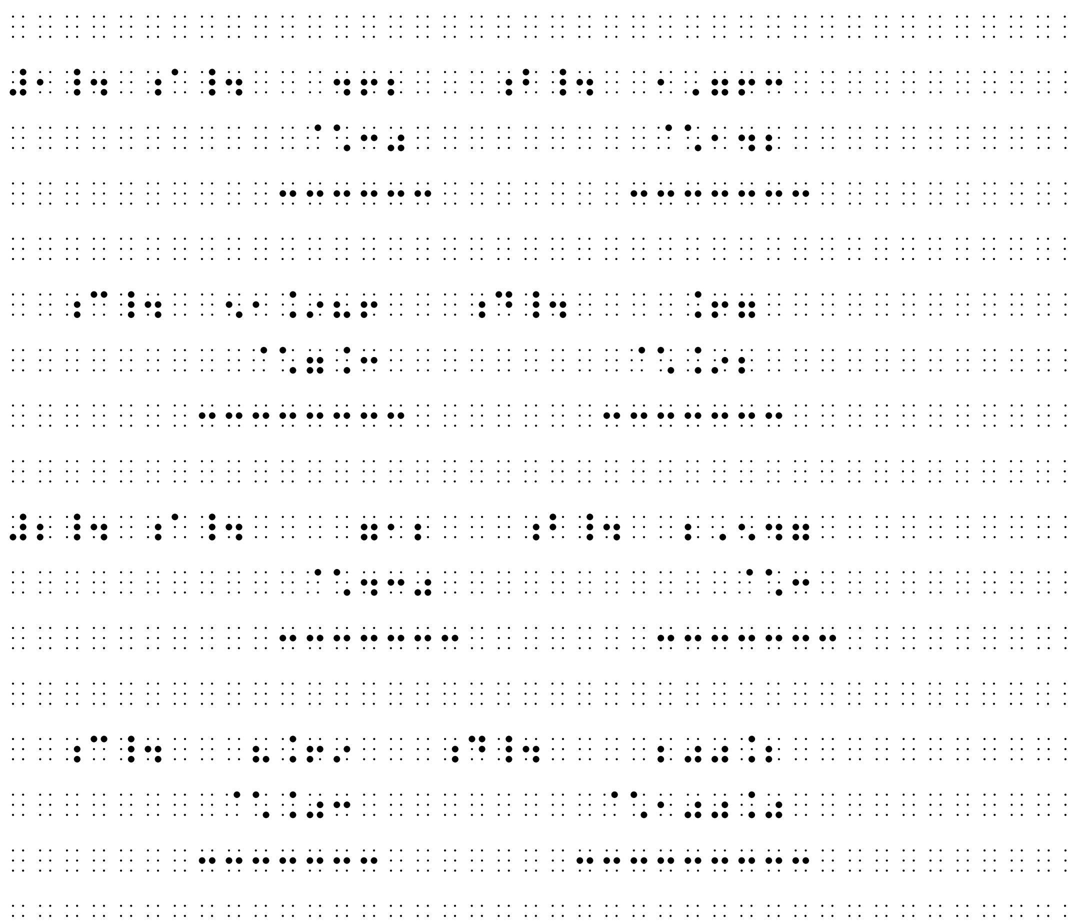
2. Multiply.

a. $\begin{array}{r} 94621 \\ \underline{567} \end{array}$ b. $\begin{array}{r} 43290 \\ \underline{380} \end{array}$ c. $\begin{array}{r} 1{,}000{,}000 \\ \underline{432} \end{array}$



Example 26-69: Lettered Problems Directly Follow Identifier

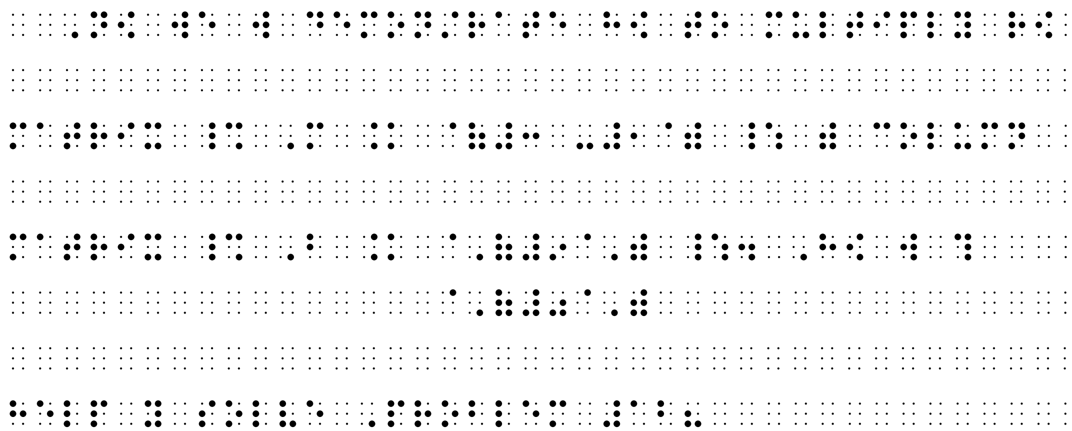
1.	a. $\begin{array}{r} 462 \\ \times 30 \\ \hline \end{array}$	b. $\begin{array}{r} 1,763 \\ \times 142 \\ \hline \end{array}$	c. $\begin{array}{r} 51.986 \\ \times \quad 7.3 \\ \hline \end{array}$	d. $\begin{array}{r} .67 \\ \times .92 \\ \hline \end{array}$
2.	a. $\begin{array}{r} 712 \\ \times 430 \\ \hline \end{array}$	b. $\begin{array}{r} 2,547 \\ \times \quad 3 \\ \hline \end{array}$	c. $\begin{array}{r} 8.69 \\ \times .03 \\ \hline \end{array}$	d. $\begin{array}{r} 200.2 \\ \times 100.0 \\ \hline \end{array}$



26.6.4 **Embedded Spatial Material.** A spatial arrangement is preceded and followed by a blank line. When the spatial material is embedded, a blank line preceding and following the material is still required. When space permits, surrounding text is aligned with the main line of the arrangement. See **26.6.2**.

Example 26-70: Embedded Matrix

Now we will demonstrate how to multiply row matrix $M = \begin{bmatrix} 3 & -1 \end{bmatrix}$ with column matrix $B = \begin{bmatrix} 9 \\ 0 \end{bmatrix}$. How will this help you solve Problem 12?



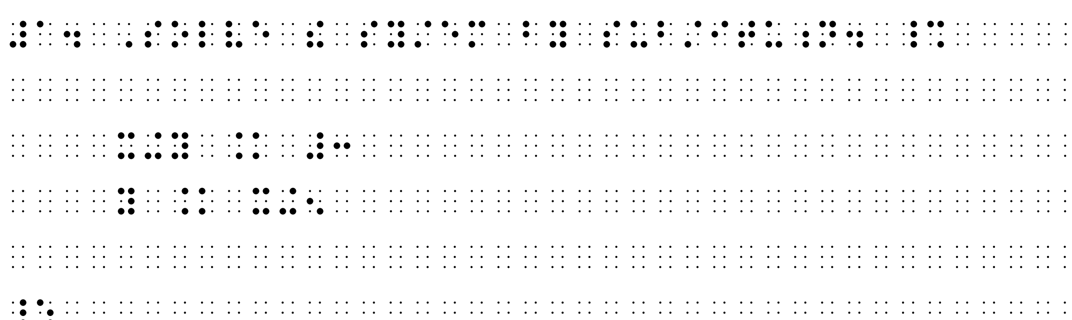
26.6.5 **Displayed Spatial Material.** Margins for displayed spatial material follow the same rules for other displayed mathematical material as described in section **26.3.3**. A blank line must precede and follow spatially arranged material.

Example 26-71: Displayed System of Equations

1. Solve the system by substitution.

$$x + y = 3$$

$$y = x + 5$$



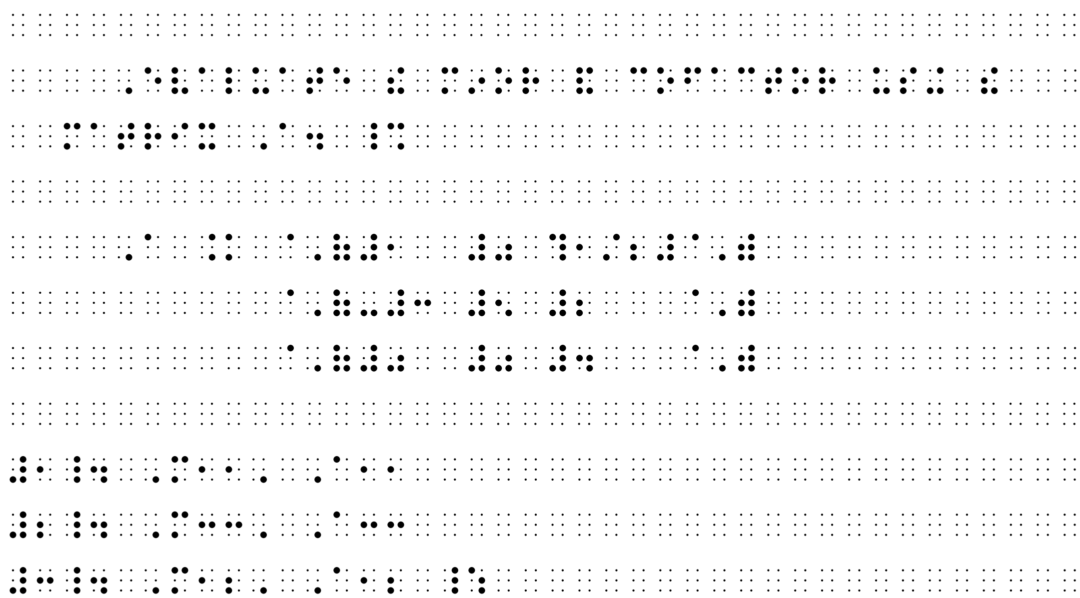
(in 1-3 itemized format, displayed material begins in cell 5)

Example 26-72: Matrix Displayed to Instructions

Evaluate the minor and cofactor using the matrix A.

$$A = \begin{bmatrix} 1 & 0 & \frac{1}{2} \\ -3 & 5 & 2 \\ 0 & 0 & 4 \end{bmatrix}$$

1. M_{11}, A_{11} 2. M_{33}, A_{33} 3. M_{12}, A_{12}



(displayed to 5-3 instructions, the anchor begins in cell 5)

26.7 Proofs and Mathematical Statements

26.7.1 A formal proof or a mathematical statement is usually introduced by a word such as *Theorem, Proposition, Lemma, Definition, Corollary, Axiom*. The following format is recommended for the transcription of such formal proofs and mathematical statements.

a. A line is skipped before the beginning of the formal proof or mathematical statement.

b. Follow print for capitalization and typeform of the paragraph heading. When print shows the paragraph heading as fully capitalized and emphasized, retain the capitalization and ignore the typeform.

c. The introductory word such as *Theorem, Proposition, Lemma, Definition, Corollary, Axiom* begins in cell 3 and the statement following this word is run over to cell 1. Alternatively, the introductory word may be changed to a cell-5 or cell-7 heading.

d. Auxiliary paragraph headings such as *Given, Hypothesis, Prove*, or *Conclusion* follow without a skipped line and begin in cell 3. Follow print for capitalization and typeform. Material associated with these headings follows the heading and is run over to cell 1. When a proof is presented by step number, a line is left blank after the caption *Proof* and the format in **26.7.2** below is followed.

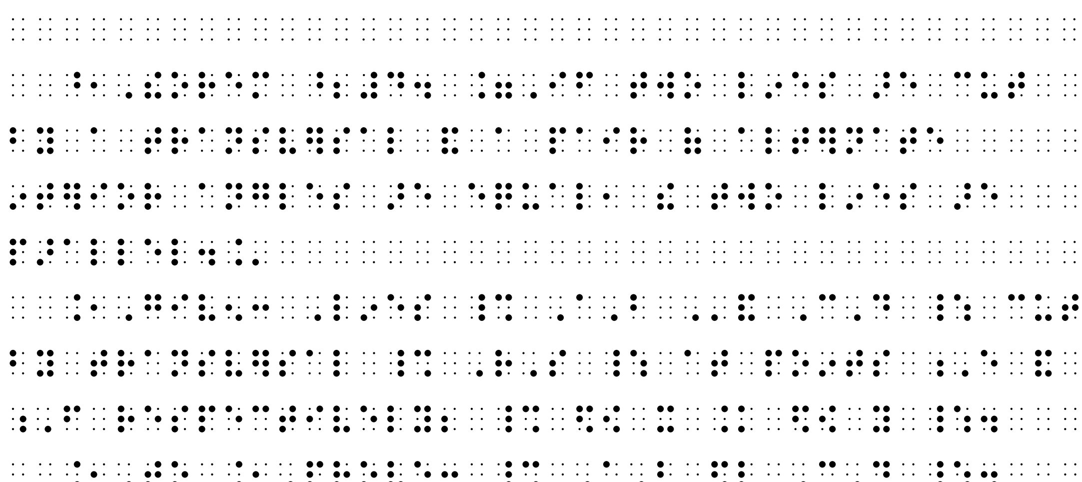
e. When the formal proof or mathematical statement is complete, a blank line is inserted before continuing with the text.

Example 26-73: Theorem with Auxiliary Captions

Theorem 4. *If two lines are cut by a transversal and a pair of alternate interior angles are equal, the two lines are parallel.*

Given: Lines AB and CD cut by transversal RS at points E and F respectively; $\angle x = \angle y$.

To Prove: AB ∥ CD.



26.7.2 **Two-Column Proof.** When a formal proof is presented by step number and is divided into two columns headed "Statement" and "Reason", the following technique is used:

a. All step numbers begin in cell 1.

b. The step number is followed by the letter "S" or "R" according as the transcription to follow is from the Statement or the Reason column. The transcription begins on the same line as the step number and runovers, if necessary, begin in cell 3. If a caption other than "Statement" or "Reason" is used, a suitable letter is used for "S" or "R".

c. Each step from the Statements column is immediately followed by the corresponding step from the Reasons column.

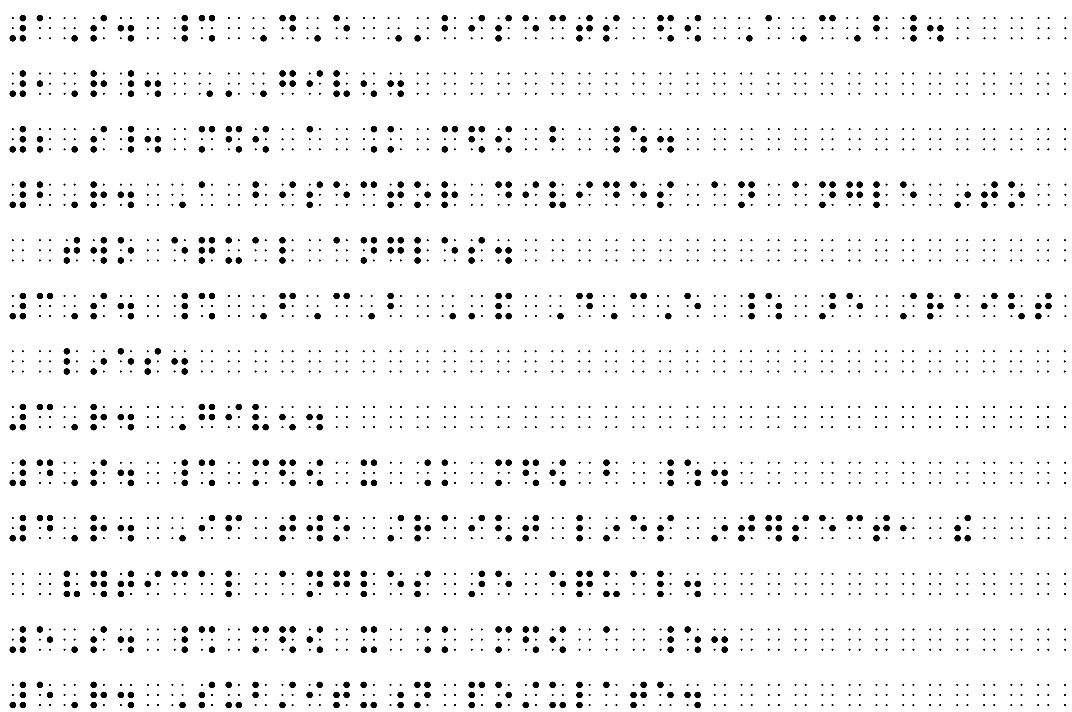
d. A transcriber’s note is included to call attention to this braille format and to specify the meaning of "S", "R", or other letters which may have been used. This note is placed at the beginning of each braille volume in which this technique is used.

Sample transcriber's note:

> Proofs printed with steps in columns headed "Statements" and "Reasons" are transcribed with an S or R immediately following the step number to show the column in which the step appears. Steps from the Statements column are immediately followed by the corresponding step from the Reasons column.

Example 26-74: Two-Column Proof

Statements	*Reasons*
1. DE bisects ∠ACB.	1. Given.
2. m∠a = m∠b.	2. A bisector divides an angle into two equal angles.
3. FCB and DCE are straight lines.	3. Given.
4. m∠x= m∠b.	4. If two straight lines intersect, the vertical angles are equal.
5. m∠x = m∠a.	5. Substitution postulate.

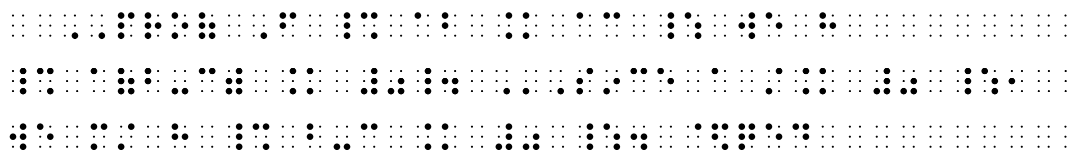


(in print, there is a Statement column and a Reason column)

26.7.3 **End of Proof.** When a shape is employed in print to indicate the end of a proof (or the end of an explanation), the symbol (⠈⠫⠟⠑⠛) is used in braille, regardless of the shape used in print. It follows the last line of text after one blank cell. See **Rule 23, Miscellaneous Signs and Symbols**. The symbol is created using the UEB transcriber-defined shape indicator and may be used in either Nemeth Code or UEB without switching.

Example 26-75: End of Proof Icon

PROOF From $ab = ac$ we have $a(b\text{-}c) = 0$. Since $a \neq 0$, we must have $b - c = 0$. ∎



26.8 Transcriber's Notes

Transcriber's notes are written outside of the Nemeth Code switch indicators, following UEB rules and *Braille Formats* guidelines. The note itself can contain mathematical material, in which case code switching occurs within the note, but Nemeth Code must be terminated before the closing transcriber's note indicator. When a transcriber's note refers to material within box lines, and all of the material within the box is in Nemeth Code, the note may be transcribed above the top box line in order to allow the insertion of switch indicators in the box lines. See example **4-68**.

26.9 Keying Technique

26.9.1 When space does not permit the inclusion of labels, headings, entries, etc., in a determinant, matrix, figure, or table as shown in print, one or more of the labels, headings, entries, etc., may be keyed. Each keyed item consists of two or three cells made up of letters, numbers, or a combination of letters and numbers, according to *Braille Formats* guidelines. See *Braille Formats* for additional keying guidance, including their placement in a transcriber's note.

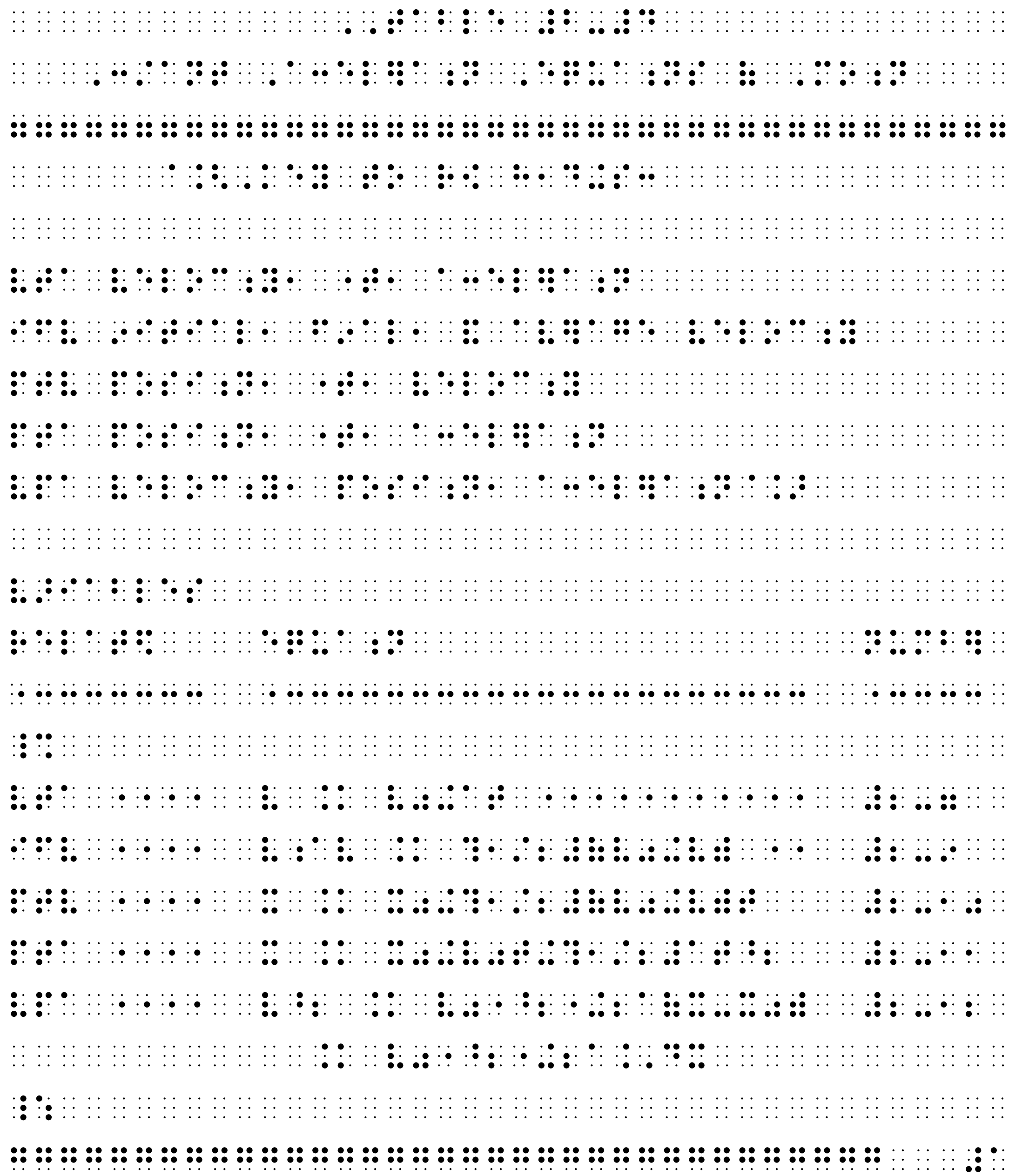
26.9.2 **Numeric Key.** A numeric key consists of one or two digits transcribed in the upper part of the braille cell. This number is preceded by the numeric indicator and is not punctuated. Numbered keys are listed in numeric order. Numeric keys may be placed either inside or outside the Nemeth switches, based on the material to which they apply.

26.9.3 **Alphabetic Key.** An alphabetic key consists of two or three lower-case English letters. Letter keys are generally listed in alphabetic order, but may, if appropriate, be listed in order of appearance. An alphabetic key is not used if any of the print entries in the table are made up of two or three lower case letters. In that case, a numeric key is used. At least one cell of a two- or three-letter key must contain a dot 3 or dot 6.

Example 26-76: Keyed Row Headings

TABLE 2-4 Constant Acceleration Equations of Motion

Variables Related	Equation	Number
velocity, time, acceleration	$v = v_o + at$	2-7
initial, final, and average velocity	$v_{av} = \frac{1}{2}\left(v_0 + v\right)$	2-9
position, time, velocity	$x = x_0 + \frac{1}{2}(v_0 + v)t$	2-10
position, time, acceleration	$x = x_0 + v_0 t + \frac{1}{2} a t^2$	2-11
velocity, position, acceleration	$v^2 = v_0{}^2 + 2a(x - x_0) = v_0{}^2 + 2a\Delta x$	2-12



(alphabetic key items are listed in order of appearance)

26.10 Stem-and-Leaf Plots

A stem-and-leaf plot is a method of showing data distribution. It is a specialized table that is transcribed using the rules for Tables and Related Columns in *Braille Formats: Principles of Print to Braille Transcription*. A stem-and-leaf plot is transcribed in Nemeth Code, even if the numbers or letters are unmodified.

A stem-and-leaf plot is made up of columns and rows which usually include a heading. The data may be shown as numbers or letters. A key is almost always provided and is transcribed beginning in cell 1 preceding the stem-and-leaf plot, which also begins in cell 1.

The symbols used in a stem-and-leaf plot do not need to be included on a Special Symbols page unless the text is an elementary math book below the 4th grade.

26.10.1 **Format.** The stem-and-leaf plot resembles a horizontal bar graph, and therefore, it is important to retain the shape. A vertical line (⠸) separates the column headings and extends to the end of the plot. One blank cell precedes and follows the vertical line. The data on the left (stem) is right justified to the vertical line and the data on the right (leaf) is left justified to the vertical line.

a. Avoid running over lines if possible. If it is necessary to runover the line, indent the line two cells to the right. *Exception:* in back-to-back plots that have a runover in the left column, the indention is two cells to the left.

b. A runover of leaves shown in print should be ignored. Use the full width of the braille column before beginning an indented row.

c. The next stem-and-leaf row entry begins on the line after the runover.

d. **Note:** Do not follow the *Braille Formats* rules for blank spaces that occur across the width of a column in tables. A blank space in a stem-and-leaf plot column is left blank and may occur in either the stem or leaf.

e. Every effort should be made to be consistent throughout a transcription.

26.10.2 **Key.** If a key is provided in print, it must precede the stem-and-leaf plot, even though it may appear in a different location in print. The portion of the key that replicates an entry in the plot is transcribed without the numeric indicator or English letter indicator and including vertical lines as it would appear within the plot. The value assigned to the key is transcribed using the numeric indicator or English letter indicator as required by the Nemeth Code. The key is formatted in cell 1 with any runover in cell 3.

When two keys are shown in print for back-to-back plots, the left column key is transcribed first, followed by the right column key. Each is transcribed beginning in cell 1 with any runover in cell 3.

26.10.3 **Numerical Data.** When the data is represented by numbers in the body of the plot:

a. omit the numeric indicator in the body of the plot,

b. braille single digit entries unspaced,

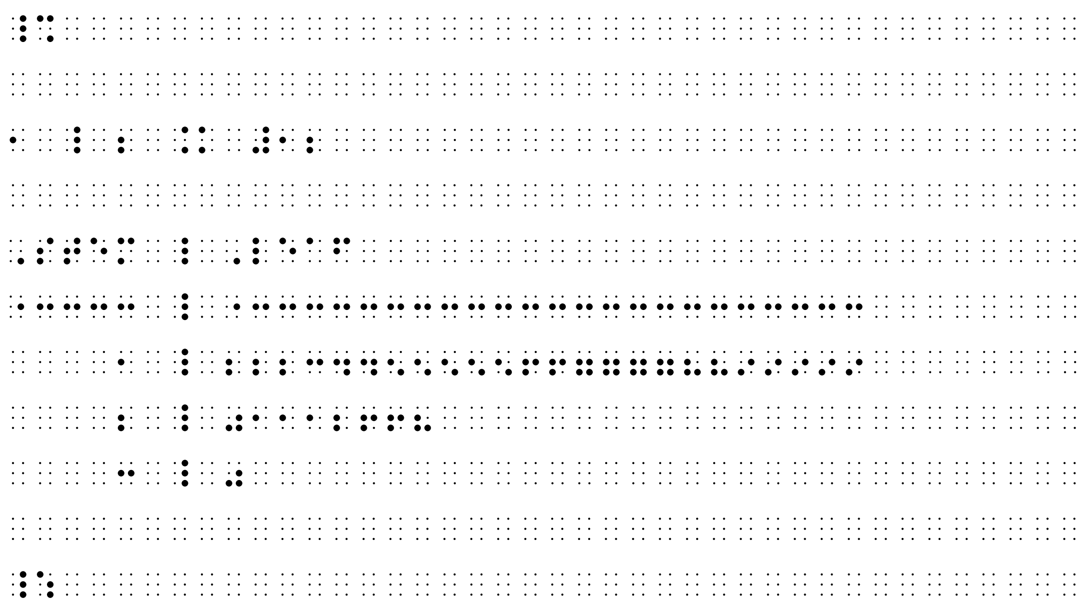
c. entries consisting of groups of two or more digits require one blank cell between entries (see example **26-79**),

d. omit a comma or other punctuation shown between units of data.

Example 26-77: Basic Stem-and-Leaf Plot

Stem	Leaf
1	2 2 2 3 4 4 5 5 5 5 5 6 6 7
	7 7 7 8 8 9 9 9 9 9
2	0 1 1 1 2 6 6 8
3	0

1 | 2 = 12



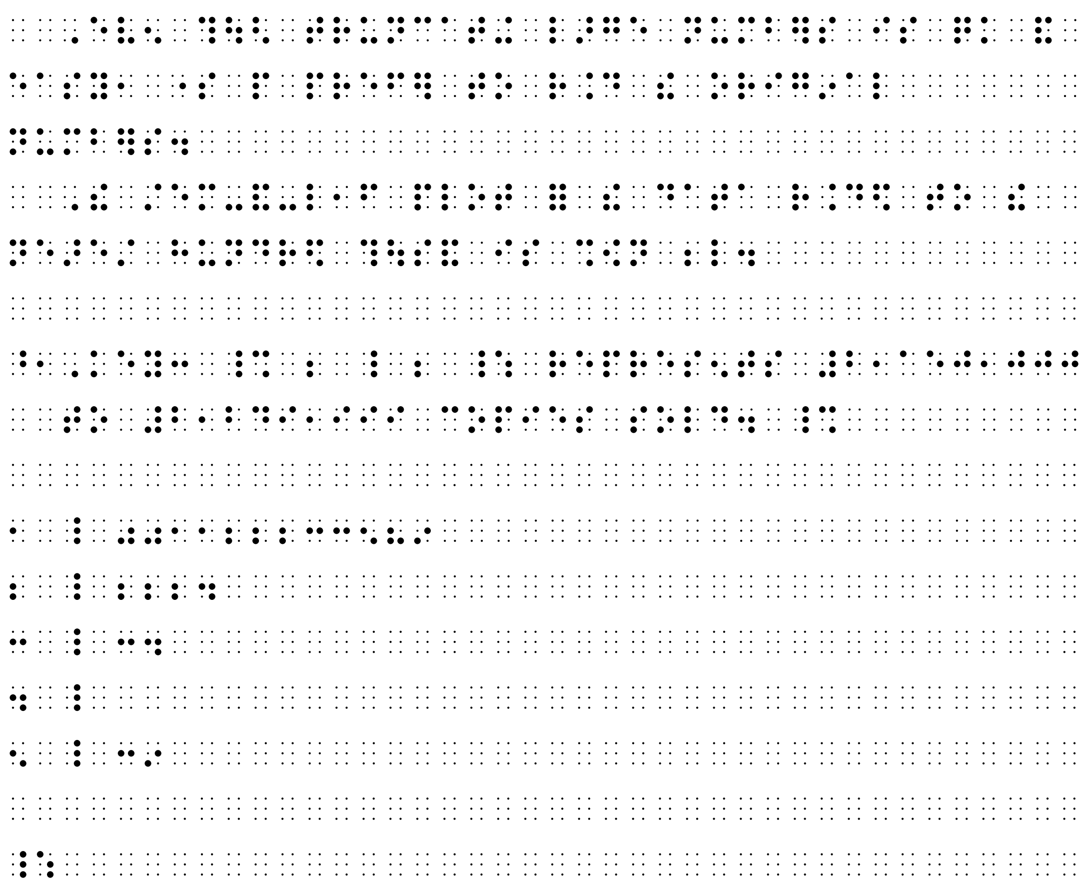
Example 26-78: Basic Stem-and-Leaf Plot

Even though truncating large numbers is quick and easy, some people prefer to round the original numbers.

The stem-and-leaf plot for the data rounded to the nearest hundred thousand is shown below.

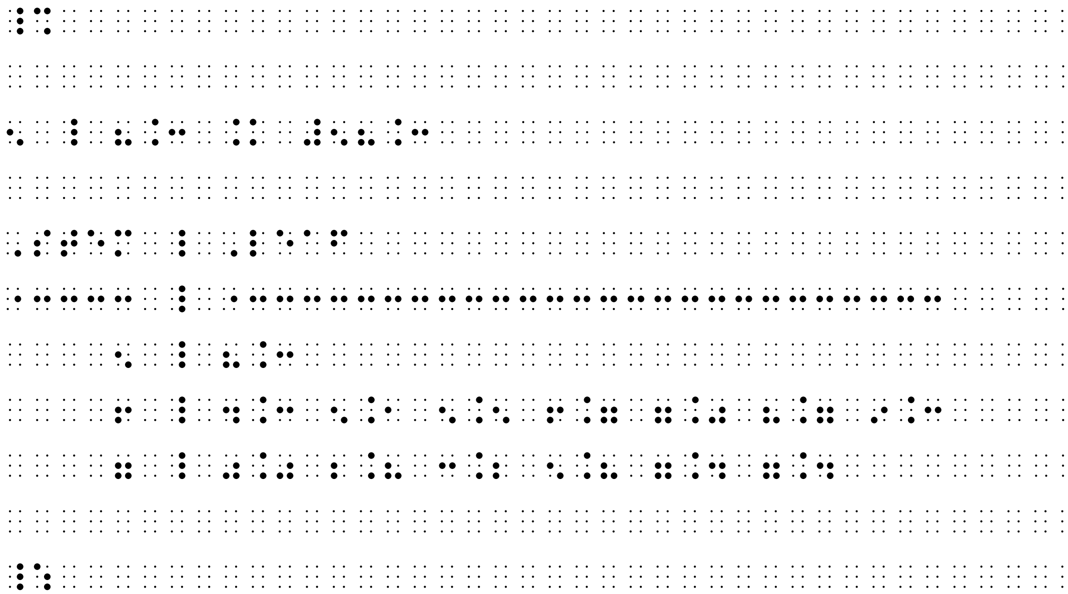
1	0 0 1 1 2 2 2 3 3 5 8 9
2	2 2 2 4
3	3 4
4	
5	3 9

Key: 2 |2 represents 2,150,000 to 2,249,999 copies sold.



Example 26-79: Numbers with Decimals

Stem	Leaf	
5	8.3	
6	4.3 5.1 5.5 6.7 7.0 8.7 9.3	
7	0.0 2.8 3.2 5.8 7.4 7.4	*5 | 8.3 = 58.3*

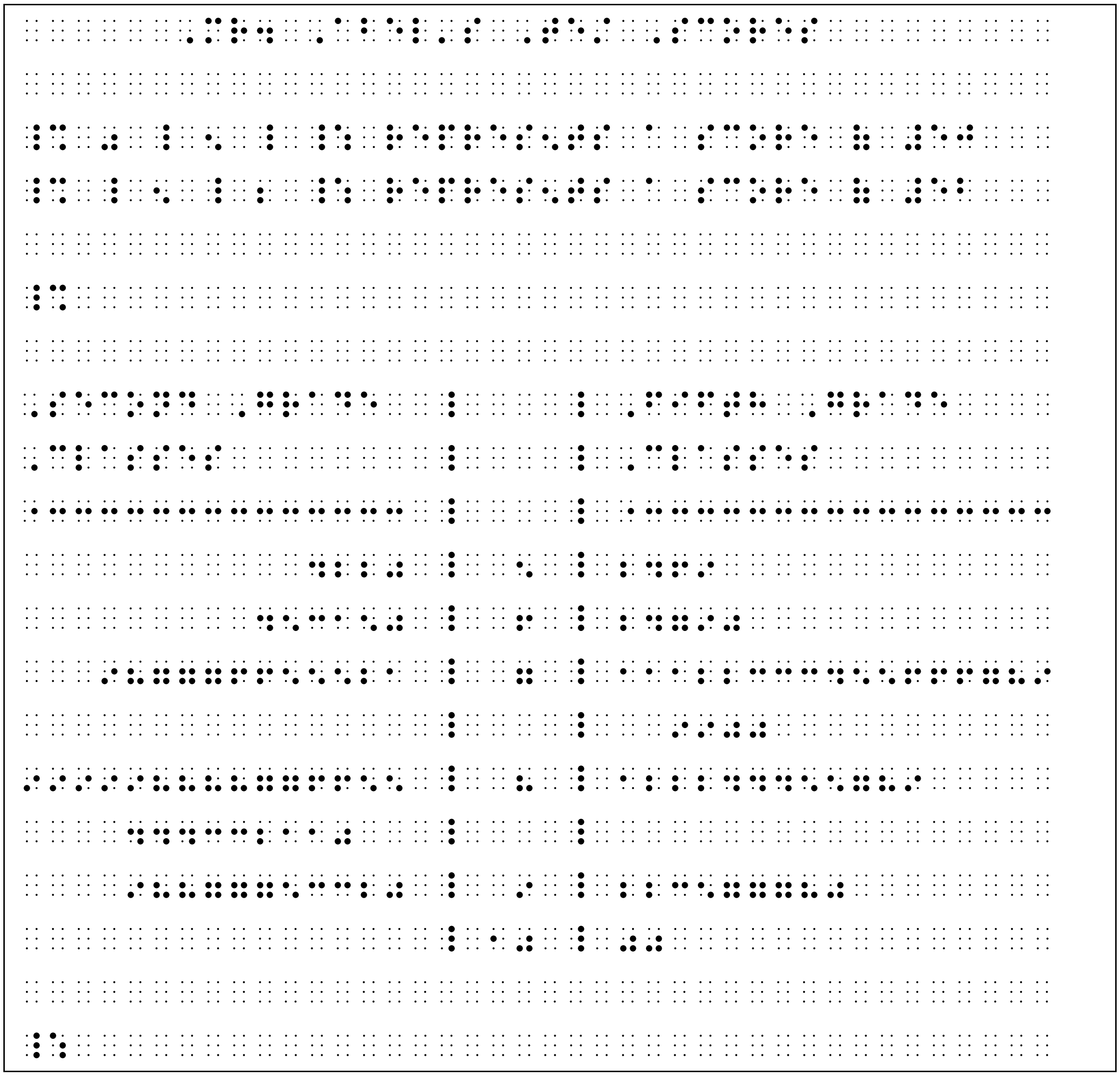


Example 26-80: Back-to-Back Stem-and-Leaf Plot

Mr. Abel's Test Scores

Second Grade Classes		Fifth Grade Classes
4220	5	2469
453150	6	24790
987776655521	7	111223334556667899900
999998888776655444332110	8	122244455789
98877753320	9	223577780
	10	00

0 | 5 | represents a score of 50 | 5 | 2 represents a score of 52



26.10.4 **Alphabetic Data.** When the data is represented by letters in the body of the plot:

a. single cap each capital letter,

b. omit the English letter indicator in the body of the plot,

c. single letters are transcribed unspaced,

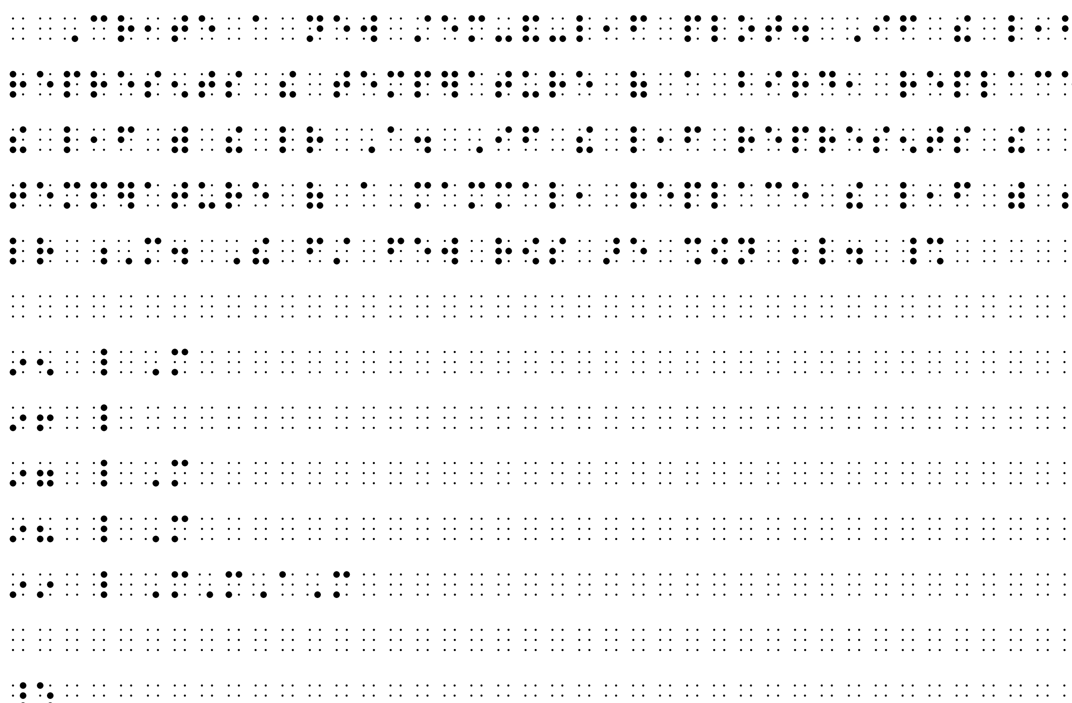
d. entries consisting of groups of two or more letters require one blank cell between each entry,

e. omit a comma or other punctuation shown between units of data.

Example 26-81: Letters in the Leaf Column

Create a new stem-and-leaf plot. If the leaf represents the temperature of a bird, replace the leaf with the letter A. If the leaf represents the temperature of a mammal, replace the leaf with the letter M. The first few rows are shown below.

95	M
96	
97	M
98	M
99	M M A M



Appendix A
Code Changes

(If your transcription has any of these items, be sure to check the rules.)

Rule 1: Changed from Orientation to Basic Principles

Rule 2: Nemeth Braille Indicators

- Changed from Braille Indicators
- Changed from Carried Number Indicator to two different Regrouping Indicators
    - Broadened to include Addition, Subtraction, Multiplication, and Division
- Added Opening and Closing Switch Indicators to the list of Indicators
    - Added Single-word Switch Indicator construction and use
- Changed the use of Opening and Closing Italic Typeform Indicators

Rule 3: Numeric Signs and Symbols

- Terminology change: "European" changed from "Continental"
- Follow print for use of American and European decimal and comma
- Transcribe Roman numerals in UEB unless they are in mathematical context.
- The option to omit the numeric indicator in tables whose entries consist entirely of numerals is restricted, to be used only as a space-saving device in order to replicate print layout.

Rule 4: Nemeth Switches

- New rule describing the use of Nemeth Code switches. Mainly approved in the Guidance published in 2018, with some additions.

- No transcriber's note indicator in Nemeth Code. TN's are transcribed in UEB.
- Abbreviated function names are mathematical (transcribe in Nemeth Code).
- A switch to Nemeth is required to transcribe a chemical element symbol.

Rule 5 Capitalization: No changes

Previous Rule IX: Contractions

- Omitted. No contractions or short-form words in Nemeth with UEB.

Rule 6: Alphabets

- Removed all references to short-form letter combinations
- Single letter chemical elements: no ELI, transcribed in Nemeth Code

Rule 7: Typeforms

- Created a symbol for Barred Type
- Use of single italic and bold; open and closing italic and bold

Rule 8: Punctuation

- Colon spacing
- Construction of the "closing single" quotation mark changed to match the UEB symbol.
- Names of quotation marks changed to reflect UEB terminology
- Directional and angle quotation marks added to the list.

Rule 9: Reference signs

- Spacing clarification
- Icons in either UEB or Nemeth

Rule 10: Abbreviations

- Non mathematical series of numbers/letters is transcribed in UEB

- Single letter abbreviation is preceded by a letter indicator, even when a period applies to the abbreviation
- No space is left before or after a multiplication dot when the second abbreviation has no related value

Rule 11: Omissions

- When a dash or underscore denotes a missing number within a larger number, the general omission symbol may be used.

Rule 12 Cancellation:

- A spatial arrangement is required when replacement is shown. If no replacement is present, the arrangement may be transcribed in either linear or spatial format.

Rule 13: Fractions

- No change

Rule 14: Superscripts and Subscripts

- Multipurpose indicator is inserted between a right superscript/subscript and a left superscript/subscript that follows unspaced in print.

Rule 15: Modifiers

- Renamed concave and convex arcs to correctly represent the shapes – see section 15.11 and symbols list in rule 17

Rule 16: Radicals

- No change

Rule 17: Shapes

- End of proof icon

Rule 18: Function Names and Abbreviations

- Terminology change: "unabbreviated function name" is now "function name"

Rule 19: Signs of Grouping

- Name of Transcriber's Grouping Symbols changed to only one Transcriber-inserted Grouping Symbol. No Nemeth symbols for transcriber's notes.

Rule 20: Operation Signs

- Division symbols and the multiplication asterisk added to the symbols list

Rule 21: Signs of Comparison

- The arc symbols are removed from the list

Rule 22: Arrows

- New items: Solid left and right arrowheads (print images)
- Nemeth arrows are used as a sign of comparison, modification, or other technical applications, not for pointing

Rule 23: Miscellaneous Signs

- Added End-of-Proof icon
- Added monetary units euro, franc, naira, and won

Rule 24: Multipurpose Indicator

- Multipurpose indicator used between a right superscript/subscript and a left superscript/subscript that follows unspaced in print

Rule 25: Spatial Arrangements

- Symbols for carried numbers (regrouping numbers) above and below the arrangement
- Augmented matrix guidance
- Systems of equations are spatial
- Symbols for vertical and diagonal ellipses
- Transcription of matrices with blank entries and single dot entries

Rule 26: Format

- TNs are written outside the Nemeth switches

- Author's comments to math equations or expressions are on the following braille line 4 cells to the right of the runover position of the expression
- Special margins now referred to as nested linked expressions
- Terminology: "mathematical statement" replaces "labeled statement"
- New guidelines for the division of math expressions
- Subitems printed across the page must each begin on a new line in braille
- Instructions may be located on preceding page when necessary
- An identifier cannot stand alone on line 25
- Format for formal proofs and mathematical statements
- End of proof icon
- New sections:
    - Embedded linked expressions
    - Displayed linked expressions
    - Itemized nested linked expressions
- Mathematical material displayed to narrative
- Spacing of abbreviations format includes its use in the UEB text
- Keying follows *Braille Formats* guidelines and may be composed of three cells.

Appendix B
Placement of Code Switch Indicators

When mathematical content occurs anywhere in a UEB transcription, the non-technical notation follows the rules of *Unified English Braille* and the technical notation follows the rules of the *Nemeth Braille Code*. Readers will assume they are reading UEB unless signaled otherwise by the use of a UEB code switch indicator, in this case, the opening Nemeth Code indicator. Between the opening Nemeth Code indicator and the Nemeth Code terminator are Nemeth symbols, following Nemeth rules. UEB symbols are not used within the Nemeth Code switch indicators. The objective within a paragraph is to keep the switch indicators on the same line as the mathematics to which they apply. Displayed material, spatial arrangements, and tables have other considerations. There are guidelines to follow when the switch indicators do not fall neatly on a line or on a page. The opening Nemeth Code indicator must be on the same braille page as the beginning of the expression to which it applies. The Nemeth Code terminator must be on the same braille page as the end of the expression to which it applies.

General Principle in Narrative Context

The opening Nemeth Code indicator is followed by a space (unless it ends a line). The Nemeth Code terminator is preceded by a space (unless it begins a line). These spaces do not represent spaces in print. Within a paragraph, a switch indicator should not stand alone on a line if there is room for it to fall on the line with the math expression to which it applies. If two or more math expressions are transcribed between the same code switch indicators, the line may wrap at the space between the expressions even if the entire Nemeth portion could fit on one line.

Switch Indicators with Itemized Material

Identifiers are transcribed according to the rules for the code in use at the time. All identifiers in a section do not need to be transcribed in the same code. To ensure that all identifiers begin in the same cell, the opening Nemeth Code indicator is placed at the end of the line of text that precedes the itemized material unless there is no room on that line, in which case it is placed in the runover position. This placement may be applied to a transcriber's note that precedes the identified material, or to headings (centered heading excepted). A code switch indicator does not take the place of the blank line that may be required preceding the itemized material.

Switch Indicators with Displayed Mathematical Material

When displayed mathematical material is both preceded and followed by UEB text, the expression and its two switch indicators may be placed all together on one line if they will fit within current margins. If more than one line is required for the expression, the opening Nemeth Code indicator is placed at the end of the text line preceding the displayed material and the Nemeth Code terminator is placed at the completion of the displayed expression. If either indicator will not fit on the current line, it is placed on the following line in the runover position.

Switch Indicators with Spatial Arrangements

Code switch indicators are placed outside of the spatial material in order not to interfere with alignment. The blank lines required before and after the arrangement are part of the spatial problem and so must be inside the Nemeth switches. The opening Nemeth Code indicator and the Nemeth Code terminator do not take the place of that required blank line. If there is not room for the opening Nemeth Code indicator at the end of the line with the preceding text, it is placed on the next line in cell 1. The required blank line is on the line following the opening switch. To close Nemeth after a spatial problem, first insert the required blank line, then place the Nemeth Code terminator in cell 1 by itself on the following line.

Switch Indicators May Stand Alone on a Line

If a math expression will fit on one line but there is not room for one or both of the switch indicators, one or both switch indicators may stand alone on a line. Keeping the mathematical expression intact on one line is the priority.

Switch Indicators and Punctuation

Punctuation that relates to the main text is placed outside of the switch indicators when the surrounding text is in UEB. There is no space between the terminator and the following punctuation. To avoid excessive code switching between mathematical items, punctuation which belongs to the sentence structure may be transcribed inside the switches. Paired punctuation (parentheses, brackets, braces, quotation marks) are transcribed inside the code switches when they enclose isolated technical material.

Switch Indicators after a Heading

An opening Nemeth Code indicator may be placed at the end of a cell-5 or cell-7 heading, or in the runover position for the heading. An opening Nemeth Code indicator cannot be placed at the end of a centered heading.

Switch Indicators and Transcriber's Notes

Transcriber's note indicators are UEB symbols and therefore must be transcribed outside of the Nemeth switches. When the note itself contains mathematical material, code switching occurs within the note. Nemeth Code must be terminated before the closing transcriber's note indicator is transcribed. No space comes between the two indicators. When itemized or spatial mathematical material follows the transcriber's note, the opening Nemeth Code indicator may be placed following the closing transcriber's note indicator only if it fits on the same line.

Switch Indicators at Page Turns

When Nemeth is in effect, Nemeth Code is not terminated by transition to a new braille page or across a page turn line.

Switch Indicators with Boxed Material

Box lines may be transcribed in either code. When literary content is followed by boxed mathematical material, if all of the material in the box is in Nemeth, the opening Nemeth Code indicator may be placed at the beginning of the top box line, followed by a blank space and the Nemeth Code terminator may be placed at the end of the bottom box line, preceded by a space.

Switch Indicators with Remarks

When short narrative comments alternate with math problems, switch indicators are used in order to transcribe the remarks in contracted braille. When switching into or out of Nemeth before a change of margins, the switch indicators are placed after the last item of the line rather than at the beginning of the next line to maintain clarity in the indented margin pattern.

When a remark applies to a spatial arrangement, the comment begins on the top line of the arrangement, to the right of the enlarged grouping symbol (if present) or a transcriber-inserted grouping symbol. When the remark contains narrative, code switching is not applied even though the words may not be part of a mathematical expression. The comment is considered part of the math text. The words are uncontracted; the single-word switch indicator is not used.

Switch Indicators with Instructions

If instructions end with an expression in Nemeth and the subsequent math problem starts with Nemeth, Nemeth Code remains in effect between the end of the instructions and the start of the problem.

Switch Indicators with Tables

When mathematical data occur in the table, code switching decisions depend upon the content of the entire table and the spacing restrictions encountered on the braille page. Each table must be individually assessed in order to determine the clearest representation in braille.

Column headings which contain words are transcribed in UEB. There may be items within the column headings that require switching to Nemeth.

It is best if a minimum of code switching is encountered within the body of the table. When a mixture of narrative entries and mathematical data occur in a table, a switch to Nemeth may be applied only where needed. However, a table may be more clearly presented by transcribing it entirely in Nemeth, even when some entries do not require a switch. For example, when only one column requires Nemeth, the opening switch and the Nemeth terminator must be applied to each entry. However, spacing restrictions may make that option unmanageable. Instead, it may be better to transcribe the entire body of the table in Nemeth, including any words.

When the entire body of the table is transcribed in Nemeth, the opening switch indicator is placed in cell 1 of the line following the column separation line (if present), and the entries begin on the next line. The Nemeth Code terminator follows the last line of entries, placed in cell 1. Words within the table, including row headings, are transcribed without contractions. If a row heading consists of one word, the single-word switch indicator is not used.

Switch Indicators with Tactile Graphics

Nemeth remains in effect for a tactile graphic if the graphic intervenes between two items in Nemeth. If the preceding text is in UEB and if a switch to Nemeth must be made for the tactile graphic, the opening switch indicator is placed at the end of the preceding text or in cell 1 on the line before the required blank line.

Appendix C
Combinations of Typeform, Alphabetic and Capitalization Indicators

Lowercase Letters

Typeform	English letters	German letters	Greek letters	Greek Letter Alternative	Hebrew letters	Russian letters
Boldface	⠸⠰	⠸⠸	⠸⠨			⠸⠈⠈
Italic	⠨⠰	⠨⠸	⠨⠨			⠨⠈⠈
Ordinary	⠰	⠸	⠨	⠨⠈	⠠⠠	⠈⠈
Sans serif	⠠⠨⠰					
Script	⠈⠰		⠈⠨		⠈⠠⠠	⠈⠈⠈

Capitalized Letters

Typeform	English letters	German letters	Greek letters	Greek Letter Alternative	Hebrew letters	Russian letters
Boldface	⠸⠰⠠	⠸⠸⠠	⠸⠨⠠			⠸⠈⠈⠠
Italic	⠨⠰⠠	⠨⠸⠠	⠨⠨⠠			⠨⠈⠈⠠
Ordinary	⠰⠠	⠸⠠	⠨⠠			⠈⠈⠠
Sans serif	⠠⠨⠰⠠					
Script	⠈⠰⠠		⠈⠨⠠			⠈⠈⠈⠠

Appendix D
Index of Nemeth Braille Symbols

The following is the list of 63 braille symbols arranged in braille order. The separation of these symbols into the usual seven lines of braille is ignored, each symbol is numbered in accordance with its rank in the list.

1	⠁	14	⠝	27	⠿	40	⠺	53	⠼
2	⠃	15	⠕	28	⠷	41	⠂	54	⠜
3	⠉	16	⠏	29	⠮	42	⠆	55	⠄
4	⠙	17	⠟	30	⠾	43	⠒	56	⠤
5	⠑	18	⠗	31	⠡	44	⠲	57	⠈
6	⠋	19	⠎	32	⠣	45	⠢	58	⠘
7	⠛	20	⠞	33	⠩	46	⠖	59	⠸
8	⠓	21	⠥	34	⠹	47	⠶	60	⠐
9	⠊	22	⠧	35	⠱	48	⠦	61	⠨
10	⠚	23	⠭	36	⠫	49	⠔	62	⠰
11	⠅	24	⠽	37	⠻	50	⠴	63	⠠
12	⠇	25	⠵	38	⠳	51	⠌		
13	⠍	26	⠯	39	⠪	52	⠬		

The items in the **Index of Nemeth Braille Symbols** are presented in braille order in accordance with the list of the 63 braille symbols above.

The English alphabet letters are not specified in this Code but have been included here for completeness.

1 ⠁ (dot 1)

Braille	Meaning	Print	Page
⠁	English a	a	
⠁	German ah	$\mathfrak{a}$	6-1
⠁	Greek alpha	α	6-3
⠁	Hebrew aleph	$\aleph$	6-2
⠁⠍⠏	amp (amplitude)		18-1
⠁⠝⠞⠊⠇⠕⠛	antilog (antilogarithm)		18-1
⠁⠗⠉	arc (arc)		18-1
⠁⠗⠛	arg (argument)		18-1

2 ⠃ (dots 12)

Braille	Meaning	Print	Page
⠃	English b	b	
⠃	German beh	$\mathfrak{b}$	6-1
⠃	Greek beta	β	6-3

3 ⠉ (dots 14)

Braille	Meaning	Print	Page
⠉	English c	c	
⠉	German tseh	$\mathfrak{c}$	6-1
⠉⠕⠇⠕⠛	colog (cologarithm)		18-1
⠉⠕⠎	cos (cosine)		18-1
⠉⠕⠎⠓	cosh (hyperbolic cosine)		18-1
⠉⠕⠞	cot (cotangent)		18-1

Braille	Meaning	Print	Page
⠉⠕⠞⠓	coth (hyperbolic cotangent)		18-1
⠉⠕⠧⠑⠗⠎	covers (coversine)		18-1
⠉⠎⠉	csc (cosecant)		18-1
⠉⠎⠉⠓	csch (hyperbolic cosecant)		18-1
⠉⠞⠝	ctn (cotangent)		18-1
⠉⠞⠝⠓	ctnh (hyperbolic cotangent)		18-1

4 ⠙ (dots 145)

Braille	Meaning	Print	Page
⠙	English d	d	
⠙	German deh	𝔡	6-1
⠙	Greek delta	δ	6-3
⠙⠑⠞	det (determinant)		18-1

5 ⠑ (dots 15)

Braille	Meaning	Print	Page
⠑	English e	e	
⠑	German eh	𝔢	6-1
⠑	Greek epsilon	ϵ or ε	6-3
⠑⠗⠋	erf (error function)		18-1
⠑⠭⠏	exp (exponential)		18-1
⠑⠭⠎⠑⠉	exsec (exsecant)		18-1

6 ⠋ (dots 124)

Braille	Meaning	Print	Page
⠋	English f	f	
⠋	German eff	𝔣	6-1
⠋	Greek phi	φ	6-3

7 ⠛ (dots 1245)

Braille	Meaning	Print	Page
⠛	English g	g	
⠛	German gheh	𝔤	6-1
⠛	Greek gamma	γ	6-3
⠛⠗⠁⠙	grad (gradient)		18-1

8 ⠓ (dots 125)

Braille	Meaning	Print	Page
⠓	English h	h	
⠓	German hah	𝔥	6-1
⠓⠁⠧	hav (haversine)		18-1

9 ⠊ (dots 24)

Braille	Meaning	Print	Page
⠊	English i	i	
⠊	German ee	𝔦	6-1
⠊	Greek iota	ι	6-3
⠊⠍	im (imaginary part)		18-1
⠊⠝⠋	inf (infimum)		18-1

10 ⠚ (dots 245)

Braille	Meaning	Print	Page
⠚	English j	j	
⠚	German yaht	𝔧	6-1

11 ⠅ (dots 13)

Braille	Meaning	Print	Page
⠅	English k	k	
⠅	German kah	𝔨	6-2
⠅	Greek kappa	κ	6-3

12 ⠇ (dots 123)

Braille	Meaning	Print	Page
⠇	English l	l	
⠇	German ell	𝔩	6-2
⠇	Greek lambda	λ	6-3
⠇	Russian ell	л	6-2
⠇⠊⠍	lim (limit)		18-2
⠇⠝	ln (natural logarithm)		18-2
⠇⠕⠛	log (logarithm)		18-2

13 ⠍ (dots 134)

Braille	Meaning	Print	Page
⠍	English m	m	
⠍	German em	𝔪	6-2
⠍	Greek mu	μ	6-3
⠍⠁⠭	max (maximum)		18-2
⠍⠊⠝	min (minimum)		18-2
⠍⠕⠙	mod (modulo)		18-2

14 ⠝ (dots 1345)

Braille	Meaning	Print	Page
⠝	English n	n	
⠝	German en	𝔫	6-2
⠝	Greek nu	ν	6-3

15 ⠕ (dots 135)

Braille	Meaning	Print	Page
⠕	barbed right full arrowhead	>	22-2
⠕	curved division sign, curving right	)	20-1, 25-1
⠕	English o	o	

Braille	Meaning	Print	Page
⠕	German oh	𝔬	6-2
⠕	Greek omicron	ο	6-3
⠕	solid right arrowhead	►	22-2

16 ⠏ (dots 1234)

Braille	Meaning	Print	Page
⠏	English p	p	
⠏	German peh	𝔭	6-2
⠏	Greek pi	π	6-3

17 ⠟ (dots 12345)

Braille	Meaning	Print	Page
⠟	English q	q	
⠟	German koo	𝔮	6-2

18 ⠗ (dots 1235)

Braille	Meaning	Print	Page
⠗	English r	r	
⠗	German err	𝔯	6-2
⠗	Greek rho	ρ	6-3
⠗⠑	re (real part)		18-2

19 ⠎ (dots 234)

Braille	Meaning	Print	Page
⠎	English s	s	
⠎	German ess	𝔰	6-2
⠎	Greek sigma	σ	6-3
⠎⠑⠉	sec (secant)		18-2
⠎⠑⠉⠓	sech (hyperbolic secant)		18-2
⠎⠊⠝	sin (sine)		18-2

Braille	Meaning	Print	Page
⠎⠊⠝⠓	sinh (hyperbolic sine)		18-2
⠎⠥⠏	sup (supremum)		18-2

20 ⠞ (dots 2345)

Braille	Meaning	Print	Page
⠞	English t	t	
⠞	German teh	𝔱	6-2
⠞	Greek tau	τ	6-3
⠞⠁⠝	tan (tangent)		18-2
⠞⠁⠝⠓	tanh (hyperbolic tangent)		18-2

21 ⠥ (dots 136)

Braille	Meaning	Print	Page
⠥	English u	u	
⠥	German oo	𝔲	6-2
⠥	Greek upsilon	υ	6-3

22 ⠧ (dots 1236)

Braille	Meaning	Print	Page
⠧	English v	v	
⠧	German fao	𝔳	6-2
⠧⠑⠗⠎	vers (versine)		18-2

23 ⠭ (dots 1346)

Braille	Meaning	Print	Page
⠭	English x	x	
⠭	German iks	𝔵	6-2
⠭	Greek xi	ξ	6-3

24 ⠽ (dots 13456)

Braille	Meaning	Print	Page
⠽	curved right full arrowhead		22-2
⠽	English y	y	
⠽	German ypsilon	𝔶	6-2
⠽	Greek psi	ψ	6-3

25 ⠵ (dots 1356)

Braille	Meaning	Print	Page
⠵	English z	z	
⠵	German tset	𝔷	6-2
⠵	Greek zeta	ζ	6-3

26 ⠯ (dots 12346)

Braille	Meaning	Print	Page
⠯	curved left full arrowhead		22-2
⠯	factorial	!	23-1
⠯	Greek chi	χ	6-3

27 ⠿ (dots 123456)

Braille	Meaning	Print	Page
⠿	blunted left full arrowhead		22-2
⠿	blunted right full arrowhead		22-2
⠿	general omission symbol		11-1

28 ⠷ (dots 12356)

Braille	Meaning	Print	Page
⠷	left parenthesis	(	19-1

29 ⠮ (dots 2346)

Braille	Meaning	Print	Page
⠮	single integral	∫	23-1
⠮⠮	double integral	∫∫	23-1
⠮⠮⠮	triple integral	∫∫∫	23-1
⠮⠈⠫⠉⠻	integral with superposed circle	∮	23-1
⠮⠈⠫⠗⠻	integral with superposed rectangle	⨒	23-1
⠮⠈⠫⠲⠻	integral with superimposed square	⨖	23-2
⠮⠈⠠⠿⠻	integral with superposed infinity	$\int\!\!\!\!\!\infty$	23-1

30 ⠾ (dots 23456)

Braille	Meaning	Print	Page
⠾	right parenthesis	)	19-1

31 ⠡ (dots 16)

Braille	Meaning	Print	Page
⠡	dot, and times	·	15-2, 20-2
⠡⠈⠸⠐⠅⠻	dot within inclusion sign	⪽	21-9
⠡⠈⠸⠨⠂⠻	dot within reverse inclusion sign	⪾	21-9
⠡⠈⠨⠅⠻	dot between bars of equal sign	⩦	21-9

32 ⠣ (dots 126)

Braille	Meaning	Print	Page
⠣	directly over indicator (first order)		2-3, 15-1
⠣	index-of-radical indicator		2-3, 16-1
⠣	makes nearer arrowhead point up		2-1, 22-1
⠣⠇⠊⠍	upper limit	$\overline{\lim}$	18-2
⠣⠮	upper integral	$\overline{\int}$	23-1
⠣⠣	directly over indicator (second order)		2-3, 15-1
⠣⠶⠶	regrouping indicator for numbers above the arrangement (varying in length)		2-3, 25-1

33 ⠩ (dots 146)

Braille	Meaning	Print	Page
⠩	directly under indicator (first order)		2-3, 15-1
⠩	makes nearer arrowhead point down		2-1, 22-1
⠩⠇⠊⠍	lower limit	$\underline{\lim}$	18-2
⠩⠮	lower integral	$\underline{\int}$	23-1
⠩⠩	directly under indicator (second order)		2-3, 15-1

Braille	Meaning	Print	Page
⠩⠶⠶	regrouping indicator for numbers below the arrangement (varying in length)		2-4, 25-1
⠩⠄⠄	vertical ellipsis	⋮	25-1

34 ⠹ (dots 1456)

Braille	Meaning	Print	Page
⠹	Greek theta	ϑ	6-3
⠹	opening simple fraction indicator		2-1, 13-1
⠹⠒⠒⠒⠼	spatial fraction line (varying in length)	______	25-1

35 ⠱ (dots 156)

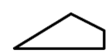
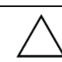
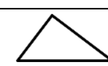
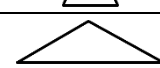
Braille	Meaning	Print	Page
⠱	Greek eta	η	6-3
⠱	horizontal bar (macron)	—	15-2
⠱	Russian sha	ш	6-2
⠱⠈⠩	bar over logical product	$\overline{\wedge}$	21-6
⠱⠈⠩⠱	bar over and bar under logical product	$\overline{\underline{\wedge}}$	21-6
⠱⠈⠩⠈⠱	bar over and equals sign under logical product	$\overline{\underset{=}{\wedge}}$	21-6
⠱⠈⠱	bar over single tilde	$\overline{\sim}$	21-7
⠱⠈⠱⠈⠱	bar over double tilde	$\overline{\approx}$	21-7
⠱⠈⠬	bar over logical sum	$\overline{\vee}$	21-6
⠱⠈⠬⠱	bar over and bar under logical sum	$\overline{\underline{\vee}}$	21-6

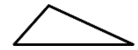
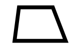
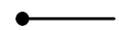
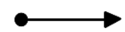
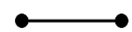
Braille	Meaning	Print	Page
⠱⠈⠬⠨⠅	bar over and equals sign under logical sum	$\underline{\underline{\overline{\vee}}}$	21-6
⠱⠈⠸⠐⠅⠻	bar through inclusion sign	$\in$	21-9
⠱⠈⠸⠨⠂⠻	bar through reverse inclusion sign	$\ni$	21-9
⠱⠸⠐⠅	bar over inclusion sign (is a subset of)	$\overline{\subset}$	21-5
⠱⠸⠨⠂	bar over reverse inclusion sign	$\overline{\supset}$	21-7
⠱⠐⠅	bar over less than sign (is equal to or less than)	$\overline{<}$ or $\leqslant$	21-5
⠱⠨⠂	bar over greater than sign (is equal to or greater than)	$\overline{>}$ or $\geqslant$	21-4

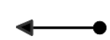
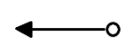
36 ⠫ (dots 1246)

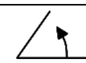
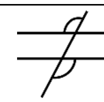
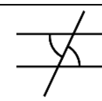
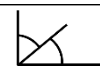
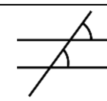
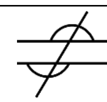
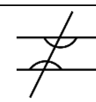
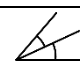
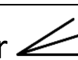
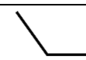
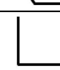
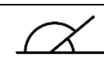
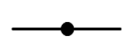
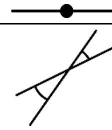
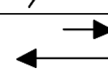
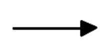
Braille	Meaning	Print	Page
⠫	shape indicator		2-4, 17-1
⠫⠁	arc concave downward	$\frown$	15-1, 17-1
⠫⠉	circle	$\bigcirc$	17-1
⠫⠉⠸⠫⠡⠻	circle with interior dot	$\odot$	17-4, 20-1
⠫⠉⠸⠫⠫⠣⠒⠒⠕⠻	circle with interior arrow pointing up	$\textcircled{\uparrow}$	17-3

Braille	Meaning	Print	Page
⠫⠉⠸⠫⠫⠣⠒⠒⠕⠐⠫⠩⠒⠒⠕⠻	circle with interior arrow pointing up followed by interior arrow pointing down		17-3
⠫⠉⠸⠫⠫⠩⠒⠒⠕⠻	circle with interior arrow pointing down		17-3
⠫⠉⠸⠫⠫⠩⠒⠒⠕⠐⠫⠣⠒⠒⠕⠻	circle with interior arrow pointing down followed by interior arrow pointing up		17-3
⠫⠉⠸⠫⠫⠪⠒⠒⠫⠒⠒⠕⠻	circle with interior arrow pointing left over interior arrow pointing right		17-3
⠫⠉⠸⠫⠫⠪⠒⠒⠻	circle with interior arrow pointing left		17-3
⠫⠉⠸⠫⠫⠒⠒⠕⠫⠪⠒⠒⠻	circle with interior arrow pointing right over interior arrow pointing left		17-3
⠫⠉⠸⠫⠫⠒⠒⠕⠻	circle with interior arrow pointing right		17-3
⠫⠉⠸⠫⠬⠻	circle with interior plus sign	⊕	17-4, 20-1
⠫⠉⠸⠫⠤⠻	circle with interior minus sign	⊖	17-4, 20-1
⠫⠉⠸⠫⠈⠡⠻	circle with interior cross	⊗	17-4
⠫⠙	diamond	◇	17-1

Braille	Meaning	Print	Page
⠫⠑	ellipse (oval)		17-1
⠫⠛	parallelogram		17-2
⠫⠓	rhombus	 or 	17-2
⠫⠓⠭	irregular hexagon		17-2
⠫⠊	intersecting lines		17-2
⠫⠅	keystroke indicator		2-4, 17-1
⠫⠇	is parallel to	$\parallel$	17-2, 21-2
⠫⠕	arrow barbed at right or right-pointing arrow (contracted form)	$\longrightarrow$	15-1, 17-1, 21-1
⠫⠏	is perpendicular to	$\perp$	17-2, 21-2
⠫⠏⠛	irregular pentagon		17-2
⠫⠟	quadrilateral		17-2
⠫⠗	rectangle		17-2
⠫⠎	star	☆	9-1, 17-2, 20-3
⠫⠞	regular triangle (equilateral)		15-2, 17-2
⠫⠞⠨⠁⠻	acute triangle		17-5
⠫⠞⠨⠊⠻	isosceles triangle		17-5
⠫⠞⠨⠕⠻	obtuse triangle		17-5
⠫⠞⠨⠗⠻	right triangle		17-5

Braille	Meaning	Print	Page
⠫⠞⠨⠎⠻	scalene triangle		17-5
⠫⠵	trapezoid		17-2
⠫⠡⠒⠒	arrow dotted at left (no barb)		15-1
⠫⠡⠒⠒⠕	arrow dotted at left and barbed at right		15-1
⠫⠡⠒⠒⠡	arrow dotted at both ends		15-1
⠫⠣⠪⠒⠒⠕	vertical two-way arrow	↕	21-1
⠫⠣⠒⠒⠕	arrow pointing up	↑	17-1, 21-1
⠫⠣⠒⠒⠕⠐⠫⠩⠒⠒⠕	up-pointing arrow followed by down-pointing arrow	↑↓	21-8
⠫⠣⠒⠒⠕⠐⠫⠩⠸⠒⠒⠕	up-pointing arrow followed by boldface down-pointing arrow	↑**↓**	21-8
⠫⠣⠸⠒⠒⠕⠐⠫⠩⠒⠒⠕	boldface up-pointing arrow followed by down-pointing arrow	**↑**↓	21-8
⠫⠣⠸⠒⠒⠕⠐⠫⠩⠸⠒⠒⠕	boldface up-pointing arrow followed by boldface down-pointing arrow	**↑↓**	21-8
⠫⠩⠒⠒⠕	down-pointing arrow	↓	17-1, 21-1
⠫⠩⠒⠒⠕⠐⠫⠣⠒⠒⠕	down-pointing arrow followed by up-pointing arrow	↓↑	21-8

Braille	Meaning	Print	Page
⠫⠩⠒⠒⠕⠐⠫⠣⠸⠒⠒⠕	down-pointing arrow followed by boldface up-pointing arrow	↓**↑**	21-8
⠫⠩⠸⠒⠒⠕⠐⠫⠣⠒⠒⠕	boldface down-pointing arrow followed by up-pointing arrow	**↓**↑	21-8
⠫⠩⠸⠒⠒⠕⠐⠫⠣⠸⠒⠒⠕	boldface down-pointing arrow followed by boldface up-pointing arrow	**↓↑**	21-8
⠫⠪	angle	∠	17-1
⠫⠪⠒⠒	arrow barbed at left	←	15-1
⠫⠪⠒⠒	left-pointing arrow	←	17-1, 21-1
⠫⠪⠒⠒⠕	arrow barbed at both ends	↔	15-1
⠫⠪⠒⠒⠕	horizontal two-way arrow	↔	21-1
⠫⠪⠒⠒⠡	arrow barbed at left and dotted at right		15-1
⠫⠪⠒⠒⠫⠸⠒⠒⠕	left-pointing arrow over boldface right-pointing arrow	← **→**	21-4
⠫⠪⠒⠒⠨⠡	arrow with hollow dot at right and barbed at left		15-2
⠫⠪⠸⠫⠫⠁⠻	angle with interior arc	∡	17-3

Braille	Meaning	Print	Page
⠫⠪⠸⠫⠫⠪⠢⠔⠻	angle with interior clockwise arrow		17-3
⠫⠪⠸⠫⠫⠢⠔⠕⠻	angle with interior counterclockwise arrow		17-3
⠫⠪⠸⠁⠑⠻	alternate exterior angles		17-4
⠫⠪⠸⠁⠊⠻	alternate interior angles		17-4
⠫⠪⠸⠉⠏⠻	complementary angles		17-4
⠫⠪⠸⠉⠻	corresponding angles		17-4
⠫⠪⠸⠑⠻	exterior angles		17-5
⠫⠪⠸⠊⠻	interior angles		17-5
⠫⠪⠸⠚⠻	adjacent angles	 or 	17-4
⠫⠪⠸⠕⠻	obtuse angle		17-5
⠫⠪⠸⠗⠻	right angle		17-5
⠫⠪⠸⠎⠏⠻	supplementary angles		17-5
⠫⠪⠸⠎⠻	straight angle		17-5
⠫⠪⠸⠧⠻	vertical angles		17-5
⠫⠒⠕⠫⠪⠒⠒⠒	short right-pointing arrow over long left-pointing arrow		21-4
⠫⠒⠒⠕	arrow barbed at right or right-pointing arrow (uncontracted form)		15-1, 17-1, 21-1

Braille	Meaning	Print	Page
⠫⠒⠒⠕⠫⠪⠒⠒	right-pointing arrow over left-pointing arrow	⇄	21-4
⠫⠒⠒⠕⠫⠸⠪⠒⠒	right-pointing arrow over boldface left-pointing arrow	⇄	21-4
⠫⠒⠒⠡	arrow dotted at right (no barb)	—●	15-1
⠫⠒⠒⠒⠕⠫⠪⠒	long right-pointing arrow over short left-pointing arrow	⇄	21-4
⠫⠒⠒⠈⠕⠫⠠⠪⠒⠒	right-pointing arrow with upper barb over left-pointing arrow with lower barb	⇌	21-4
⠫⠒⠒⠨⠡	arrow with hollow dot at right (no barb)	—○	15-2
⠫⠲	square	□	17-2, 20-3
⠫⠲⠸⠫⠡⠻	square with interior dot	⊡	17-4
⠫⠲⠸⠫⠱⠻	square with interior horizontal bar	⊟	17-4
⠫⠲⠸⠫⠳⠻	square with interior vertical bar	◫	17-4
⠫⠲⠸⠫⠢⠻	square with interior northwest-southeast diagonal	⧅	17-4
⠫⠲⠸⠫⠢⠈⠔⠻	square with interior diagonals	⊠	17-4

Braille	Meaning	Print	Page
⠫⠲⠸⠫⠔⠻	square with interior southwest-northeast diagonal	⧄	17-4
⠫⠢	regular pentagon	⬠	17-2
⠫⠖	regular hexagon	⬡	17-2
⠫⠄	arc concave upward	‿	15-1, 17-1
⠫⠸⠪⠒⠒⠫⠒⠒⠕	boldface left-pointing arrow over right-pointing arrow	⇆	21-4
⠫⠸⠪⠒⠒⠫⠸⠒⠒⠕	boldface left-pointing arrow over boldface right-pointing arrow	⇆	21-4
⠫⠸⠒⠒⠕⠫⠪⠒⠒	boldface right-pointing arrow over left-pointing arrow	⇄	21-4
⠫⠸⠒⠒⠕⠫⠸⠪⠒⠒	boldface right-pointing arrow over boldface left-pointing arrow	⇄	21-4
⠫⠸⠲	filled-in square	■	20-2
⠫⠨⠡⠒⠒	arrow with hollow dot at left (no barb)	o——	15-2
⠫⠨⠡⠒⠒⠕	arrow with hollow dot at left and barbed at right	o——▸	15-2
⠫⠨⠡⠒⠒⠨⠡	arrow with hollow dot at both ends	o——o	15-2

37 ⠻ (dots 12456)

Braille	Meaning	Print	Page
⠻	closing cancellation indicator		2-1, 12-1
⠻	termination indicator		2-4, 15-1, 16-1, 17-1

38 ⠳ (dots 1256)

Braille	Meaning	Print	Page
⠳	single vertical bar as a sign of grouping	|	19-1
⠳	straight left full arrowhead	├	22-2
⠳	straight right full arrowhead	┤	22-2
⠳	vertical bar (is a factor, divides)	|	20-3
⠳	vertical bar (not a tally mark)	|	15-2, 23-2
⠳	vertical bar (such that)	|	21-2
⠳⠳	double vertical bar as a sign of grouping	||	19-1
⠳⠈⠫⠪⠒⠒⠻	vertical bar through shaft of left-pointing arrow	⇷	21-9
⠳⠈⠫⠒⠒⠕⠻	vertical bar through shaft of right-pointing arrow	⇸	21-9

39 ⠪ (dots 246)

Braille	Meaning	Print	Page
⠪	barbed left full arrowhead	<	22-1
⠪	contraction for comma and optional space at superscript or subscript level		14-1
⠪	curved division sign, curving left	(	20-1, 25-1
⠪	opening cancellation indicator		2-1, 12-1
⠪	solid left arrowhead	◄	22-1

40 ⠺ (dots 2456)

Braille	Meaning	Print	Page
⠺	English w	w	
⠺	German veh	𝔴	6-2
⠺	Greek omega	ω	6-3

41 ⠂ (dot 2)

Braille	Meaning	Print	Page
⠂	literary comma	,	8-1
⠂	numeral 1	1	3-1
⠂⠂⠂	dotted arrow shaft	•••	22-1

42 ⠆ (dots 23)

Braille	Meaning	Print	Page
⠆	numeral 2	2	3-1
⠆	semicolon	;	8-2

43 ⠒ (dots 25)

Braille	Meaning	Print	Page
⠒	colon	:	8-1
⠒	numeral 3	3	3-1
⠒	short single arrow shaft	__	22-1
⠒⠀⠒	dashed arrow shaft	_ _	22-1
⠒⠒	ordinary single arrow shaft	___	22-1
⠒⠒⠒	long single arrow shaft	____	22-1
⠒⠒⠒	separation line used in spatial arrangements (varying in length)	_______	25-1
⠒⠒⠒⠒⠒	horizontal fraction line used in spatial arrangements (varying in length)		13-2

44 ⠲ (dots 256)

Braille	Meaning	Print	Page
⠲	numeral 4	4	3-1
⠲	period	.	8-1

45 ⠢ (dots 26)

Braille	Meaning	Print	Page
⠢	numeral 5	5	3-1
⠢⠔	curved arrow shaft	⌒ or ◡	22-1

46 ⠖ (dots 235)

Braille	Meaning	Print	Page
⠖	exclamation point	!	8-1
⠖	numeral 6	6	3-1

47 ⠶ (dots 2356)

Braille	Meaning	Print	Page
⠶	numeral 7	7	3-1
⠶	short double arrow shaft	═	22-1
⠶⠶	ordinary double arrow shaft	══	22-1
⠶⠶⠶	long double arrow shaft	═══	22-1

48 ⠦ (dots 236)

Braille	Meaning	Print	Page
⠦	left-pointing double angle quotation mark	«	8-2
⠦	numeral 8	8	3-1
⠦	opening directional quotation mark	“	8-1
⠦	opening double quotation mark	"	8-1
⠦	question mark	?	8-1

49 ⠔ (dots 35)

Braille	Meaning	Print	Page
⠔	numeral 9	9	3-1
⠔⠒⠢	wavy arrow shaft	∼	22-1

50 ⠴ (dots 356)

Braille	Meaning	Print	Page
⠴	closing directional quotation mark	”	8-1
⠴	closing double quotation mark	"	8-1
⠴	numeral 0	0	3-1

Braille	Meaning	Print	Page
⠴	right-pointing double angle quotation mark	»	8-2

51 ⠌ (dots 34)

Braille	Meaning	Print	Page
⠌	horizontal simple fraction line		13-1
⠌	negation sign	\ or / or |	21-10
⠌⠫⠇	is not parallel to	$\nparallel$	17-2
⠌⠫⠏	is not perpendicular to	$\not\perp$	17-2
⠌⠨⠅	is not equal to	$\neq$ or $\not=$	21-1
⠌⠠⠡	therefore negated (it does not follow that)	$/\!\therefore$	23-2

52 ⠬ (dots 346)

Braille	Meaning	Print	Page
⠬	regular plus	+	20-2
⠬⠤	plus or minus	±	20-2
⠬⠐⠤	regular plus followed by regular minus	+ –	20-2
⠬⠐⠸⠤	regular plus followed by boldface minus	+ **–**	20-2

53 ⠼ (dots 3456)

Braille	Meaning	Print	Page
⠼	closing simple fraction indicator		2-1, 13-1
⠼	numeric indicator		2-3, 3-1

54 ⠜ (dots 345)

Braille	Meaning	Print	Page
⠜	radical (square root)	√	16-1
⠜	square root sign used in spatial arrangement	√	25-1

55 ⠄ (dot 3)

Braille	Meaning	Print	Page
⠄	apostrophe	'	8-1
⠄	prime	'	23-2
⠄⠄⠄	ellipsis or horizontal ellipsis	…	8-1, 25-1

56 ⠤ (dots 36)

Braille	Meaning	Print	Page
⠤	hyphen	-	8-1
⠤	regular minus	−	20-1
⠤⠬	minus or plus	∓	20-2
⠤⠤	short dash	–	8-1
⠤⠤⠤⠤	long dash	—	8-1, 11-2
⠤⠐⠬	regular minus followed by regular plus	− +	20-2
⠤⠐⠤	minus followed by minus	−−	20-1
⠤⠐⠸⠬	regular minus followed by boldface plus	− **+**	20-2

57 ⠈ (dot 4)

Braille	Meaning	Print	Page
⠈	script type indicator		2-4, 7-1

Braille	Meaning	Print	Page
⠈	superposition indicator		2-3, 15-1
⠈⠁	at	@	23-1
⠈⠉	cent	¢	23-2
⠈⠙	partial derivative (round d)	∂	23-2
⠈⠑	euro	€	23-2
⠈⠑	membership (is an element of, belongs to)	∈	21-1
⠈⠋	French franc	₣	23-2
⠈⠓	crossed h (h-bar)	ℏ	23-1
⠈⠇	pound sterling	£	23-2
⠈⠝	naira	₦	23-2
⠈⠕	barbed right upper arrowhead	⦣	22-2
⠈⠎	dollar	$	23-2
⠈⠽	curved right upper arrowhead	◝	22-2
⠈⠽	yen or yuan	¥	23-2
⠈⠯	curved left upper arrowhead	◜	22-2
⠈⠯	universal quantifier (for all, for each, for every)	∀	23-2
⠈⠿	blunted left upper arrowhead	⌈	22-2
⠈⠿	blunted right upper arrowhead	⌉	22-2
⠈⠿	existential quantifier (there exists, for some)	∃	23-2

Braille	Meaning	Print	Page
⠈⠿⠳	existential quantifier (there exists uniquely for exactly one)	∃| or ∃!	23-2
⠈⠷	left square bracket	[	19-1
⠈⠾	right square bracket	]	19-1
⠈⠡	cross (Cartesian product, multiplication sign)	×	20-2
⠈⠣⠠⠣	equivalence (is equivalent to)	≎	21-5
⠈⠩	logical product (and, meet)	∧	20-1
⠈⠩⠱	bar under logical product	$\underline{\wedge}$	21-6
⠈⠩⠨⠅	equals sign under logical product	$\overset{\wedge}{=}$	21-6
⠈⠱	simple tilde (as modifier)	~	15-2
⠈⠱	simple tilde (is related to, is similar)	~	20-3, 21-2
⠈⠱⠱	bar under single tilde	≃	21-7
⠈⠱⠈⠱	double tilde	≈	21-7
⠈⠱⠈⠱⠱	bar under double tilde	≊	21-7
⠈⠱⠈⠱⠨⠅	equals sign under double tilde	$\overset{\approx}{=}$	21-7
⠈⠱⠨⠅	equals sign under single tilde	≅	21-7
⠈⠫⠟⠑⠙	end of proof	▌ or □	23-1, 26-48

Braille	Meaning	Print	Page
⠈⠻	general reference indicator		2-2, 9-1
⠈⠳	straight left upper arrowhead	⌊	22-2
⠈⠳	straight right upper arrowhead	⌋	22-2
⠈⠪	barbed left upper arrowhead	∠	22-1
⠈⠺	Won	₩	23-2
⠈⠢	reverse membership (contains the element)	∋	21-2
⠈⠴	percent sign	%	23-2
⠈⠴⠴	per mille sign	‰	23-2
⠈⠌	since (because)	∵	23-2
⠈⠬	logical sum (join, or)	∨	20-1
⠈⠬⠱	bar under logical sum	⊻	21-6
⠈⠬⠨⠅	equals sign under logical sum	⩣	21-6
⠈⠼	asterisk	∗	9-1, 20-1
⠈⠈	Russian (Cyrillic) letter indicator		2-1, 6-1, C-1
⠈⠈⠈	lowercase script Russian letter indicator		C-1
⠈⠈⠈⠠	capital script Russian letter indicator		C-1
⠈⠈⠠	capital Russian letter indicator		C-1

Braille	Meaning	Print	Page
⠈⠘⠷	upper left half bracket	⌈ or ⌜	19-2
⠈⠘⠾	upper right half bracket	⌉ or ⌝	19-2
⠈⠘⠠⠷	upper left enlarged half bracket	⌈ or ⌜	19-2
⠈⠘⠠⠾	upper right enlarged half bracket	⌉ or ⌝	19-2
⠈⠸⠷	left barred bracket	⟦	19-1
⠈⠸⠾	right barred bracket	⟧	19-1
⠈⠸⠠⠷	left enlarged barred bracket	⟦	19-1
⠈⠸⠠⠾	right enlarged barred bracket	⟧	19-1
⠈⠨	lowercase script Greek letter indicator		C-1
⠈⠨⠇	crossed lambda (lambda-bar)	ƛ	23-1
⠈⠨⠠	capital script Greek letter indicator		C-1
⠈⠰	lowercase script English letter indicator		C-1
⠈⠰⠷	lower left half bracket	⌊ or ⌞	19-2
⠈⠰⠾	lower right half bracket	⌋ or ⌟	19-2
⠈⠰⠠	capital script English letter indicator		C-1
⠈⠰⠠⠷	lower left enlarged half bracket	⌊ or ⌞	19-2

Braille	Meaning	Print	Page
⠈⠰⠠⠾	lower right enlarged half bracket	⌋ or ⌟	19-2
⠈⠠⠁	angstrom unit	Å	23-1
⠈⠠⠏	paragraph mark	¶	20-2
⠈⠠⠗	crossed R (R-bar)	℞	23-1
⠈⠠⠎	single section mark	§	20-2
⠈⠠⠷	enlarged left square bracket	[	19-1
⠈⠠⠾	enlarged right square bracket	]	19-1
⠈⠠⠱	extended tilde	~~	15-2, 20-3, 21-2
⠈⠠⠠	lowercase script Hebrew letter indicator		C-1

58 ⠘ (dots 45)

Braille	Meaning	Print	Page
⠘	elevates nearer arrowhead by 45 degrees		2-1, 22-1
⠘	superscript indicator		2-2, 14-1
⠘⠄⠄	diagonal ellipsis (lower left to upper right)	⋰	25-1
⠘⠘	superscript with superscript indicator		2-2, 14-1
⠘⠘⠘	superscript with superscript with superscript indicator		2-2, 14-1

Braille	Meaning	Print	Page
⠘⠘⠰	superscript with superscript with subscript indicator		2-2, 14-1
⠘⠰	superscript with subscript indicator		2-2, 14-1
⠘⠰⠘	superscript with subscript with superscript indicator		2-2, 14-1
⠘⠰⠰	superscript with subscript with subscript indicator		2-2, 14-1

59 ⠸ (dots 456)

Braille	Meaning	Print	Page
⠸	boldface arrow type		2-1, 22-1
⠸	boldface type indicator		2-4, 7-1
⠸	filled-in shape indicator		2-4, 17-1
⠸	German letter indicator		2-1, 6-1, C-1
⠸	punctuation indicator		2-3, 8-1
⠸	single word boldface type indicator		2-4, 7-1
⠸	slanted division sign	/ or \	20-1, 25-1
⠸	straight division sign	|	20-1, 25-1
⠸	tally mark (not a vertical bar)	|	23-2

Braille	Meaning	Print	Page
⠸	vertical line used in spatial division arrangements (varying in length)	|	25-1
⠸⠇	identity (is congruent to, is identical to)	≡	21-1
⠸⠯	ampersand (and, logical product)	&	20-1
⠸⠿	variation (varies as)	∝	21-2
⠸⠡	backslash (divides, is a factor of)	\	20-1
⠸⠣	caret (circumflex, hat)	∧	15-2, 23-1
⠸⠩	inverted caret	∨	15-2
⠸⠩	opening Nemeth Code indicator		2-3, 4-1
⠸⠹	opening fractional part of mixed number indicator		2-2, 13-1
⠸⠱	Nemeth Code terminator		2-3, 4-1
⠸⠫	interior shape-modification indicator		2-4, 17-1
⠸⠻	single dagger	† or †	9-1, 20-1
⠸⠳	boldface single vertical bar	**|**	19-1
⠸⠳⠸⠳	boldface double vertical bar	**||**	19-1
⠸⠦	question mark (as a modifier)	?	15-2

Braille	Meaning	Print	Page
⠸⠴	empty set (represented by zero with oblique bar through it)	∅	23-1
⠸⠌	diagonal fraction line or slash		13-1
⠸⠌	slash (per, over, divided by)	/	20-2
⠸⠬	boldface plus	**+**	20-2
⠸⠬⠐⠤	boldface plus followed by regular minus	**+** −	20-2
⠸⠬⠐⠸⠤	boldface plus followed by boldface minus	**+ −**	20-2
⠸⠼	closing fractional part of mixed number indicator		2-2, 13-1
⠸⠤	boldface minus	**−**	20-1
⠸⠤⠐⠬	boldface minus followed by regular plus	**−** +	20-2
⠸⠤⠐⠸⠬	boldface minus followed by boldface plus	**− +**	20-2
⠸⠈⠷	boldface left square bracket	**[**	19-1
⠸⠈⠾	boldface right square bracket	**]**	19-1
⠸⠈⠈	lowercase boldface Russian letter indicator		C-1

Braille	Meaning	Print	Page
⠸⠈⠈⠠	capital boldface Russian letter indicator		C-1
⠸⠸	lowercase boldface German letter indicator		C-1
⠸⠸⠻	double dagger	‡ or ǂ	9-1, 20-1
⠸⠸⠠	capital boldface German letter indicator		C-1
⠸⠐⠅	inclusion sign (is contained in, is a subset of)	⊂	21-1
⠸⠐⠅⠱	bar under inclusion sign (is a subset of)	⊆	21-5
⠸⠐⠅⠈⠨⠅⠻	inclusion sign through equals sign	⋲	21-9
⠸⠐⠅⠨⠅	equals sign under inclusion sign (is a subset of)	⫅	21-5
⠸⠨	lowercase boldface Greek letter indicator		C-1
⠸⠨	single word bold italic type		2-4, 7-1
⠸⠨⠅	boldface equals sign	**=**	21-1
⠸⠨⠂	reverse inclusion sign (contains, in logic implies)	⊃	21-2
⠸⠨⠂⠱	bar under reverse inclusion sign	⊇	21-7

Braille	Meaning	Print	Page
⠸⠨⠂⠈ ⠨⠅⠻	reverse inclusion sign through equals sign	⋑̸=	21-9
⠸⠨⠂ ⠨⠅	equals sign under reverse inclusion sign	⫆	21-7
⠸⠨⠠	capital boldface Greek letter indicator		C-1
⠸⠰	lowercase boldface English letter indicator		C-1
⠸⠰⠠	capital boldface English letter indicator		C-1
⠸⠠	capital German letter indicator		C-1
⠸⠠⠄	closing boldface type indicator for two or more words, phrases, and mathematical statements		2-4, 7-1

60 ⠐ (dot 5)

Braille	Meaning	Print	Page
⠐	baseline indicator		2-2, 14-1
⠐	multipurpose indicator		2-3, 15-1, 24-1
⠐⠅	less than sign (normal)	<	21-1
⠐⠅⠱	bar under less than sign (is less than or equal to)	≤ or ⩽	21-5

Braille	Meaning	Print	Page
⠐⠅⠈⠐⠅⠻	nest of two less than signs with straight sides (is small compared with)	$\ll$	21-9
⠐⠅⠐⠨⠅⠐⠨⠂	less than sign followed by equals sign followed by greater than sign	$< \; = \; >$	21-8
⠐⠅⠐⠨⠂	less than sign followed by greater than sign	$< \; >$	21-8
⠐⠅⠨⠅	equals sign under less than sign (is less than or equal to)	$\leqq$ or $\leqslant$	21-6
⠐⠱⠣⠸⠣⠻	caret over horizontal bar	$\overset{\wedge}{-}$	21-3
⠐⠱⠩⠡⠻	dot under horizontal bar	$\underset{\cdot}{-}$	21-3
⠐⠱⠩⠸⠣⠻	caret under horizontal bar (is perspective to, perspective correspondence)	$\underset{\wedge}{-}$	21-3
⠐⠂	ratio sign (is to)	:	21-2
⠐⠈⠱⠩⠡⠻	dot under simple tilde	$\underset{\cdot}{\sim}$	21-3
⠐⠨⠅⠣⠡⠻	dot over equals sign (is approximately equal to)	$\doteq$	21-3
⠐⠨⠅⠣⠫⠾⠻	equilateral triangle over equals sign	$\triangleq$	21-3

Braille	Meaning	Print	Page
⠐⠨⠅⠣⠳⠻	vertical bar over equals sign	$\overset{\vert}{=}$	21-3
⠐⠨⠅⠣⠸⠣⠻	caret over equals sign	≙	21-3
⠐⠨⠅⠣⠸⠩⠻	inverted caret over equals sign	≚	21-3
⠐⠨⠅⠣⠸⠦⠻	question mark over equals sign	≟	21-3
⠐⠨⠅⠣⠨⠡⠻	degree sign over equals sign (is equal in degrees to)	≗	21-3
⠐⠨⠅⠣⠰⠣⠻	left-pointing caret over equals sign	$\overset{<}{=}$	21-3
⠐⠨⠅⠣⠰⠩⠻	right-pointing caret over equals sign	$\overset{>}{=}$	21-3
⠐⠨⠅⠩⠡⠡⠣⠡⠡⠻	two dots over and two dots under equals sign	$\overset{..}{\underset{..}{=}}$	21-3
⠐⠨⠅⠩⠡⠣⠡⠻	dot over and dot under equals sign	≑	21-3
⠐⠨⠅⠩⠸⠣⠻	caret under equals sign (is projective to, projective correspondence)	$\underset{\wedge}{=}$	21-3

61 ⠨ (dots 46)

Braille	Meaning	Print	Page
⠨	decimal point (American)	.	3-1
⠨	first inner radical indicator		2-3, 16-1

Braille	Meaning	Print	Page
⠨	Greek letter indicator for standard letters		2-1, 6-1, C-1
⠨	italic type indicator		2-4, 7-1
⠨	mathematical comma (European)	.	3-1
⠨	shaded shape indicator		2-4, 17-1
⠨	single word italic type indicator		2-4, 7-1
⠨	structural shape-modification indicator		2-4, 17-1
⠨⠅	regular equals sign (is equal to)	$=$	21-1
⠨⠅⠈⠩	equals sign over logical product	$\overset{=}{\wedge}$	21-6
⠨⠅⠈⠩⠱	equals sign over and bar under logical product	$\underline{\overset{=}{\wedge}}$	21-6
⠨⠅⠈⠩⠨⠅	equals sign over and equals sign under logical product	$\underset{=}{\overset{=}{\wedge}}$	21-6
⠨⠅⠈⠱	equals sign over single tilde	$\overset{=}{\sim}$	21-7
⠨⠅⠈⠱⠈⠱	equals sign over double tilde	$\overset{=}{\approx}$	21-7
⠨⠅⠈⠬	equals sign over logical sum	$\overset{=}{\vee}$	21-6
⠨⠅⠈⠬⠱	equals sign over and bar under logical sum	$\underline{\overset{=}{\vee}}$	21-6

Braille	Meaning	Print	Page
⠨⠅⠈⠬⠨⠅	equals sign over and equals sign under logical sum	$\overset{=}{\underline{\underline{\vee}}}$	21-6
⠨⠅⠈⠸⠐⠅⠻	equals sign with superposed inclusion sign	⋲	21-9
⠨⠅⠈⠸⠨⠂⠻	equals sign with superposed reverse inclusion sign	⋺	21-9
⠨⠅⠸⠐⠅	equals sign over inclusion sign (is a subset of)	$\overset{=}{\subset}$	21-5
⠨⠅⠸⠨⠂	equals sign over reverse inclusion sign	$\overset{=}{\supset}$	21-7
⠨⠅⠐⠅	equals sign over less than (is equal to or less than)	$\overset{=}{<}$ or ⪕	21-6
⠨⠅⠨⠂	equals sign over greater than (is equal to or greater than)	$\overset{=}{>}$ or ⪖	21-5
⠨⠷	left curly brace	{	19-1
⠨⠷⠀⠨⠾	empty set (represented by facing braces)	{ }	23-1
⠨⠾	right curly brace	}	19-1
⠨⠡	hollow dot	∘	15-2, 20-1, 23-1
⠨⠩	intersection sign (cap)	∩	20-1
⠨⠩⠱	bar under intersection sign	$\underline{\cap}$	21-5

Braille	Meaning	Print	Page
⠨⠩⠨⠅	equals signs under intersection sign	$\underline{\underline{\cap}}$	21-5
⠨⠫	del (nabla, gradient)	∇ or ▽	23-1
⠨⠫	inverted triangle	▽	17-2
⠨⠂	greater than sign (normal)	>	21-1
⠨⠂⠱	bar under greater than sign (is greater than or equal to)	≥ or ⩾	21-5
⠨⠂⠈⠨⠂⠻	nest of two greater than signs with straight sides (is large compared with)	≫	21-9
⠨⠂⠐⠐⠅	greater than sign followed by less than sign	> <	21-8
⠨⠂⠐⠨⠅⠐⠐⠅	greater than sign followed by equals sign followed by less than sign	> = <	21-8
⠨⠂⠨⠅	equals sign under greater than (is greater than or equal to)	≧ or ⫺	21-5
⠨⠌	division sign (divided by)	÷	20-1
⠨⠬	union sign (cup)	∪	20-3
⠨⠬⠱	bar under union sign	$\underline{\cup}$	21-7
⠨⠬⠨⠅	equals sign under union sign	$\underline{\underline{\cup}}$	21-7

Braille	Meaning	Print	Page
⠨⠼	number sign, crosshatch, pound sign	#	20-2
⠨⠤	minus with dot over (proper difference)	∸	20-2
⠨⠈	Greek letter indicator for alternative letters		2-1, 6-1, C-1
⠨⠈⠈	lowercase italic Russian letter indicator		C-1
⠨⠈⠈⠠	capital italic Russian letter indicator		C-1
⠨⠸	lowercase italic German letter indicator		C-1
⠨⠸⠷	barred left brace	⦃	19-2
⠨⠸⠾	barred right brace	⦄	19-2
⠨⠸⠠	capital italic German letter indicator		C-1
⠨⠸⠠⠷	enlarged left barred brace	⦃	19-2
⠨⠸⠠⠾	enlarged right barred brace	⦄	19-2
⠨⠐⠅	less than sign with curved sides	≺	21-1
⠨⠐⠅⠈⠨⠐⠅⠻	nest of two less than signs with curved sides	≪	21-9
⠨⠨	second inner radical indicator		2-3, 16-1

Braille	Meaning	Print	Page
⠨⠨	lowercase italic Greek letter indicator		C-1
⠨⠨⠷	left angle bracket (angular parenthesis)	<	19-1
⠨⠨⠾	right angle bracket (angular parenthesis)	>	19-1
⠨⠨⠐	greater than sign with curved sides	≻	21-1
⠨⠨⠐⠈⠨⠨⠐⠻	nest of two greater than signs with curved sides	⪼	21-9
⠨⠨⠨	third inner radical indicator		2-3, 16-1
⠨⠨⠠	capital italic Greek letter indicator		C-1
⠨⠨⠠⠷	left enlarged angle bracket (angular parenthesis)	<	19-1
⠨⠨⠠⠾	right enlarged angle bracket (angular parenthesis)	>	19-1
⠨⠰	lowercase italic English letter indicator		C-1
⠨⠰⠠	capital italic English letter indicator		C-1
⠨⠠	capital Greek letter indicator		C-1
⠨⠠⠷	left enlarged curly brace	{	19-1

Braille	Meaning	Print	Page
⠨⠠⠾	right enlarged curly brace	}	19-1
⠨⠠⠄	closing italic type indicator for two or more words, phrases, and mathematical statements		2-4, 7-1

62 ⠰ (dots 56)

Braille	Meaning	Print	Page
⠰	depresses nearer arrowhead by 45 degrees		2-1, 22-1
⠰	English letter indicator		2-1, 6-1, C-1
⠰	subscript indicator		2-2, 14-1
⠰⠣	left-pointing caret	<	15-2
⠰⠩	right-pointing caret	>	15-2
⠰⠆	proportion sign (as)	::	21-2
⠰⠄⠄	diagonal ellipsis (upper left to lower right)	⋱	25-1
⠰⠘	subscript with superscript indicator		2-2, 14-1
⠰⠘⠘	subscript with superscript with superscript indicator		2-2, 14-1
⠰⠘⠰	subscript with superscript with subscript indicator		2-2, 14-1

Braille	Meaning	Print	Page
⠰⠰	subscript with subscript indicator		2-2, 14-1
⠰⠰⠘	subscript with subscript with superscript indicator		2-2, 14-1
⠰⠰⠰	subscript with subscript with subscript indicator		2-2, 14-1
⠰⠠	capital English letter indicator		C-1

63 ⠠ (dot 6)

Braille	Meaning	Print	Page
⠠	decimal point (European)	,	3-1
⠠	mathematical comma (American)	,	3-1, 8-1
⠠	single capitalization indicator		2-1, 5-1
⠠⠕	barbed right lower arrowhead	↗	22-2
⠠⠗	relation (is related to)	R	21-2
⠠⠽	curved right lower arrowhead	)	22-2
⠠⠯	curved left lower arrowhead	(	22-2
⠠⠿	blunted left lower arrowhead	[	22-2
⠠⠿	blunted right lower arrowhead	]	22-2
⠠⠿	infinity	∞	23-1

Braille	Meaning	Print	Page
⠠⠷	left enlarged parenthesis	(	19-1
⠠⠾	right enlarged parenthesis	)	19-1
⠠⠡	therefore (normal)	∴	23-2
⠠⠹	opening complex fraction indicator		2-2, 13-1
⠠⠳	single enlarged vertical bar	|	19-1
⠠⠳	straight left lower arrowhead	⌈	22-2
⠠⠳	straight right lower arrowhead	⌉	22-2
⠠⠳⠠⠳	double enlarged vertical bar	||	19-1
⠠⠪	barbed left lower arrowhead	⟍	22-1
⠠⠦	opening single quotation mark	'	8-1
⠠⠴	closing single quotation mark	'	8-1
⠠⠌	horizontal complex fraction line		13-1
⠠⠼	closing complex fraction indicator		2-2, 13-1
⠠⠄	ditto mark	"	23-1
⠠⠄	single-word switch indicator		2-3, 4-1
⠠⠄	typeform terminator		2-5, 7-1
⠠⠄⠾	transcriber-inserted grouping symbol		19-2

Braille	Meaning	Print	Page
⠠⠄⠸	opening boldface type indicator for two or more words, phrases and mathematical statements		2-4, 7-1
⠠⠄⠨	opening italic type indicator for two or more words, phrases and mathematical statements		2-4, 7-1
⠠⠸	barred type (Blackboard or double struck) indicator		2-4, 7-1
⠠⠸⠌	diagonal complex fraction line or slash		13-1
⠠⠨	sans serif type indicator		2-4, 7-1
⠠⠨⠰	lowercase sans serif English letter indicator		C-1
⠠⠨⠰⠠	capital sans serif English letter indicator		C-1
⠠⠠	double capitalization indicator		2-1, 5-1
⠠⠠	Hebrew letter indicator		2-1, 6-1, C-1
⠠⠠⠹	opening hypercomplex fraction indicator		2-2, 13-1

Braille	Meaning	Print	Page
⠠⠠⠌	horizontal hypercomplex fraction line		13-2
⠠⠠⠼	closing hypercomplex fraction indicator		2-2, 13-1
⠠⠠⠸⠌	diagonal hypercomplex fraction line or slash		13-2